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Sensitivity Analysis of JLSP Inventory Model with Ordering Cost inclusive of a Freight Cost under Trade Credit in a Two-stage Supply Chain

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Abstract

This study analyzes the distributor's inventory model in a two-stage supply chain consisting of the supplier, the distributor and the end customer. The supplier will allow a credit period before the distributor settles the account with him in order to stimulate the demand for the product he produces. It is also assumed that the distributor pays the shipping cost for the order and hence, the distributor's ordering cost consists of a fixed ordering cost and the shipping cost that depend on the order quantity. The availability of the delay in payments from the supplier enables discount of the distributor's selling price from a wider range of the price option in anticipation of increased customer's demand. As a result, the availability of a credit transaction leads to an increase in inventory levels. On the other hand, in the case of deteriorating products in which the utility of the product perish over time, the deterioration rate with time plays a role in reducing inventory levels. In this regard, we analyze the effect of the length of the credit period and the degree of product deterioration on the distributor's inventory level. For the analysis, we formulate the distributor's annual net profit and analyze the effect of the length of credit period and deterioration rate of the product on inventory policy numerically.

Keywords: *Credit Period, Deterioration , Pricing, Lot-Sizing, Sensitivity Analysis*

1. Introduction

The classical Economic Order Quantity (EOQ) model was analyzed under the basic assumption that the customer pays for the product immediately upon delivery of the ordered product from the supplier. However, the supplier may also allow a delay in payment of product sales for a certain period as a means of price differentiation from competitors. Such credit transactions are expected to have a significant impact on inventory policies, as it is possible to temporarily use inventory investment costs. From this perspective, there are lots of research work under credit transactions. Goyal [1], Chung [2] and Teng et al. [3] analyzed the inventory model assuming that the customer's demand rate was constant under credit transactions. However, as Mehta [4] mentioned in his paper, one of the main reasons suppliers can accept credit from customers is that they can expect to increase demand for products through credit transactions. The supplier is able to make up for the loss of capital incurred during the credit transaction period from the gains from increasing sales volume. Therefore, it can be seen that the duration of credit transactions affects customer demand, and the positive effect of credit transactions on customer demand is limited in analysis by analyzing the inventory model under the assumption that customer demand is constant.

The purpose of this study is to analyze the inventory model on the distributor's point of view in a two-stage supply chain consisting of the supplier, the distributor, and his customer. In the case of such supply chain, the

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delay in payment has the effect of reducing the stock investment cost of the distributor. Since the customer's demand of the product is influenced by selling price of the distributor, the distributor can adjust his selling price in anticipation of the increasing demand of the customer according to the length of the credit period allowed by the supplier. In general, the demand of the customer is influenced by the sales price and therefore, the distributor may choose his selling price from wider range of the price option depending on the length of the credit period expect to increase the customer's demand. Since the distributor's lot size is influenced by his customer's demand, it must be consider the problem determining the distributor's lot size and the selling price simultaneously. We will call this problem as the joint lot size and price (JLSP) problem. In this regard, many studies have been published on the JLSP problem under credit transactions. Chang et al. [5], Dye and Ouyang [6] and Ouyang et al. [7] analyzed the JLSP problem under the assumption that the demand of the customer is a constant price elasticity function of the selling price. Avinadav et al. [8] and Shi et al. [9] analyzed the JLSP problem under the assumption that demand is a linearly decreasing function of sales price.

All the studies mentioned above analyzed the model under the assumption that the life of the product remains constant regardless of the passage of time. This assumption is valid for products whose characteristics do not deterioration over time, but in many products, it can be seen that the characteristics of the product change over time and become unusable. In this case, the inventory held may be depleted not only by customer demand but also by deterioration over time. In the case of deterioration, the deterioration rate over time will be a factor in reducing the inventory level, as a result, reduce the distributor's lot size. From this point of view, Cohen [10] analyzed the JLSP problem for the perishable products that deteriorate at a constant rate. In addition, Tsao and Sheen [11] analyzed the JLSP problem for the perishable products that deteriorate at a constant rate under credit transactions. All of the above studies analyzed the assumption that the order cost of the product was a fixed cost without considering the shipping cost associated with the product order. However, in the case of many practical problems, the order cost consists of a fixed ordering cost and a shipping cost depending on the order quantity. Aucamp [12] and Lee [13] examined a study on the inventory model under the assumption that the order cost consisting of a fixed ordering cost and shipping cost charged by the size of the order quantity. Recently, Shinn [14, 15] analyzed the JLSP problem assuming that the transportation of the product is carried in a certain unit, such as pallets, boxes, containers, trucks, etc., in a situation where the supplier permits a delay in payment. According to the result of Shinn [14], in the case of trade credit, if the payment of the product is delayed for a certain length of time, the distributor will have the effect of reducing the inventory investment cost during the grace period in payment. It can be seen that the savings appear as a result of discounting his selling price to the customer in anticipation of increasing demand from the customer. As a result, it was found that the order quantity of the distributors increased, and the inventory level also increased. The positive effects of such credit transactions on the inventory policy of the distributor were able to achieve the same results in the case of deterioration [15]. However, in the case of deterioration, the amount of deterioration appears in proportion to the level of inventory, so it can be seen that the inventory level of the distributor is reduced compared to the case of no deterioration as stated by Shinn [15]. From the above results, we can know the fact that the trade credit and the deterioration of products have a great influence on the inventory policy of the distributor. Therefore, in this study, we will examine the effects of the length of the credit period and the deterioration rate on the distributor's inventory policy based on the research results of Shinn [14, 15]. In Section 2, we formulate and analyze the appropriate JLSP model. Sensitivity analyses to evaluate the length of credit period and the deterioration rate on the distributor's inventory policy are performed numerically in Section 3, which is followed by conclusions.

2. Mathematical Model and Model Analysis

Based on the JLSP problem under credit transactions considered by Shinn [14, 15], this study aims to analyze the effect of credit transaction period and deterioration rate on the lot size and sales price of the distributor. Therefore, the assumptions and notations used in this paper are the same as those of Shinn [14, 15].

Assumptions:

(1) The customer's demand rate is represented by a constant price elasticity function of the distributor's

selling price.

(2) Shortages are not allowed.

(3) The supplier permits a certain credit period and sales revenue generated during the period is deposited in an interest with rate *I*. At the end of the period, the product price is settled and the buyer begins paying the capital opportunity cost for the products in inventory with rate R ($R \geq I$).

(4) The customer pays the freight cost for the transportation of the quantity purchased.

(5) In the case of decay, decay follows exponential distribution with parameter λ.

Notations:

D	= the customer′s annual demand rate, as a function of the buyer′s selling price, $D = KP^{-e}$.
K	= Scaling factor.
e	= the index of price elasticity.
P	= the buyer′s sales price, $P < P_u$.
C	= purchase cost per unit.
Q	= order quantity.
T	= replenishment cycle time.
tc	= credit period set by the supplier.
H	= inventory holding cost without the capital opportunity cost.
I	= earned interest rate (as a percentage).
R	= capital opportunity cost (as a percentage).
$S(Q)$	= ordering cost for Q, $(j-1)U < Q \leq jU$, $j = 1, 2, \cdots, n$; $A + F_j$.
A	= fixed ordering cost.
F_j	= freight cost for Q, $(j-1)U < Q \leq jU$, $j = 1, 2, \cdots, n$; $S_0 + (j-1)S, S_0 \geq S$.
λ	= a positive number representing the inventory decaying rate.
$q(t)$	= inventory level at time t.

As stated by Shinn [14, 15], the distributor's annual net profit consists of five elements as follows:

$\Pi(P,T)$ = Sales revenue − Purchasing cost − Ordering cost − Inventory holding cost − Capital opportunity cost.

First, we formulate $\Pi(P,T)$ when the deterioration does not occur. According to the results of Shinn [14], when $L_j = jU/D$, depending on the relative size of *tc* and *T*, $\Pi(P,T)$ can be expressed by the following two equations.

(1) Case 1($tc \leq T$)

$$\Pi_1(T,P) = PD - CD - \frac{A+F_j}{T} - \frac{HDT}{2} - \left(\frac{C(R-I)Dtc^2}{2T} + \frac{CRDT}{2} - CRDtc\right),\ L_{j-1} < T \leq L_j,\ j = 1, 2, \cdots, n, \quad (1)$$

(2) Case 2($tc > T$)

$$\Pi_2(T,P) = PD - CD - \frac{A+F_j}{T} - \frac{HDT}{2} - \left(\frac{CIDT}{2} - CIDtc\right),\ L_{j-1} < T \leq L_j,\ j = 1, 2, \cdots, n. \quad (2)$$

For the case of deterioration product, as stated by Shinn [15], $\Pi(P,T)$ also has two different expressions depending on the relative size of *tc* to *T* where $L_j = \frac{1}{\lambda}\ln\left(\frac{\lambda}{D}(jU) + 1\right)$ as follows:

(1) Case 1($tc \leq T$)

$$\Pi_1(T,P) = PD - \frac{CD(e^{\lambda T}-1)}{\lambda T} - \frac{A+F_j}{T} - \frac{HD(e^{\lambda T}-\lambda T-1)}{\lambda^2 T} - \left(\frac{CRD(e^{\lambda(T-tc)}-\lambda(T-tc)-1)}{\lambda^2 T} - \frac{CIDtc^2}{2T}\right)$$
$$,\ L_{j-1} < T \leq L_j,\ j = 1, 2, \cdots, n, \quad (3)$$

(2) Case 2($tc > T$)

$$\Pi_2(T,P) = PD - \frac{CD(e^{\lambda T}-1)}{\lambda T} - \frac{A+F_j}{T} - \frac{HD(e^{\lambda T}-\lambda T-1)}{\lambda^2 T} - \left(\frac{CIDT}{2} - CIDtc\right)$$
$$,\ L_{j-1} < T \leq L_j,\ j = 1, 2, \cdots, n, \quad (4)$$

In this case, although $\Pi(P,T)$ is differentiable, the resulting equation is not easy to deal with mathematically; that is it is impossible to find an optimal solution in explicit form. Thus, it is possible to solve the model approximately through a truncated Taylor series expansion for an exponential term, i.e.,

$$e^{\lambda T} \approx 1 + \lambda T + \frac{1}{2}\lambda^2 T^2 \qquad (5)$$

which is a valid approximation for smaller values of λT. By approximation, $\Pi(P,T)$ can be written as

(1) Case 1($t_c \leq T$)

$$\Pi_1(T,P) = PD - CD - \frac{A+F_j}{T} - \frac{(H+C\lambda)DT}{2} - \left(\frac{C(R-I)Dt_c^{\,2}}{2T} + \frac{CRDT}{2} - CRDt_c\right), \quad L_{j-1} < T \leq L_j,\ j = 1,2,\cdots,n, \qquad (6)$$

(2) Case 2($t_c > T$)

$$\Pi_2(T,P) = PD - CD - \frac{A+F_j}{T} - \frac{(H+C\lambda)DT}{2} - \left(\frac{CIDT}{2} - CIDt_c\right),\ L_{j-1} < T \leq L_j,\ j = 1,2,\cdots,n. \qquad (7)$$

Looking at the structure of the approximate equations (6) and (7), there is no difference except that H in equations (1) and (2), which is the distributor's annual net profit without deterioration, is replaced by $H + C\lambda$. Also, note that when $\lambda = 0$, that is, degeneration does not occur, it can be seen that equations (6) and (7) show the same results as equations (1) and (2) when deterioration does not occur. As a result, both deterioration and non-deterioration cases can be analyzed using equations (6) and (7). Therefore, based on the results of Shinn [15], the distributor's optimal replenishment cycle time T^* and the selling price P^* can be obtained by the following solution algorithm.

$$\max_{T,P} \Pi(P,T) = \left\{ \max_{P\in PRT_{1,j}} \Pi_{1,j}^0(P),\ \max_{P\in PRT_{2,j}} \Pi_{1,j}^0(P),\ \max_{P\in PRL_{1,j}} \Pi_{1,j}(L_j,P),\ \max_{P\in PRL_{2,j}} \Pi_{2,j}(L_j,P) \right\} \qquad (8)$$

Solution Algorithm

Step 1. For each P, $P \in PRT_{1,j}$, $PRT_{1,j} = \{P3_j \leq P < P2_j, P1_j \leq P\}$, compute $\Pi_{1,j}^0(P) = \Pi_{1,j}(T_{1,j}(P),P)$ and find $P_{1,j}$ which maximize $\Pi_{1,j}^0(P)$.

Step 2. For each P, $P \in PRL_{1,j}$, $PRL_{1,j} = \{P6_j \leq P, P < P3_j\}$, compute $\Pi_{1,j}(L_j,P)$ and find $P_{1,j}$ which maximize $\Pi_{1,j}(L_j,P)$.

Step 3. For each P, $P \in PRT_{2,j}$, $PRT_{2,j} = \{P5_j \leq P < P4_j, P < P1_j\}$, compute $\Pi_{2,j}^0(P) = \Pi_{2,j}(T_{2,j}(P),P)$ and find $P_{2,j}$ which maximize $\Pi_{2,j}^0(P)$.

Step 4. For each P, $P \in PRL_{2,j}$, $PRL_{2,j} = \{P < P6_j, P < P5_j\}$, compute $\Pi_{2,j}(L_j,P)$ and find $P_{2,j}$ which maximize $\Pi_{2,j}(L_j,P)$.

Step 5. Select the distributor's optimal replenishment cycle time (T^*) and sales price (P^*) which gives the maximum annual net profit among those obtained in steps 1, 2, 3 and 4.

where

$$P1_j = \left(K(H + C\lambda + CI)t_c^{\,2}/2(A+F_j)\right)^{1/e}, \qquad (9)$$

$$P2_j = \left(\frac{KC(R-I)t_c^{\,2}}{\sqrt{(A+F_j)^2 + C(R-I)H_1 t_c^{\,2}((j-1)U)^2} - (A+F_j)}\right)^{1/e}, \qquad (10)$$

$$P3_j = \left(\frac{KC(R-I)t_c^{\,2}}{\sqrt{(A+F_j)^2 + C(R-I)H_1 t_c^{\,2}(jU)^2} - (A+F_j)}\right)^{1/e}, \qquad (11)$$

$$P4_j = \left(\frac{2K(A+F_j)}{H_2((j-1)U)^2}\right)^{1/e}, \tag{12}$$

$$P5_j = \left(\frac{2K(A+F_j)}{H_2(jU)^2}\right)^{1/e}, \tag{13}$$

$$P6_j = \left(\frac{K(e^{\lambda tc}-1)}{\lambda jU}\right)^{1/e} \quad \text{if } \lambda > 0, \tag{14}$$

$$= \left(\frac{K tc}{jU}\right)^{1/e} \quad \text{if } \lambda = 0. \tag{15}$$

3. Sensitivity Analysis

Now, we analyze how much effect the length of credit period and the deterioration rate have on the distributor's inventory policy considering the ordering cost, which includes shipping costs proportionately depending on the amount of product transportation, under trade credit. Unfortunately, the structure of equations (6) and (7) do not allow sensitivity analysis for tc and λ analytically. Therefore, the same example problems are solved using some different values of tc and λ to answer the above question. The following example problem is applied to analyze the effects of the credit period and the deterioration rate on the buyer's inventory policy. That is, A = 250[$/unit], K = 250,000, e = 2.5, C = 3[$/unit], H = 0.15[$/unit·year], R = 0.15(=15%), I = 0.1(= 10%), U = 500[unit], S_0 = 15[$] and S = 13[$].

First, in order to analyze the effect of the credit period on the distributor's lot size and the selling price, the example problems are solved by fixing the deterioration rate to 0.2 and applying the credit period (years) of five levels of 0.05, 0.1, 0.15, 0.2, and 0.3. The following facts can be confirmed from the results in Table 1.

(1) As the length of the credit period increases, the distributor's selling price decreases while the distributor's annual net profit increases.

(2) The decrease in sales price due to the increase in the credit period is a factor in the increase in demand for his customers, and has been shown to increase the distributor's lot size.

Table 1. Results with various values of *tc* at $\lambda = 0.2$

tc	$\Pi(T^*, P^*)$	D	P^*	T^*	Q^*
0.05	7,334	3,832	5.32	0.36	1,415
0.10	7,414	3,897	5.28	0.35	1,432
0.15	7,492	3,961	5.25	0.35	1,453
0.20	7,567	4,019	5.22	0.35	1,476
0.30	7,708	4,124	5.16	0.35	1,535

From the above results, when a trade credit is allowed from a supplier, the delayed payment of the product during the credit period appears an effect of reducing the stock investment cost of the distributor, and this reduction of the stock investment cost eventually results in the sale price of the distributor. In addition, since the demand of the customer is directly influenced by the selling price of the distributor, it can also be confirmed that the selling price can be adjusted while expecting an increase in the demand of his customer according to the length of the credit period.

Next, for analysis of the effect of the product's deterioration rate on the distributor's inventory policy, the credit transaction period is fixed to 0.2 while applying the deterioration rate of five levels of 0, 0.05, 0.1, 0.2 and 0.3 are applied. The following facts can be confirmed from the results in Table 2.

(1) When the deterioration rate (λ) increases, the distributor's selling price increases, and as a result, his

customer's demand rate decreases.

(2) Also, due to the decrease in the customer's demand, the distributor's lot size tended to decrease gradually.

Table 2. Results with various values of λ at $tc = 0.2$

λ	$\Pi(T^*, P^*)$	D	P^*	T^*	Q^*
0.00	8,047	4,207	5.12	0.48	2,000
0.05	7,900	4,050	5.20	0.37	1,500
0.10	7,787	4,045	5.20	0.36	1,500
0.20	7,567	4,019	5.22	0.35	1,476
0.30	7,367	3,929	5.27	0.32	1,322

In the case of deterioration, the amount of deterioration with time appears in proportion to the inventory level at that time, so it can be seen that even in the case of credit transactions, the distributor's lot size gradually decreases as the deterioration rate increases. In particular, it can be seen that it shows the same result as the JLSP problem under credit transactions without deterioration.

Finally, in order to examine the effect of the credit period and the deterioration rate at the same time, five levels of *tc* are adopted, *tc* = 0.05, 0.1, 0.15, 0.2 and 0.3. For each level of *tc*, five levels of λ, λ= 0, 0.05, 0.1, 0.2 and 0.3 are solved. The results are shown in Table 3, and it can be seen that the results showed the same tendencies as in Tables 1 and 2.

Table 3. Results with various values of *tc* and λ

		tc														
		0.05			0.10			0.15			0.20			0.30		
		$\Pi(T^*, P^*)$	P^*	Q^*	$\Pi(T^*, P^*)$	P^*	Q^*	$\Pi(T^*, P^*)$	P^*	Q^*	$\Pi(T^*, P^*)$	P^*	Q^*	$\Pi(T^*, P^*)$	P^*	Q^*
λ	0.00	7793	5.22	2000	7880	5.18	2000	7965	5.15	2000	8047	5.12	2000	8205	5.08	2000
	0.05	7662	5.29	1500	7745	5.26	1500	7824	5.23	1500	7900	5.20	1500	8055	5.08	1974
	0.10	7550	5.29	1500	7632	5.26	1500	7711	5.23	1500	7787	5.20	1500	7929	5.16	1500
	0.20	7334	5.32	1415	7414	5.28	1432	7492	5.25	1453	7567	5.22	1476	7708	5.16	1500
	0.30	7148	5.44	1000	7222	5.41	1000	7295	5.30	1301	7367	5.27	1322	7499	5.22	1372

4. Conclusion

In this paper, we analyzed the effects of the credit period and the deterioration rate on the distributor's lot size and selling price. For the analysis, it was assumed that the distributor's order cost consists of the fixed ordering cost and the shipping cost charged by the transport unit. In order to evaluate the sensitivity analysis for the length of the credit period and the deterioration rate to the inventory decision of the distributor, we formulated the distributor's annual net profit function. Unfortunately, the structure of mathematical expressions do not allow sensitivity analysis for the credit period and deterioration rate analytically and therefore, we solve the same example problems using some different values of *tc* and λ. According to the results of the analysis, when credit transactions are permitted from suppliers, the delayed payment of product during the credit period has been shown to have the effect of reducing the stock investment cost of the distributor. It was found that it affects the distributor's selling price. In addition, since the demand rate of the customer is directly influenced by the selling price of the distributor, it can also be confirmed that the selling price can be adjusted while expecting an increase in his customer's demand rate according to the length of the credit period. In addition, in the case of deterioration, the amount of deterioration over time is proportional to the inventory level at that time. And therefore, it can be seen that if the deterioration rate increases, the lot size of the distributors tends to decrease gradually.

References

[1] S. K. Goyal, "Economic order quantity under conditions of permissible delay in payments," *Journal of Operational Research Society*, Vol.36, No.4, pp.335-338, 1985.

[2] K. J. Chung, "A theorem on the determination of economic order quantity under conditions of permissible delay in payments," *Computers & Operations Research*. Vol.25, No.1, pp.49-52, 1998.

[3] J. T. Teng, C. T. Chang, M. S. Chern and Y. L. Chan, “Retailer’s optimal ordering policies with trade credit financing,” *International Journal of Systems Science*, Vol.38, No.3, pp.269-278, 2007.
[4] D. Mehta, “The formulation of credit policy models,” *Management Science*, Vol.15, pp.B30-B50, 1968.
[5] H. C. Chang, C. H. Ho, L. Y. Ouyang and C. H. Su, “The optimal pricing and ordering policy for an integrated inventory model when trade credit linked to order quantity,” Applied Mathematical Modeling, Vol. 33, pp.2978-2991, 2009.
[6] C. Y. Dye and L. Y. Ouyang, “A particle swarm optimization for solving joint pricing and lot-sizing problem with fluctuating demand and trade credit financing,” *Computers & Industrial Engineering*, Vol.60, pp.127-137, 2011.
[7] L. Y. Ouyang, C. H. Ho, and C. H. Su, “An optimization approach for joint pricing and ordering problem in an integrated inventory system with order-size dependent trade credit,” *Computers & Industrial Engineering*, Vol.57, pp.920-930, 2009.
[8] T. Avinadav, A. Herbon and U. Spiegel, “Optimal ordering and pricing policy for demand functions that are separable into price and inventory age,” *International Journal of Production Economics*, Vol.155, pp.406-417, 2014.
[9] J. Shi, R. Y. K. Fung and J. Guo, “Optimal ordering and pricing policies for seasonal products: Impacts of demand uncertainty and capital constraint,” Discrete Dynamics in Nature and Society, Vol. 2016, Article ID 1801658, pp. 1-13, 2016.
[10] M. A. Cohen, “Joint pricing and ordering policy for exponentially decaying inventory with known demand,” Naval Research Logistics Quarterly, Vol. 24, pp.257-268, 1977.
[11] Y. C. Tsao and G. J. Sheen, “Joint pricing and replenishment decisions for deteriorating items with lot-size and time-dependent purchasing cost under credit period,” International Journal of Systems Science, Vol. 38, No.7, pp.549 -561, 2007.
[12] D. C. Aucamp, “Nonlinear freight costs in the EOQ problem,” European Journal of Operational Research. Vol. 9, pp. 61-63, 1982.
[13] C. Y. Lee, “The economic order quantity for freight discount costs,” IIE Transactions, Vol. 18, pp. 318-320, 1986.
[14] S. W. Shinn “Distributor's Lot-sizing and Pricing Policy with Ordering Cost inclusive of a Freight Cost under Trade Credit in a Two-stage Supply Chain,” International Journal of Advanced Culture Technology, Vol. 8 No. 1, pp. 62 -70, 2020.
https://doi.org/10.17703//IJACT2020.8.1.62.
[15] S. W. Shinn “Joint Price and Lot-size Determination for Decaying items with Ordering Cost inclusive of a Freight Cost under Trade Credit in a Two-stage Supply Chain,” The Journal of the Convergence on Culture Technology, Vol. 6 No. 2, pp.151-162, 2020.
https://dx.doi.org/10.17703//JCCT2019.5.1.