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A Study on Optimal PID Controller Design Ensure the Absolute Stability

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절대안정도를 보장하는 최적 PID 제어기 설계에 관한 연구

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Abstract In this paper, an optimal controller design that guarantees absolute stability is proposed. The order of application of the thesis determines whether the delay time is included, and if the delay time is included, the delay time is approximated through the Pade approximation method. Then, the open loop transfer function for the process model and the controller transfer function is obtained, and the absolute stability interval is calculated by the Routh-Hurwitz discrimination method. In the last step, the optimal Proportional and Integral and Derivative(PID) control parameter value is calculated using a genetic algorithm using the interval obtained in the previous step. As a result, it was confirmed that the proposed method guarantees stability and is superior to the existing method in performance index by designing an optimal controller. If we study the compensation method for the delay time in the future, it is judged that better performance indicators will be obtained.

Key Words : PID Controller, Stability, Pade approximation, Delay time, Optimization

요 약 본 논문에서는 절대 안정도를 보장하는 최적의 제어기 설계에 대해 제안하였다. 논문의 적용 순서는 지연시간의 포함여부를 판단하고, 지연시간이 포함되었을 경우 Pade 근사법을 통해서 지연시간을 근사화 한다. 그 다음 공정모델과 제어기 전달함수에 대한 개루프 전달함수를 구하며, Routh-Hurwitz 판별법에 의해서 절대 안정도 구간을 계산한다. 마지막 단계에서는 앞 단계에서 구한 구간을 활용하여 유전자 알고리즘으로 최적의 PID 제어 파라미터 값을 구한다. 그 결과 제안 된 방법은 안정성이 보장되며, 최적의 제어기를 설계하여 기존의 방법보다 성능 지표에서 우월함을 확인하였다. 향후 지연시간에 대한 보상방법이 연구된다면 더욱 좋은 성능지표를 얻을 것으로 판단된다.

주제어 : PID 제어기, 안정도, Pade 근사법, 지연시간, 최적화

1. Introduction

The PID (Proportional and Integral and Derivative) controller has a simple structure, so it is easy to analyze and many theories for design have been studied. It is implemented in the industrial field and is highly reliable, so it is widely used in the process industry.

The method of determining the parameters of the PID controller has developed rapidly since the research results of the Ziegler-Nichols

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Rule were published, Internal Model Control(IMC) methods are commonly used in chemical plants[1-5]. However, the PID tuning method known as the linear control method is often performed by experience, and there are methods that operate only in a specific model rather than a general model[6-8]. Therefore, methods that can be applied universally are being studied, and a representative method among them is a method of determining controller parameters using a genetic algorithm[9-14].

In order to apply this method, research on setting the parameter range must be preceded. In this paper, the Routh-Hurwitz discrimination method was applied for this preceding study. As a result, a range of control parameters could be obtained, and this value was used as a parameter range of a genetic algorithm to obtain an optimal control parameter. There is a drawback that the Routh-Hurwitz discrimination method can be applied to the model including the delay time. However, in this paper, in order to solve these shortcomings, the Pade approximation was applied and the delay time was changed to the additional form of poles and zeros. This paper proposes an optimal controller design method by applying Pade approximation, Routh-Hurwitz stability discrimination method, and genetic algorithm to models with first and second delay times. The composition of this paper is in the order of determination of control parameters based on the Routh-Hurwitz stability determination method, controller design, and conclusions.

2. Controller tuning algorithm

2.1 PI parameters tuning of the first-order model

The most important thing in designing a PI controller is to determine the values of the control parameters kp and Ti. Among the tuning methods currently studied, the PI tuning algorithm using genes is widely applicable to all systems. In order to apply this method, the range of the PI parameter must be determined in advance, but in most cases it is determined by the designer's experience. We present a method to determine the range of PI control parameters based on stability.

The transfer function of the PI controller is shown in equation (1) and the control process transfer function is shown in equation (2).

$$C(s) = k_c(1+\frac{1}{sT_i}) \tag{1}$$

$$G(s) = \frac{k_p}{1+s\,\tau}e^{-sL} \tag{2}$$

Here kp and Ti are control parameters.

The open-loop transfer function is obtained from equations (1) and (2).

$$C(s)\,G(s) = \frac{k_c k_p\,(1+sT_i)}{sT_i\,(1+s\tau)}e^{-sL} \tag{3}$$

The approximate equation of Pade is applied to the delay time of equation (3), and the characteristic equation is obtained as in equation (4).

$$\frac{T_i\tau L}{2}s^3+\left(T_i\tau+\frac{LT_i}{2}-\frac{k_c k_p T_i L}{2}\right)s^2+\left(T_i+k_c k_p T_i-\frac{k_c k_p L}{2}\right)s+k_c k_p=0 \tag{4}$$

Table 1 shows the range of PI control parameters considering the stability of equation (4).

Table 1. PI parameter range considering stability

k_c	T_i	
	$y>0$	$y<0$
$0<k_c<\frac{(2\tau+L)}{k_pL}$	$0<T_i<-\frac{x}{y}$	$T_i>-\frac{x}{y}$

$$x = \left(k_c k_p L\tau + \frac{k_c k_p L^2}{4} - \frac{k_c^2 k_p^2 L^2}{4}\right),$$
$$y = \left(\frac{k_c^2 k_p^2 L}{2} - \frac{L}{2} - \tau - k_c k_p \tau\right)$$

2.2 PID parameters tuning of the second -order model

For processes with first order delay time, a PI controller alone can achieve good enough results, but for higher order systems, a PID controller must be used.

A process with a second delay time cannot obtain satisfactory results with a PI controller and must be designed with a PID controller. This chapter presents a method of determining the parameter range of a PID controller based on stability for a control process with a second delay time.

The controller and process transfer function are as shown in Equations (5) and (6).

$$G_c(s) = k_c + \frac{T_i}{s} + T_d s = k\left(\frac{As^2 + Bs + C}{s}\right) \quad (5)$$

$$G_p(s) = \frac{e^{-sL}}{as^2 + bs + c} \quad (6)$$

Here, $A = (T_d/k), B = (k_c/k), C = (T_i/k)$

From equations (5) and (6), the open-loop transfer functions $G_c(s)$ and $G_p(s)$ are as in equation (7).

$$G_c(s)\,G_p(s) = \frac{ke^{-sL}}{s + ke^{-sL}} \quad (7)$$

Here, if A=a, B=b, and C=c, the characteristic equation can be obtained as Equation (8).

$$s + ke^{-sL} = 0 \quad (8)$$

The delay time of Equation (8) can be expressed using the Pade approximation equation as follows.

$$e^{-sL} \cong \frac{1 - \frac{L}{2}s}{1 + \frac{L}{2}s} \quad (9)$$

Substituting Equation (9) into Equation (8) and arranging for s, the characteristic equation is as Equation (10).

$$LS^2 + (2 - Lk)s + 2k = 0 \quad (10)$$

Table 2 shows the range of PID control parameters considering the stability of Equation (10).

Table 2. PID parameter range considering stability

k_c	T_i	T_d
kB	kC	kA
$0 < k < \frac{2}{L}$		

3. Simulation and consideration

For the primary system with large and small delay time through simulation, optimal control parameters were determined through a method of determining a control parameter range based on stability and a genetic algorithm. Then, the existing method and the proposed method were compared and analyzed through the graph.

3.1 PI controller design for the first order system with delay time

The first order system with a large delay time is shown in Equation (11).

$$G(s) = \frac{e^{-10s}}{s+1} \quad (11)$$

For Equation (11), the range of parameters based on the Routh-Hurwitz discrimination and the optimal parameter values using genetic algorithm are shown in Table 3.

Table 3. PID parameter range considering stability

Range of k_c	Range of T_i		k_c	T_i
	k_c	T_i		
$0 < k_c < 1.2$	1.199	$T_i > 547.9$	0.574	5.095

In Fig. 1, the control parameters obtained by the proposed method for the model of equation (2) are compared with the existing method [7], and it can be confirmed that the method suggested is fine.

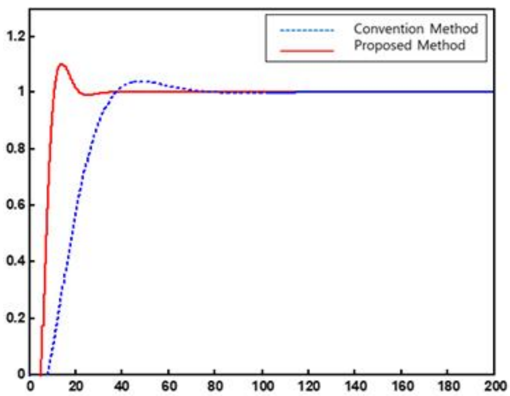

Fig. 1. PI control response curve

Table 4 compares the performance index in terms of Over-Shoot and ISE, confirming that the proposed method is superior to the existing method.

Table 4. Performance index comparison

	Conventional Method	Proposed Method
Over-Shoot	1.0411	1.1025
ISE	162.52	113.19

3.2 PID controller design for the second order system with delay time

The second order system with a delay time is shown in Equation (12).

$$G(s) = \frac{1}{6.86s^2 + 32.15s + 25} e^{-0.607s} \quad (12)$$

For the control process with the first delay time, it is possible to obtain sufficiently good results with only the PI controller, but the control process with the high-order delay time must be controlled using the PID controller.

For Equation (12), the parameter range based on the Routh-Hurwitz discrimination and the optimal parameter values using the genetic algorithm are shown in Table 5.

Table 5. Range of PID control parameters

k_c	T_i	T_d
27.51	21.40	5.78
$k = 0.8828$		

In Fig. 2, the control parameters obtained by the proposed method for the model of equation (11) are compared with the conventional method, and it can be seen that the proposed method is fine.

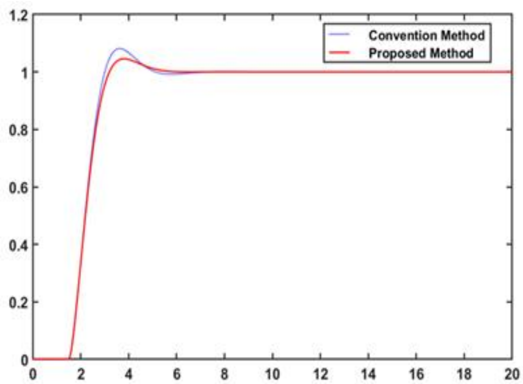

Fig. 2. PID control response curve

Table 6 compares the performance index in terms of Over-Shoot and ISE, and it can be confirmed that the method suggested is excellent to the existing method.

Table 6. Performance index comparison

	Conventional Method	Proposed Method
Over-Shoot	1.03	1.06
ISE	100.48	100.05

3.3 PID control for higher order models

The higher-order system is shown in equation (13).

$$G(s) = \frac{(s+2)}{(s+1)(s+5)(0.5s+1)} \quad (13)$$

In order to apply the proposed method, the higher order system should be reduced to a model with a second order delay time. Equation (14) is a reduced model.

$$G(s) = \frac{1}{0.377s^2+3.90s+7.5}e^{-0.0186} \quad (14)$$

For Equation (14), the range of parameters based on Routh-Hurwitz discrimination and optimal parameter values using genetic algorithm are shown in Table 7.

Table 7. Range of PID control parameters

k_c	T_i	T_d
8.58	16.5	0.83
k=2.20		

In Fig. 3, it can be seen that it cannot be obtained by the conventional method, but can be obtained by the proposed method.

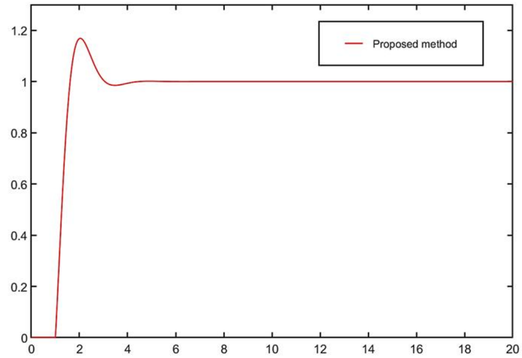

Fig. 3. Equation 13 for the model PID control response curve

Table 8. shows the performance index in terms of Over-Shoot and ISE.

Table 8. Performance index comparison

	Conventional Method	Proposed Method
Over-Shoot	No Solution	1.169
ISE		197.8

4. Conclusion

In this paper, the PI control parameter range was determined based on the Routh-Hurwitz stability discrimination method for the system with the first order delay, and then the optimal PI control parameter value was determined using a genetic algorithm. And for a system with a second delay time, the controller transfer function and the pole and zero points of the process transfer function are canceled and converted into a system with the first delay time. After determining, the optimal k value was calculated using a genetic algorithm, and the PID parameter value was calculated. The proposed method has the advantage that it can be widely applied to models with 1st and 2nd order delay times. Through simulation, it was confirmed that the range of stability-based control parameters and optimal control parameters were determined through genetic algorithms, and that the proposed method was superior to the existing method in terms of control performance. In future research, research is needed on the control parameter tuning part of this paper with genetic algorithms.

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