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Impact of Channel Estimation Errors on BER Performance of Single-User Decoding NOMA System

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Abstract

In the fifth generation (5G) and beyond 5G (B5G) mobile communication, non-orthogonal multiple access (NOMA) has attracted great attention due to higher spectral efficiency and massive connectivity. We investigate the impacts of the channel estimation errors on the bit-error rate (BER) of NOMA, especially with the single-user decoding (SUD) receiver, which does not perform successive interference cancellation (SIC), in contrast to the conventional SIC NOMA scheme.

First, an analytical expression of the BER for SUD NOMA with channel estimation errors is derived. Then, it is demonstrated that the BER performance degrades severely up to the power allocation less than about 20%. Additionally, we show that for the fixed power allocation of 10% in such power allocation range, the signal-to-noise (SNR) loss owing to channel estimation errors is about 5 dB. As a consequence, the channel estimation error should be considered for the design of the SUD NOMA scheme

Keywords: *B5G, NOMA, Superposition coding, Successive interference cancellation, Power allocation.*

1. Introduction

As one of the state-of-the-art technologies in the fifth generation (5G) and beyond mobile networks, non-orthogonal multiple access (NOMA) [1-3] has attracted a lot of attention, compared to existing orthogonal multiple access (OMA) in the fourth generation (4G) mobile communications, such as long term evolution advanced (LTE-A) [4, 5]. Even though the conventional NOMA has provided the super-low latency, the successive interference cancellation (SIC), which is the main processing of the NOMA receiver, together with superposition coding (SC) in the transmitter, is still big complexity and latency [6, 7]. Recently, single-user decoding (SUD), which does not perform SIC at the NOMA receiver, have been investigated in discrete-input Lattice-based NOMA [8-11]. In this paper, we investigate the impacts of channel estimation errors on the bit-error rate (BER) performance in NOMA, especially for the SUD receiver. We consider mainly the SUD receiver with imperfect channel state information (CSI). The remainder of this paper is organized as follows. In Section 2, the system and channel model are described. The closed-form expression for the BER with imperfect CSI is derived in Section 3. The results are presented and discussed in Section 4. Finally, the conclusions are presented in Section 5.

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The main contributions of this paper is summarized as follows:

- 1. We derive an analytical expression of the BER performance for the SUD NOMA, which does not perform SIC at the NOMA receiver.
- 2. We show that the BER performance of the SUD receiver degrades severely up to the power allocation less than about 20%, whereas above the power allocation range greater than 20%, the degradation of the BER performance of the SUD receiver is tolerable.
- 3. Furthermore, to investigate the signal-to-noise (SNR) loss owing to channel estimation errors in the power allocation range of the severe BER performance degradation, we also calculate the SNR loss, which is about 5 dB at the power allocation of 10%.

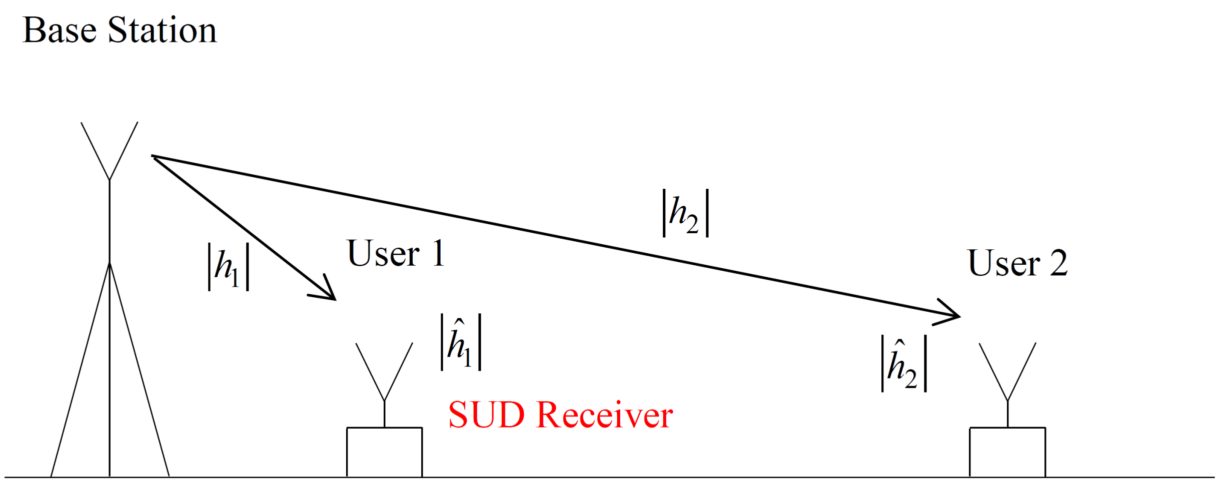

Figure 1. System and Channel Model.

2. System and Channel Model

We consider a cellular downlink NOMA transmission system, in which two users are paired from a base station within the cell. The block diagram for the system and channel model is illustrated in Fig. 1. The Rayleigh fading channel between the *m*th user and the base station is denoted by $h_m \sim \mathcal{CN}(0,\Sigma_m)$, $m=1,2$, where $\mathcal{CN}(\mu,\sigma^2)$ represents the distribution of circularly-symmetric complex Gaussian (CSCG) random variable (RV) with mean μ and variance Σ. The channels are sorted as $\Sigma_1 > \Sigma_2$. The base station sends the superimposed signal $x=\sqrt{\alpha P}s_1+\sqrt{(1-\alpha)P}s_2$, where s_m is the message for the *m*th user with unit power, $\mathbb{E}[s_m s_m^*]=\mathbb{E}[|s_m|^2]=1$, α is the power allocation factor, with $0 \le \alpha \le 1$, and P is the total transmitted power at the base station. The observation at the *m*th user is given by

$$r_m = |h_m| x + n_m, \tag{1}$$

where $n_m \sim \mathcal{N}(0, N_0/2)$ is additive white Gaussian noise (AWGN). We consider the binary phase shift keying (BPSK) modulation, with $s_m \in \{+1,-1\}$.

3. BER Derivation for SUD Receiver with Imperfect CSI for Stronger Channel Gain User

We consider the power allocation range, $0 \le \alpha \le 0.5$, to ensure the user fairness. Then, we define the percentage channel estimation error,

$$e_m = \frac{|h_m| - |\hat{h}_m|}{|h_m|}, \tag{2}$$

where $|\hat{h}_m|$ is the channel estimation.

For the second user, the decision boundary is unchanged by the channel estimation error $|\hat{h}_m|$, because it does not include the channel gain,

$$r_2^{(\text{db})} = 0 \tag{3}$$

Therefore, the BER performance with imperfect channel state information (CSI) is the same as that with perfect CSI, for the second user. However, for the first user, since the decision boundary includes the channel estimation error $|\hat{h}_m|$, the BER performance with imperfect CSI degrades, compared to that with perfect CSI. Now, we derive the BER expression. First, the decision boundary with perfect CSI for the first user is given by

$$r_1^{(\text{db})} \simeq 0, \ \pm |h_1| \sqrt{P(1-\alpha)}. \tag{4}$$

Meanwhile, the decision boundary with imperfect CSI for the first user is given by

$$r_1^{(\text{db})} \simeq 0, \ \pm |\hat{h}_1| \sqrt{P(1-\alpha)}. \tag{5}$$

Then, the conditional BER of the SUD receiver with imperfect CSI can be expresses by

$$
\begin{aligned}
&P_{e\|h_1|}^{(1;\,\mathrm{SUD})} \\
&\left.\begin{aligned}
&\simeq \frac{1}{2}Q\left(\sqrt{\frac{|h_1|^2 P\left(\sqrt{\alpha}+e_1\sqrt{(1-\alpha)}\right)^2}{N_0/2}}\right) \\
&+\frac{1}{2}Q\left(\sqrt{\frac{|h_1|^2 P\left(\sqrt{\alpha}-e_1\sqrt{(1-\alpha)}\right)^2}{N_0/2}}\right)
\end{aligned}\right\} \mathit{terms\ for}\ r_1^{(\mathrm{db})} \simeq |\hat{h}_1|\sqrt{P(1-\alpha)} \\
&\left.\begin{aligned}
&-\frac{1}{2}Q\left(\sqrt{\frac{|h_1|^2 P\left(\sqrt{(1-\alpha)}+\sqrt{\alpha}\right)^2}{N_0}}\right) \\
&+\frac{1}{2}Q\left(\sqrt{\frac{|h_1|^2 P\left(\sqrt{(1-\alpha)}-\sqrt{\alpha}\right)^2}{N_0/2}}\right)
\end{aligned}\right\} \mathit{terms\ for}\ r_1^{(\mathrm{db})} \simeq 0 \\
&\left.\begin{aligned}
&+\frac{1}{2}Q\left(\sqrt{\frac{|h_1|^2 P\left(2\sqrt{(1-\alpha)}+\sqrt{\alpha}-e_1\sqrt{(1-\alpha)}\right)^2}{N_0/2}}\right) \\
&-\frac{1}{2}Q\left(\sqrt{\frac{|h_1|^2 P\left(2\sqrt{(1-\alpha)}-\sqrt{\alpha}-e_1\sqrt{(1-\alpha)}\right)^2}{N_0/2}}\right).
\end{aligned}\right\} \mathit{terms\ for}\ r_1^{(\mathrm{db})} \simeq -|\hat{h}_1|\sqrt{P(1-\alpha)}
\end{aligned}
\tag{6}
$$

where $Q(x)=\int_x^\infty \frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}\,dz$. Then we can use the well-known Rayleigh fading integration formula

$$\int_0^\infty Q\left(\sqrt{2\gamma}\right)\frac{1}{\gamma_b}e^{-\frac{\gamma}{\gamma_b}}\,d\gamma=\frac{1}{2}\left(1-\sqrt{\frac{\gamma_b}{1+\gamma_b}}\right), \tag{7}$$

where the RV γ is exponentially distributed with the mean γ_b of γ, which is given by

$$\gamma_b=\mathbb{E}\left[\gamma\right]. \tag{8}$$

Therefore, the average BER of the SUD receiver with imperfect CSI can be expressed by

$$
\begin{aligned}
&P_e^{(1;\ \mathrm{SUD};\ \mathrm{imperfect\ CSI})} \\
&\left.\begin{aligned}
&\simeq \frac{1}{2}F\left(\frac{\Sigma_1 P\left(\sqrt{\alpha}+e_1\sqrt{(1-\alpha)}\right)^2}{N_0}\right)\\
&+\frac{1}{2}F\left(\frac{\Sigma_1 P\left(\sqrt{\alpha}-e_1\sqrt{(1-\alpha)}\right)^2}{N_0}\right)
\end{aligned}\right\} terms\ for\ r_1^{(\mathrm{db})} \simeq \left|\hat{h}_1\right|\sqrt{P(1-\alpha)}\\
&\left.\begin{aligned}
&-\frac{1}{2}F\left(\frac{P}{N_0}\left(\sqrt{(1-\alpha)}+\sqrt{\alpha}\right)^2\Sigma_1\right)\\
&+\frac{1}{2}F\left(\frac{P}{N_0}\left(\sqrt{(1-\alpha)}-\sqrt{\alpha}\right)^2\Sigma_1\right)
\end{aligned}\right\} terms\ for\ r_1^{(\mathrm{db})} \simeq 0 \qquad (9)\\
&\left.\begin{aligned}
&+\frac{1}{2}F\left(\frac{P}{N_0}\left(2\sqrt{(1-\alpha)}+\sqrt{\alpha}-e_1\sqrt{(1-\alpha)}\right)^2\Sigma_1\right)\\
&-\frac{1}{2}F\left(\frac{P}{N_0}\left(2\sqrt{(1-\alpha)}-\sqrt{\alpha}-e_1\sqrt{(1-\alpha)}\right)^2\Sigma_1\right).
\end{aligned}\right\} terms\ for\ r_1^{(\mathrm{db})} \simeq -\left|\hat{h}_1\right|\sqrt{P(1-\alpha)}
\end{aligned}
$$

where

$$F\left(\gamma_b\right)=\frac{1}{2}\left(1-\sqrt{\frac{\gamma_b}{1+\gamma_b}}\right). \qquad (10)$$

4. Numerical Results and Discussions

We compare the BER $P_e^{(1;\ \mathrm{SUD};\ \mathrm{imperfect\ CSI})}$ with the imperfect CSI to the BER $P_e^{(1;\ \mathrm{SUD};\ \mathrm{perfect\ CSI})}$ with the perfect CSI, which is given by

$$
\begin{aligned}
&P_e^{(1;\ \mathrm{SUD};\ \mathrm{perfect\ CSI})} \simeq F\left(\frac{\Sigma_1 P\alpha}{N_0}\right) \longrightarrow terms\ for\ r_1^{(\mathrm{db})} \simeq \left|h_1\right|\sqrt{P(1-\alpha)}\\
&\left.\begin{aligned}
&-\frac{1}{2}F\left(\frac{P}{N_0}\left(\sqrt{(1-\alpha)}+\sqrt{\alpha}\right)^2\Sigma_1\right)\\
&+\frac{1}{2}F\left(\frac{P}{N_0}\left(\sqrt{(1-\alpha)}-\sqrt{\alpha}\right)^2\Sigma_1\right)
\end{aligned}\right\} terms\ for\ r_1^{(\mathrm{db})} \simeq 0 \qquad (11)\\
&\left.\begin{aligned}
&+\frac{1}{2}F\left(\frac{P}{N_0}\left(2\sqrt{(1-\alpha)}+\sqrt{\alpha}\right)^2\Sigma_1\right)\\
&-\frac{1}{2}F\left(\frac{P}{N_0}\left(2\sqrt{(1-\alpha)}-\sqrt{\alpha}\right)^2\Sigma_1\right).
\end{aligned}\right\} terms\ for\ r_1^{(\mathrm{db})} \simeq -\left|h_1\right|\sqrt{P(1-\alpha)}
\end{aligned}
$$

It is assumed that $\Sigma_1 = 1.5$ and $\Sigma_2 = 0.5$. And we assume $e_1 = 0.2$, i.e., 20% channel estimation error.

First, we consider the constant total transmitted signal power to noise power ratio (SNR) $P / N_0 = 40$ dB. In Fig. 1, the BER for the SUD NOMA scheme with imperfect CSI is compared to that with perfect CSI, for the first user, with the range of the power allocation, $0 \le \alpha \le 0.5$, under user fairness.

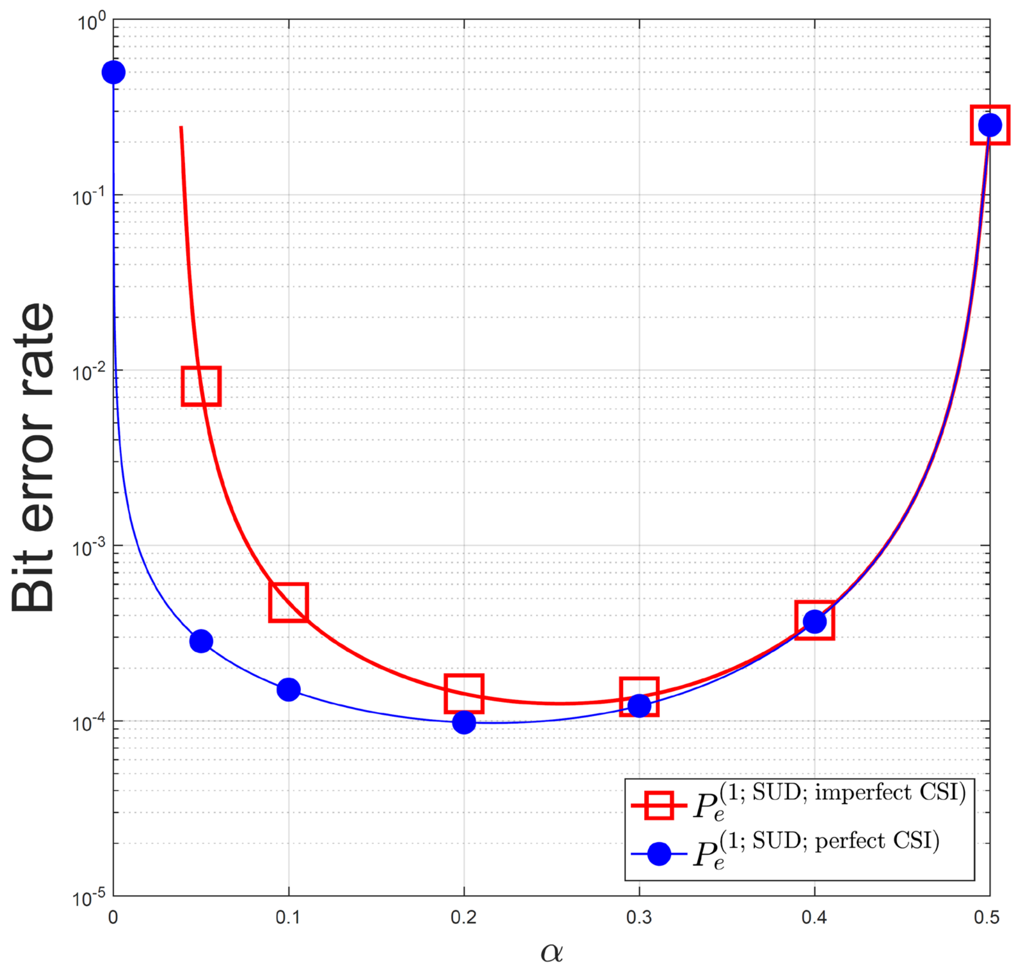

Figure 2. Comparison of BERs for SUD NOMA with perfect CSI and imperfect CSI NOMA ($0 \le \alpha \le 0.5$).

As shown in Fig. 1, the severe BER degradation owing to imperfect channel estimation is observed for the power allocation factor less than about $\alpha \simeq 0.2$. For the power allocation greater than $\alpha \simeq 0.2$, the severe BER degradation owing to imperfect channel estimation becomes negligible, because for such power allocation range, the inter-user interference (IUI) becomes dominant.

Second, in order to investigate the SNR loss owing to the imperfect channel estimation, the BER is depicted versus the SNR, for the fixed power allocation $\alpha = 0.1$.

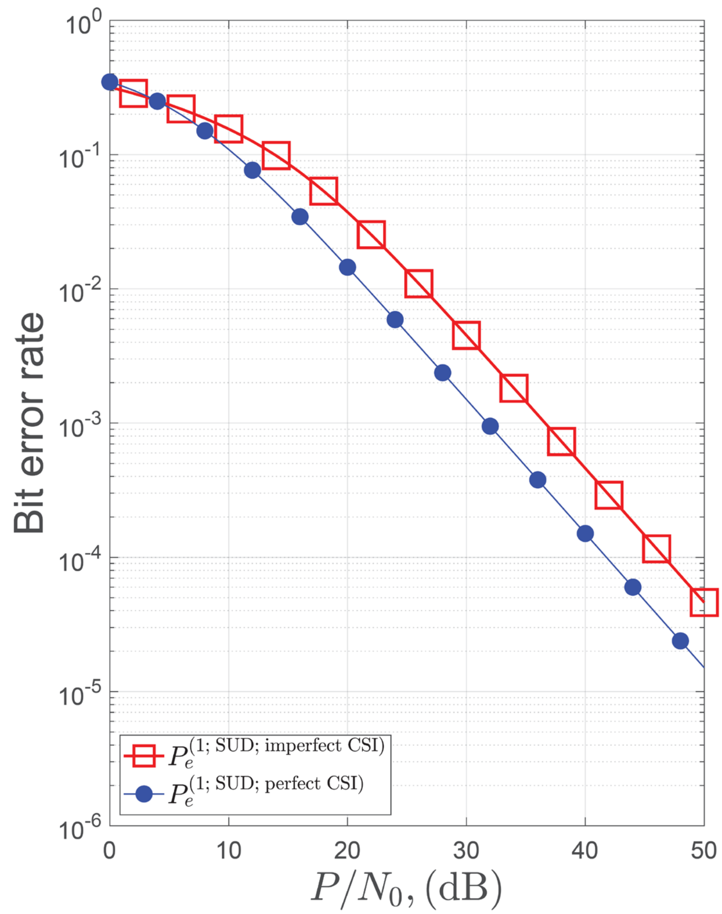

Figure 3. Comparison of BERs for SUD NOMA with perfect CSI and imperfect CSI NOMA, ($\alpha = 0.1$).

As shown in Fig. 2, at the BER of 10^{-4}, the SNR loss of about 5 dB is observed, with the power allocation of $\alpha = 0.1$, and the percent channel estimation error of $e_1 = 0.2$.

Therefore, we surmise two things with such results; one is that in SUD NOMA, the impact of the channel estimation error on the BER is severe up to the power allocation less than about $\alpha \simeq 0.2$. The other is that for such range, the channel estimation error results in the considerable SNR loss.

5. Conclusion

In this paper, we derived an analytical expression for the BER of SUD NOMA with channel estimation errors. Then the impact of channel estimation errors on the BER was investigated. It was shown that the BER degradation is severe, less than the power allocation, about $\alpha \simeq 0.2$, whereas for the power allocation range greater than about $\alpha \simeq 0.2$, the BER performance degradation is tolerable, because in such power allocation range, the IUI is dominant for the BER degradation.

To investigate the SNR loss due to imperfect CSI, we also demonstrated that the SNR loss owing to the channel estimation errors is about 5 dB, at the BER of 10^{-4}, with the fixed power allocation of 10%.

In result, the channel estimation error should be considered in design of the SUD NOMA system, especially for the receiver of the stronger channel user without SIC, because the power allocation range of the severe

BER degradation is a significant operating range for the user-fairness.

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