J. Korean Soc. Math. Educ., Ser. D, Res. Math. Educ. http://doi.org/10.7468/jksmed.2020.23.3.149
Vol. 23, No. 3, 149-164, September, 2020

Instructional Alignment Observation Protocol (IAOP) for Implementing the CCSSM: Focus on the Practice Standard, "Model with Mathematics"

Jihyun Hwang (Professor)
Kangwon National University, jihyun-hwang@kangwon.ac.kr

(Received September 7, 2020; Revised September 23, 2020; Accepted September 28, 2020)

This study aimed to establish an observation protocol for mathematical modeling as an alternative way to examine instructional alignment to the Common Core State Standards for Mathematics. The instructional alignment observation protocol (IAOP) for mathematical modeling was established through careful reviews on the fidelity of implementation (FOI) framework and prior studies on mathematical modeling. I shared the initial version of the IAOP including 15 items across the structural and instructional critical components as the FOI framework suggested. Thus, the IAOP covers what teachers should do and know for practices of mathematical modeling in classrooms and what teachers and students are expected to do. Based on the findings in this study, validity and reliability of the IAOP should be evaluated in follow-up studies.

Keywords: mathematical modeling, observation protocol, instructional alignment. CCSSM.
MESC Classification: D50
MSC2010 Classification: 97D50

I. INTRODUCTION

The U.S. mathematics education have experienced significant changes triggered by the development of the Common Core State Standards for Mathematics (CCSSM; National Governors Association Center for Best Practices [NGAC] & Council of Chief State School Officers [CCSSO], 2010). Some sensitive aspects in the U.S. education system like assessment and textbook development have immediately responded to the establishment of the CCSSM. On the other hand, mathematics teachers might have gradually change their instructions while full implementation of the CCSSM into instructions were expected to be accomplished until the 2014-2015 academic year in forty-three states adopting the CCSSM at present (Common Core State Standards

Initiative, 2015).

It is crucial to examine alignment between instructions and the CCSSM not only for students' better opportunities to learn mathematics with the CCSSM but also for evaluating the effects of the CCSSM on students' mathematics achievement in a proper manner. The CCSSM have been established to ensure students' college readiness as well as their opportunities to learn with high-quality mathematics curriculum (Schmidt & Houang, 2012). However, high-quality standards do not necessarily lead to high mathematics achievement (Roach, Niebling, & Kurz, 2008). In the middle of the CCSSM and students' learning mathematics, mathematics teachers are required to implement the CCSSM into their classroom successfully so that students can have opportunity to satisfy the CCSSM. Success or failure of the CCSSM to improve students' learning mathematics should not be concluded grounded on only student performance in the CCSSM-aligned assessments like the Smarter Balanced Assessment. Without instructions aligned to the CCSSM, it is impossible to help failing to reasonably argue connections between students' assessed learning and development of the CCSSM. In other words, students' performance in the assessments would not be mainly attributed to establishment of the CCSSM if instructions were not aligned to the CCSSM.

Unfortunately, the definition of standards-based mathematics instructions is uncertain (Schoenfeld & Kilpatrick, 2013). Polikoff (in press) argued that educators and teachers have experienced numerous challenges to implement the CCSSM into their instructions. He argued that the primary issue in these challenges is alignment of educational components including assessments materials, instructions, and professional development. Some studies (e.g., Polikoff, in press; Porter, McMaken, Hwang, & Yang, 2011) recently investigated alignment of assessments and textbooks to the CCSSM separately from what textbook publishers and assessment developers argued about alignment. However, only few studies have attempted to examine alignment between instructions and the CCSSM using a specific quantitative method, the Survey of Enacted Curriculum (SEC; Blank, Porter, & Smithson, 2001).

Martone and Sireci (2009) argued that the SEC approach is the unique way to investigate alignment of each of assessment, textbooks, and instructions to the CCSSM providing reliable and comprehensive data. However, Cobb and Jackson (2011) and Beach (2011) criticized reliability and validity of the SEC approach as a method to examine instructional alignment (Polikoff & Porter, in press) to the CCSSM because this approach relies on teachers' self-reporting. Researchers agreed with necessity of alternative approaches to conceptualizing and measuring alignment of instructions (Porter et al., 2011), but no further research has been found.

The purpose of this research is to establish an alternative way to examine for alignment between instructions and the CCSSM grounded on the framework for the

Fidelity of Implementation (FOI framework; Century, Rudnick, & Freeman, 2010). Particularly, the Instructional Alignment Observation Protocol (IAOP) is suggested as the alternative method for instructional alignment. The practice standard in the CCSSM, "modeling with mathematics" (p. 7) is the center standard of IAOP developed in this research.

II. THEORETICAL FRAMEWORK

1. RELATIONSHIP BETWEEN INSTRUCTIONS AND THE CCSSM

In order to scrutinize alignment of mathematics instructions to the CCSSM, it is necessary to consider comprehensively what mathematics instructions and the CCSSM are as well as what alignment between them means. When I review the definition of alignment first, some researchers have attempted to define alignment of other educational components to standards. Webb (1997) defined alignment between standards and assessments as "the degree to which expectations [e.g., the CCSSM] and assessment are in agreement and serve in conjunction with one another to guide the system toward students learning what they are expected to *know and do* [italic added]" (p. 3). To examine alignment between textbooks and the CCSSM, Polikoff (in press) referred alignment as "agreement on both topic and cognitive demand" (p. 2). These definitions indicate that alignment of textbooks or assessments to the CCSSM could be analyzed with the two dimensions; contents and cognitive demands (what student should know and what they should do).

I might expand Webb's (1997) definition to alignment of instructions. However, when I consider the nature of relationships between the CCSSM and instructions, I cannot expand Webb's (1997) to instructional alignment straightforwardly. Cobb, Stephan, McClain, and Gravemeijer (2011) suggested the interpretative framework to analysis individual and collective mathematical learning with a combination of social and psychological perspectives. Instructions involve in cultural factors, classroom and school environment (social perspective) as well as students' individual characteristics (psychological perspective). The CCSSM are interpreted and implemented into the classroom micro-culture (Cobb et al., 2011) while students learn mathematics by participating in a classroom community (Barab & Duffy, 2000). Therefore, instructions differ from large-scale assessments, textbooks and the CCSSM in terms of whether it is situated (contextualized) or not.

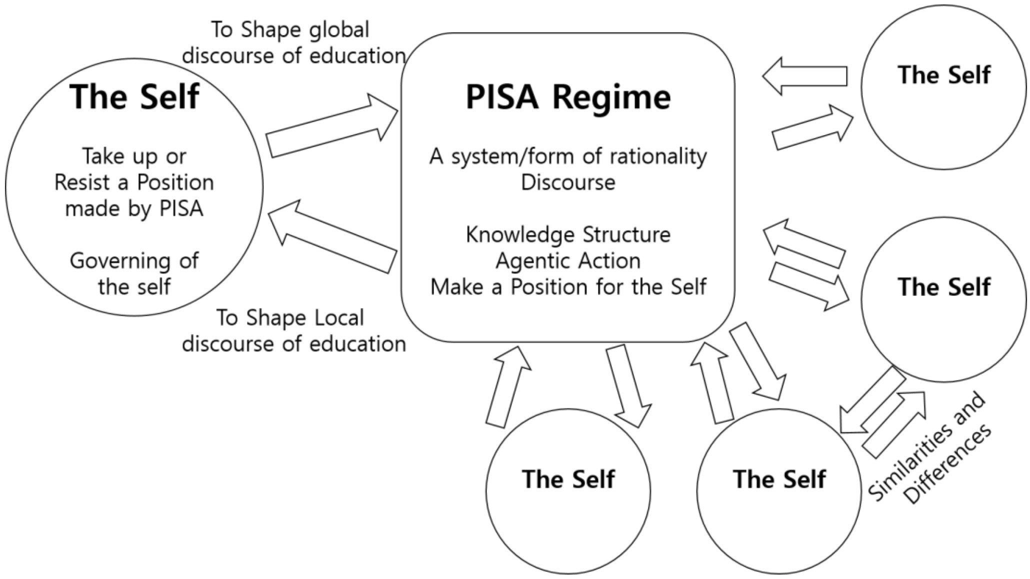

Figure 1. The Relation between PISA and Selves

To articulate relationships between instructions and the CCSSM, *subjectivation* in Foucault's method (Kanes, Morgan, & Tsatsaroni, 2014) could be employed. His method was suggested to understand "how it enables recruited by or enabled to stand aside from knowledge effects" (Kanes et al., p. 161). Foucault addressed two processes: objectivation and subjectivation. The first is a process to see how knowledge is formed by the formal rules related with conversational practices, and the second focused on the deliberate reinsertion of subjects. Subjects are capable to reflect and shape themselves based on the potential to resist and challenge power relations caused by knowledge acquisition. Furthermore, Kanes et al. (2014) discussed Foucault's method as a theoretical lens to analyze bidirectional practices of "the self" and the PISA mathematics regime.

Based on the argument of Kanes et al. (2014), our concept of the relationship between the selves and the PISA regime is seen in Figure 1. PISA assessments contribute to shape global discourses of education from practices of the selves while each self either takes up or resists a position made by PISA assessments. To be specific, when PISA suggested the term mathematical literacy, the self can interpret and adopt this concept based on its educational goals, systems and social values. However, how teachers respond to the CCSSM resemble to how the self (teacher) reinsert the PISA mathematics regime (the CCSSM) to its own educational system (a classroom community). The self in Foucault' method differs from teachers because the self can take up or resist a position made by PISA. However, teachers as the self could contextualize the CCSSM into their teaching in a similar way of the self who adopts PISA’s position.

Standards-based instructions are shaped under teachers’ perspectives on the CCSSM and their classroom cultures so that students have opportunities to learn what they are expected to know and do. Even school environments (e.g., diversity and curriculum; White, 2003) and special student populations (e.g., students with disabilities; Saunders, Bethune, Spooner, & Browder, 2013) could be considered when teachers implement the CCSSM into instructions. How teachers subjectivate the well-documented standards into their instructions should be noted and fundamental of the alternative methods for instructional alignment to the CCSSM.

Teachers’ subjectivation of the CCSSM necessitates the alternative way in addition to the SEC approach. Because the SEC method does not include contextual factors explicitly in its alignment index, comparing content coverage and cognitive demands in the SEC approach allow us to examine alignment between two decontextualized components. However, for instructional alignment, this method could not provide significant information about whether or not instructions actually provide students opportunities to learn aligned to the CCSSM in their classroom communities.

The alternative approach for instructional alignment should be flexible based on which set of standards teachers select for lessons that alignment evaluators observe. Certainly, it is impossible to cover all standards in a couple of lessons. Based on school curriculum and teachers’ lesson plans, several content and practice standards are emphasized in a lesson (Barab & Duffy, 2000). Instructional alignment to the CCSSM is essentially investigated with a set of several practice and content standards while the entire the CCSSM can be involved in alignment of assessment or textbooks. Evaluators might want to generalize multiple observations to acquire overall instructional alignment to the CCSSM. However, this generalized instructional alignment still differs from alignment of assessments because overall instructional alignments are interpreted with common classroom contexts through observation. For example, if teachers teach gifted children during evaluated lessons, students’ average achievement could be accounted for in interpreting overall instructional alignment. In addition, teachers teach the same mathematics to two different classes, teachers might higher alignment in the later class than the former class.

Instructional alignment to the CCSSM is defined in this research as following; the degree to which instructions facilitates students to learn what they are expected to know and do in the CCSSM and students actually do that during instructions. With this definition, how can a person know if a mathematics instruction is aligned to the CCSSM? This is the main question to develop an observation protocol for instructional alignment.

2. FRAMEWORK FOR THE FIDELITY OF IMPLEMENTATION

I adopt the framework for the fidelity of Implementation (FOI framework; Century et al., 2010) to construct an observation protocol. According to Century and the colleagues, FOI of a program is defined as "[t]he extent to which an enacted program is consistent with the intended program model" (p. 202). This definition of FOI indicates that the FOI framework could be appropriate to measure alignment between teachers' enacted curriculum and the intended curriculum of the CCSSM.

Table 1. Original descriptions of critical components in the FOI framework (Century et al., 2010, p. 205)

Category of Critical Components		Description
Structural Critical Component	Procedural Critical Component	The program developers' understanding what the user (teacher) should do (e.g., the basic steps of the procedure)
	Educative Critical Component	The program developers' understanding on what the user (teacher) need to know (required knowledge to enact the intervention as intended
Instructional Critical Component	Pedagogical Component	Actions, behavior, and interactions that the user (teacher) is expected to engage in when enacting the intervention, including the user's interactions with the participants/recipients (students)
	Student Engagement Component	Actions, behavior, and interactions the recipients (students) are expected to engage in when participating in the enactment of the intervention

The FOI framework includes two main components to evaluate FOI of a certain program originally; the structural critical components and the instructional critical components. Table 1 shows the original descriptions of each component in the FOI framework of a certain program. The structural critical component represent what teachers should do and know for aligned instructions to the CCSSM while the instructional critical component denotes actions, behavior, and interactions that teachers and students are expected to in instructions aligned to the CCSSM.

As seen in Table 1, the instructional component represent what teachers and students are expected to *do,* which closely related to the practice standards in the CCSSM. Thus, within the FOI framework, the practice standards at which teachers aim in instructions is the center of our alternative method. Particularly, "model with mathematics" is selected in this research. Teachers might have lack of understanding on modeling compared to other practice standards while teachers cannot distinguish modeling from traditional problem solving (Zawojewski, 2013). These could be why I select "modeling with mathematics" to establish the alternative method for alignment of instructions.

III. RESEARCH METHODLOGY

1. ALIGNEMENT STUDY

Before development of the CCSSM, several studies have investigated alignment between instructions and standards other than the CCSSM. Jacobs, Hiebert, Givvin, and Hollingsworth (2006) scrutinized how teachers implement the principles and standards suggested by the National Council of Teachers of Mathematics (NCTM, 2000). Jacobs and the colleagues found that teachers' actual instructions were not well-aligned with the NCTM standards although 80% of teachers reported their familiarity with the ideas of the NCTM standards. Thus, this research suggested us to examine instructional alignment to the CCSSM in two folds; understanding of the CCSSM and actual teaching practices.

There have been attempts to create instruments such as surveys or observation protocols to evaluate instructional alignment. Ross, Mcdougall, Hogaboam-Gray, and LeSage (2003) developed a 20-item survey based on nine dimensions of standards-based teaching. This research checked reliability and predictive validity of the survey. That survey could be a costly instrument to assess standards-based teaching. However, that article did not clearly mentioned what standards-based teaching is, which results in confusion about what the survey examines.

Interestingly, the California Department of Education (2015, April 21) provides the implementation survey with 25 items. That survey have six sections; general implementation, professional learning, instructional materials, assessment, communications and outreach, and implementation comments. As a part of the the CCSSM implementation plan for California, the survey is accessible online, but with regards of research, more discussions are necessary to ensure reliability and validity of that survey.

Sawada et al. (2002) established the Reformed Teaching Observation Protocol (RTOP) to measuring three aspects of reformed teaching; (a) standards based, (b) inquiry oriented, and (c) student centered. Sawada et al. showed RTOP was reliable and effective. Its development procedure and detailed items could be beneficial to establish observation protocol for instructional alignment to the CCSSM.

In addition to development of instruments, Polikoff (2013) attempted to reveal significant predictors for instructional alignment to mathematics, language, or science standards. The SEC approach assessed teachers' instructional alignment and the alignment index was used as a dependent variable in a linear regression analysis. A variety of teachers' and classrooms' characteristics were considered as predictors of instructional alignment. As a result of the regression analysis, years of teaching experiences and the number of content course teachers had took were the important predictors. However, the predictors could explain only 7% of variances in the instructional alignment index. Thus, complex aspects of instructions might be overlooking in this quantitative data analysis.

2. MATHEMATICAL MODELING

Mathematical modeling is defined as a cognitive skill to generate a system presenting complex systems using mathematical symbols and concepts (English, 2006). According to the CCSSM, modeling is referred as "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions" in the CCSSM (NGAC & CCSSO, 2010, p. 72). Lesh and Fennewald (2013) argued that students can experience mathematical modeling through multiple cycles of expressing → testing → revising ways of thinking. Because modeling empathizes students' thinking process, this repeated construction of generalized and reusable systems to solving complex situations necessitate reasoning and making senses together (Doerr & English, 2003).

Modeling is essential for students to transfer their learning in classrooms to everyday life and their future professions (Lesh & Fennewald, 2013; NGAC & CCSSO, 2010) because interactions between students' thinking and real events are fundamental in modeling process (Hestenes, 2010). If teachers employ complex real situations that students possibly experience in the future, modeling could promote students' transfer of learning.

It should be noted that students' modeling strongly depend on modeling tasks and complex situations that students face. The important role of tasks allows us to distinguish modeling from problem solving. According to Schoenfeld and Kilpatrick (2013), problem solving means "engaging in a task whose solution method is not known in advance" in school mathematics (p.905). Because of unawareness of a solution method, what problem solvers know is significant in problem solving. However, situations for modeling could be challenging to even experts. For example, precise prediction of tomorrow weather is based on peoples' models on weather changes. Normal people could predict rain by observing movement of clouds while experts could use complex numerical weather models with a number of variables. If their predictions were inaccurate, they would consider revising their models. This example shows that modeling is not the matter of awareness of solution methods, rather what how people create and change their models. Furthermore, the example indicate that one modeling situation can be flexible in changing the complexity by how many factors are included in this prediction modeling tasks.

IV. ESTABLISHMENT OF THE FOI FRAMEWORK FOR MODELING

Establishment of IAOP begins with careful review on the FOI framework to modify for instructional alignment. I constantly compare analysis of the modeling standards in the CCSSM and literature review to the original FOI framework (Century et al., 2010). The FOI framework for modeling leads to create observation protocols. After several raters evaluate several lessons focusing on mathematics modeling, I will analyze reliability and validity of this protocol in following studies. The Principal Factor Analysis (PFA) is applied to check score validity and Cohen's kappa and Cronbach's alpha will be calculated for inter-rater reliability and internal consistency. Particularly, IAOP mainly relies on the expert validity but it would be better to have other types of validity. Thus, I will attempt to reveal correlation between scores from IAOP and the SEC alignment index for convergence validity because the SEC approach is the only existing method to measure instructional alignment. Checking reliability and validity leads to revise the FOI framework for modeling and specifics in the observation protocol if necessary. The final version of IAOP results from the circular process of reviewing the FOI framework, establishing details in the observation protocol, and checking reliability and validity. However, in this study, I will report only the initial version before the PFA.

This study begins with revision of the FOI framework in terms of mathematical modeling in the CCSSM. Although the FOI framework provides details about each component, I still clarify which should be included in each component in the FOI framework to evaluate instructional alignment.

First, the CCSSM mainly indicate what students are expected to know and do, I primarily focus on the student engagement component. Thus, I identify what students are expected to engage in the description of "model with mathematics" (NGAC & CCSSO, 2010, p. 7) as following;

- Solve problems arising in everyday life, society, and the workplace
- Making assumptions and approximations to simplify a complicated situation

- Analyze relationships between quantities mathematically to draw conclusion
- Interpret mathematical results in the context of the situation
- Reflect on whether the results make sense
- Revise and improve the model if necessary

An important issue for the other components is that the CCSSM do not obviously mention what teachers are expected to know for and engage in for each standard. Thus, I review previous research on mathematical modeling to reveal pedagogical components for the modeling standard. The pedagogical component certainly corresponds to the student engagement component because teachers are expected to help students do elements in the student engagement component. Thus, the preliminary elements in the pedagogical component is following;

- Facilitation of making sense of mathematics (Speiser & Chuck, 2013)
- Facilitation of creating and critiquing conjectures (Lesh & Fennewald, 2013)
- Teachers' questions triggered divergent modes of thinking
- Using appropriate materials (modeling tasks) and tools (Zawojewski, 2013)

Compared to the instructional critical component, the structural critical component is relevant to teachers' perceptions, knowledge, use of textbooks, and curriculum. To be specific, the educative critical component is relevant to teachers' knowledge about content standards that teachers target. This component could be measured using assessments for the Mathematical Knowledge for Teaching (MKT; Ball, Thames, & Phelps, 2008) even though these assessments cannot provide any information about teachers' MKT in terms of the CCSSM. Furthermore, teachers' understanding on mathematical modeling is critical to instructional alignment because teachers' learning goal is the core of instruction cycle suggested by (Simon, 1995). Thus, the educative critical component for the modeling standard includes understanding of what the target content standards as well as what the modeling standard indicate (e.g., what is students' goal performance after the lesson).

Lastly, the procedural critical component includes what teachers should do to teach mathematical modeling in instructions aligned with the CCSSM. In terms of mathematical modeling, tasks have crucial roles in students' learning experience of modeling (Zawojewski, 2013). Tasks implemented in classrooms should allow students to make conjectures and revise their conjectures. If tasks focus on finding only correct answers or students already know how to solve questions, students cannot have opportunities of modeling. Thus, modeling task design and preparation are essential in the procedural critical component. Moreover, teachers are required to offer enough time to explore mathematics tasks to students. Teachers need to prepare assessment and criteria to determine whether students meet the goals provided by the CCSSM.

Table 2 shows the framework to evaluate instructional alignment to the CCSSM, particularly the modeling practice standard. Certainly, the FOI framework for modeling should be revised continuously because it is fundamental and significant to establish observation protocol.

Table 2. Details of each component in the FOI framework of modeling

Structural Critical Component		Instructional Critical Component	
Procedure	Educative	Pedagogical	Student Engagement
Order -Modeling tasks is presented in appropriate time in the lesson (lack of time or too much time can be assigned) **Pre-lesson** -Modeling tasks development	Perception/understanding of modeling	**Facilitating student engagement with content** -Facilitation of making sense of mathematics -Using appropriate materials and tools for modeling tasks **Facilitating student role as learner** -Teacher facilitation of students' creating and critiquing conjectures -The teacher's questions triggered divergent modes of thinking **Pedagogical strategies** -Teachers use of modeling tasks or materials in clear ways -Teacher use of proper assessments to inform instruction	**Students engage with others** -Solve problems arising in everyday life, society, and the workplace -Making assumptions and approximations to simplify a complicated situation -Analyze relationships between quantities mathematically to draw conclusion -Interpret mathematical results in the context of the situation -Reflect on whether the results make sense -Revise and improve the model if necessary **Students use the materials** -Students do/complete essential activities

V. ESTABLISHMENT OF THE OBSERVATION PROTOCOL FOR MODELING

The next step is to construct an instrument, which evaluates details of each components in the FOI framework. The structural critical component is necessarily evaluated before instructions using rubrics, survey, and the MKT assessment while the instructional critical component is assessed during classroom observations. The focus of this research is to establish observation protocol for the instructional critical component. This research is a part of establishing the full instrument to measure instructional alignment. I need more research to create a survey to reveal teachers' understanding of the modeling standards as well as rubrics to examine the procedural critical component.

The initial IAOP contains 15 items with four-point Likert scales. I report the IAOP in Figure 2. I highlight that the IAOP in Figure 2 is the initial version. I require following revisions of IAOP until it is possible to ensure validity and reliability of IAOP. The FOI framework for modeling might fail to capture implementation of mathematical modeling comprehensively. The revisions might need changes in descriptions of the IAOP questions to increase reliability. Furthermore, it is necessary to find some instructions for mathematical modeling. Finally, I will apply statistical methods to analyze reliability and validity after evaluating modeling lessons. This step is the most important for providing systematic evidence of why and how IAOP works to measure instructional alignment to the modeling standard.

Instructional Alignment Observation Protocol (IAOP)

Observer:	________________________		**School:**	_______________	**Video:**	**Y**	**N**
Teacher Code:	________________________		**Observation Date:**	___________	**Length of Observation:**	_________	
Grade: ____	**Video Semester:**	**FA** **SP**	**Signature:**	_________________________________			

Direction 1 (Scoring). Each of the items is to be rated on a scale ranging from 0 to 3. Choose "0" if in your judgment, the characteristic never occurred in the lesson, not even once. If it did occur, even if only once, "1" or higher should be chosen. Choose "3" only if the item was very descriptive of the lesson you observed. Intermediate ratings do not reflect the number of times an item occurred, but rather the degree to which that item was characteristic of the lesson observed. Scoring is determined by observations using the above characteristics of effective classroom teaching in a general, regardless of method or approach. Ratings should reflect the degree in which these elements are present in the observed classroom and not a specific number of times. The following are the scoring elements related to this scale.
0 – Not Present; 1 – Low Presence; 2 – Medium Presence; 3 – Highly Present

Direction 2 (Component). The Pedagogical Component represents actions, behavior, and interactions that the teacher is expected to engage in instructions, including the user's interactions with students. The Student Engagement Component represents actions, behavior, and interactions that students are expected to engage in when participating in the lessons.

	Target Standards
Content Standards	(Write standards in detail)
Practice Standards	4. Model with Mathematics

Code	Component	Observed Criteria	Score (0-3)
P1	**Pedagogical Component**	The teacher clearly set up a modeling situation that allows students to make sense of the problem	
P2		The teacher use appropriate media and tools to present a modeling task	
P3		The teacher's questions triggered divergent modes of thinking for mathematical modeling (Did the teacher ask open-ended questions that promote reasoning and critiquing to the whole class and groups of students?)	
P4		The lesson (task) encouraged students to seek and value making conjecture, critiquing conjectures. (Was this valued within groups? Was there any whole class discussion?)	
P5		The teacher provided students opportunity to make sense of mathematics in the modeling process	
P6		The teacher use proper assessments of students' modeling to inform instructions	
P7		The teacher use proper assessments of students' understanding of concepts to inform instructions	
S1	**Student Engagement Component**	Students were actively engaged in thought-provoking problems arising in everyday life, society, and the workplace	
S2		Students made assumptions and approximations to simplify a complicated situation	
S3		Students analyzed relationships between quantities mathematically to draw conclusion	
S4		Students interpreted mathematical results in the context of the situation	
S5		Students were reflective about whether the results make sense	
S6		Students revised and improved the model if necessary	
S7		Students actually completed essential activities	
S8		Students showed evidence for what the target content standards indicate No evidence, 1 – less than 30% of students, 2 – −0) between 30% and 70%, 3 – Greater than 70%)	
		Total in Pedagogical Component	/24
		Total in Student Engagement Component	/24
		Total Score	/45
		Average Score	

Figure 2. Instructional Alignment Observation Protocol (IAOP)

REFERENCES

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389–407. doi:10.1177/0022487108324554

Barab, S. A., & Duffy, T. M. (2000). From practice field to communities of practice. In D. Jonassen & S. Land (Eds.), *Theoretical foundations of learning environments* (pp. 25–56). Mahwah, NJ: Lawrence Erlbaum Associates

Beach, R. W. (2011). Issues in analyzing alignment of Langague Arts Common Core Standards with state standards. *Educational Researcher, 40*(4), 179–182. doi:10.3102/0013189X11410055

Blank, R. K., Porter, A. C., & Smithson, J. (2001). *New tools for analyzing teaching, curriculum and standards in mathematics & science*. (National Science Foundation REC98-03080). Washington, DC: Council of Chief State School Officers.

California Department of Education. (2015, April 21). Common core state standards implementation survey. Retrieved from http://www.cde.ca.gov/re/cc/survey.asp

Century, J., Rudnick, M., & Freeman, C. (2010). A framework for measuring fidelity of implementation: A foundation for shared language and accumulation of knowledge. *American Journal of Evaluation, 31*(2), 199–218.

Cobb, P., & Jackson, K. (2011). Assessing the quality of the common core state standards for mathematics. *Educational Researcher, 40*(4), 183–185. doi:10.3102/0013189X11409928

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2011). Participating in classroom mathematics practices. In E. Yackel, K. Gravemeijer, & A. Sfard (Eds.), *A journey in mathematics education research: Insights from the work of Paul Cobb* (pp. 117–163). New York, NY: Srpinger.

Common Core State Standards Initiative. (2015). *Standards in your state*. Retrieved 2015, October 12, from http://www.corestandards.org/standards-in-your-state/

Doerr, H. M., & English, L. D. (2003). A modeling perspective on students' mathematical reasoning about data. *Journal for Research in Mathematics Education, 34*(2), 110-136. doi:10.2307/30034902

English, L. D. (2006). Mathematical modeling in the primary school: Children's construction of a consumer guide. *Educational Studies in Mathematics, 63*, 303–323. doi:10.1007/s10649-005-9013-1

Hestenes, D. (2010). Modeling theory for math and science education. In R. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies* (pp. 13–41). New York, NY: Springer.

Jacobs, J. K., Hiebert, J., Givvin, K. B., & Hollingsworth, H. (2006). Does eighth-grade mathematics teaching in the United States align with the NCTM standards? Results from the TIMSS 1995 and 1999 Video Studies. *Journal for Research in Mathematics Education, 37*(1), 5–32.

Kanes, C., Morgan, C., & Tsatsaroni, A. (2014). The PISA mathematics regime: knowledge structures and practice of the self. *Educational Studies in Mathematics, 87*, 145–165. doi:10.1007/s10649-014-9542-6

Lesh, R., & Fennewald, T. (2013). Intruction to Part I modeling: What it is? Why do it? In R. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies: ICTMA 13* (pp. 5–10). New York, NY: Springer.

Martone, A., & Sireci, S. G. (2009). Evaluating alignment between curriculum, assessment, and instruction. *Review of Educational Research, 79*(4), 1332–1361. doi:10.3102/0034654309341375

National Council of Teachers of Mathematics [NCTM]. (2000). *Principals and standards for mathematics*. Reston, VA: NCTM.

National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). *Common core state standards for mathematics.* Retrieved from http://www.corestandards.org/the-standards/mathematics

Polikoff, M. S. (2013). Teacher education, experience, and the practice of aligned instruction. *Journal of Teacher Education, 64*(3), 212–225. doi:10.1177/0022487112472908

Polikoff, M. S. (in press). How well aligned are textbooks to the Common Core Standards in mathematics? *American Educational Research Journal*. doi:10.3102/0002831215584435

Polikoff, M. S., & Porter, A. C. (in press). Instructional alignment as a measure of teaching quality. *Educational Evaluation and Policy Analysis*. doi:10.3102/0162373714531851

Porter, A. C., McMaken, J., Hwang, J., & Yang, R. (2011). Assessing the common core standards: Opportunities for improving measures of instruction. *Educational Researcher, 40*(4), 186–188. doi:10.3102/0013189X11410232

Porter, A. C., McMaken, J., Hwang, J., & Yang, R. (2011). Common core standards: The new U.S. intended curriculum. *Educational Researcher, 40*(3), 103–116. doi:10.3102/0013189X11405038

Roach, A. T., Niebling, B. C., & Kurz, A. (2008). Evaluating the alignment among curriculum, instruction, and assessment: Implications and applications for research and practice. *Psychology in the Schools, 45*(2), 158–176. doi:10.1002/pits.20282

Ross, J. A., Mcdougall, D., Hogaboam-Gray, A., & LeSage, A. (2003). A survey measuring elementary teachers' implementation of standards-based mathematics teaching. *Journal for Research in Mathematics Education, 34*(4), 344–363.

Saunders, A. F., Bethune, K. S., Spooner, F., & Browder, D. (2013). Solving the common core equation: Teaching mathematics CCSS to students with moderate and severe disabilities. *Teaching Exceptional Children, 45*(3), 24–33.

Sawada, D., Piburn, M. D., Judson, E., Turley, J., Falconer, K., Benford, R., & Bloom, I. (2002). Measuring reform practices in science and mathematics classrooms: The reformed teaching observation protocol. *School Science and Mathematics, 102*(6), 245–253.

Schmidt, W. H., & Houang, R. T. (2012). Curricular coherence and the common core state standards for mathematics. *Educational Researcher, 41*(8), 294–308. doi:10.3102/0013189X12464517

Schoenfeld, A. H., & Kilpatrick, J. (2013). A US perspective on the implementation of inquiry-based learning in mathematics. *ZDM Mathematics Education, 45*, 901–909. doi:10.1007/s11858-013-0531-5

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2), 114–145.

Speiser, B., & Chuck, W. (2013). Models as tools, especially for making sense of problems. In R. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies: ICTMA 13* (pp. 167–172). New York, NY: Springer.

Webb, N. L. (1997). *Criteria for alignment of expectations and assessments in mathematics and science education (Research Monograph No. 6)*. Washington, DC: Council of Chief State School Officers.

White, D. Y. (2003). Promoting productive mathematical classroom discourse with diverse students. *Journal of Mathematical Behavior, 22*, 37–53. doi:10.1016/S0732-3123(03)00003-8

Zawojewski, J. (2013). Problem solving versus modeling. In R. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies: ICTMA 13* (pp. 237–244). New York, NY: Springer.