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Reliability analysis on fatigue Strength for Certification of Aircraft Composite Structures

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Abstract

Reliability of fatigue strength on Aircraft Composites(GFRP) Structures was assessed in this paper. Fatigue strength of GFRP was used through the existing fatigue test data with Monte Carlo method. The S_a-N_f curve of composites fatigue strength was assumed as normal distribution and reliability was analyzed using SSIT model. Fatigue stress was designed IAW ASTM F3114-15 with special safety factor of S_{sf}=1.2~2.0. Reliability was calculated by analytic method and FORM. Sensitivity for the effect of mean and standard deviation of fatigue strength as well as fatigue stability was evaluated. This result can be usefully applied to reliability and fatigue design for composite structures of light weight aircraft.

Key Words : Certification, Fatigue Strength, Composite Structures, Stress-Strength Interference Theory, Performance Function, Reliability Index, FORM, Sensitivity, Stability

1. Introduction

Composites material for aircraft structures has been steeply increased since 1980s. Composites material was to be important function due to be satisfied with lightness and stiffness for aircraft structures. However during design, manufacturing, inspection procedures, quality certification has been depended on process of the existing Al-alloy or metal alloy. In 1999, DOT and FAA suggested that the composites materials of aircraft be considered as fatigue, damage tolerance, corrosion and categorized as A-Base as well as B-Base[1-2]. For this method what is applied to design safety factor and special safety factor has been similar concepts to the metal materials. Compliance inspection for quality certification of composite structures was reviewed as the technical level of the current aluminum and metal materials. This environment of engineering for composites has developed and studied the inspection conformity with higher expert basement of NDI until recently[3].

On 2009 FAA, Title 14 CFR Part 23(Small aircraft certification procedures) and AC No. 20-107B(Adversary Circular) recommended that structural test, specimen test and full scale test for composites to standard strength test[4-5]. XIE and LU[6] reported the certification requirements for composites aircraft structures. It includes Building Block concepts for AGATE(Advance general aviation transport experiments) and NCAMP(Nation center for advanced material performance). The report recommended that experimental data, test specimen, part test, component test, structural test related to airworthiness certification for composites be owned in public. FAA regulation suggest to follow critical load, fracture mode, limit load, fabrication methods, design factor and strength properties, special safety and inspection of composites structures. However on 2018, new suggestion was reported to address the current issues for aircraft composite structures by North America and Europe engineering industry with relation to composites. Regulation for composites structural design is to follow ASTM F3114-15[8] and this suggests safety factor and special safety factor. Choi et.al[9-12] reported test and evaluation results for fatigue properties and damage tolerance application of Helicopter Rotor Blade. Regulation for AC is shown in Table 1.

Table 1 Composites for Transport Category

FAR	Contents
25.305 25.307	AC Composites Critical Load, Failure Mode, Ultimate Load

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25.603	Material
25.605	Fabrication Method
25.609	Protection Structure
25.613	Material Strength Properties and Material design Value
25.619 25.621	Special factor 1.2 ~ 2.0 Inspection

AC : Airworthiness Certification

In this study static safety factor and special safety factor of composites(GFRP) based on ASTM F3114-15[8] is confirmed as fatigue life properties (S_a-N_f) through statistical reliability approach. Stress-Strength Interference Theory was applied and reliability sensitivity and stability was assessed.

2. Reliability of Composites Fatigue Strength

2.1 Fatigue of Composites

Fatigue strength of composites by cyclic bending load was modeled as Basquin suggested (Basquin, 1914), which is given as follows.

$$S_a = \sigma_f' \bullet (N_f)^C \qquad (1)$$

Where each of those for S_a, σ_f', N_f, C is fatigue stress, fatigue strength, fatigue life and exponent resulted from fatigue experiment. Special safety factor is S_{sf} = 1.2 ~ 2.0, and fatigue strength and stress was assumed as normal distribution for statistical analysis.

Static strength of composites in design stage is given for total safety factor, S_{tf} as shown in Fig. 1 As service hours is increased for aircraft composite structures, fatigue damage is accumulated and residual strength is decreased, fatigue strength is decreased as $S_1 \rightarrow S_2 \rightarrow S_3$ and finally to be unstable by the effect of fatigue damage according to fatigue stress interference. The range of fatigue stress as three times of standard deviation of $3\sigma_i$ was applied and so that is a little bit more severe condition comparing to A-Base. Standard deviation of mean stress of fatigue with the conditioned lower level was 10%of fatigue stress was applied on empirical ground. Fatigue result data was referred from load control fatigue test and transformed to probability distribution as shown Fig.2.

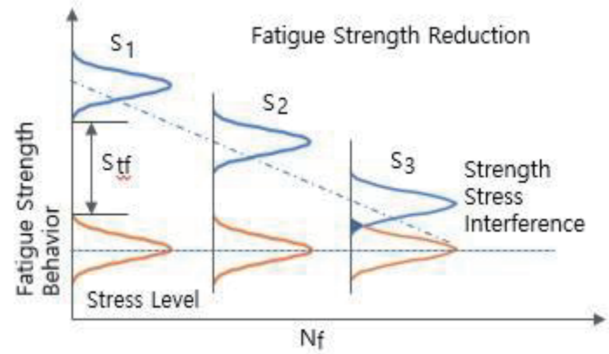

Fig. 1 Fatigue Strength Reduction of Composite

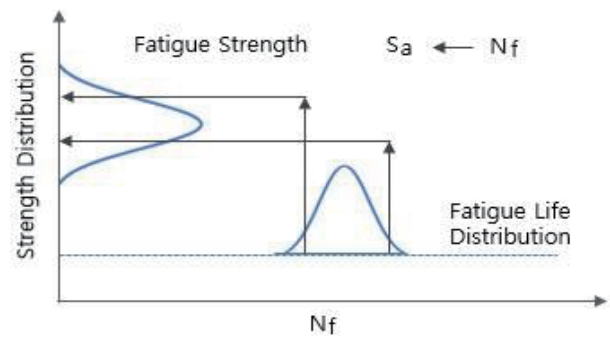

Fig. 2 Fatigue Strength Distribution of Composite

Fatigue strength probability density function(PDF) is derived as for fatigue cycle using Eq(1) as shown Fig.2. Fatigue strength data of composites was referred from the existing one of experiments[13] and Monte Carlo simulation applied regarding the data range in this study. PDF of fatigue strength is given by

$$f\,(S_i) = \frac{1}{\sqrt{2\pi\sigma_i}} exp[-\frac{1}{2}(\frac{S_i - S_a}{\sigma_i})^2] \qquad (2)$$

Where for fatigue cycles N_f, S_i is mean fatigue strength, σ_i is standard deviation of fatigue strength at each cycle and the same concept for fatigue stress.

2.2 Stress-Strength Interference Model

When fatigue strength of composites Y and applied fatigue stress X are as random variable, Reliability of the composites for fatigue strength is as follows.

$$R = P(\,X \le Y\,) \qquad (3)$$

Where R is reliability and X, Y independent variables for each. For this variable PDF $f(x)$, $f(y)$ are shown as in Fig.3.

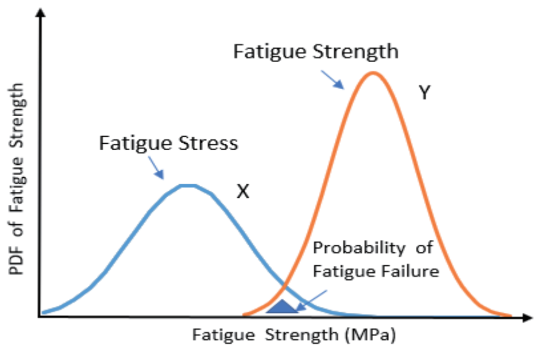

Fig. 3 Stress-Strength Interference Theory

Reliability of fatigue strength for composites is obtained using SSIT as interfered area of Fig. 3 for X, Y variables and probability of failure is defined $P_f = 1 - R$ and Reliability R is given by[14-15].

$$R = \int_{-\infty}^{+\infty} [\int_{y}^{+\infty} f(x)dx]\, g(y)dy$$

$$= \int_{-\infty}^{+\infty} F(y)g(y)dy \qquad (4)$$

or

$$R = \int_{-\infty}^{+\infty} [\int_{-\infty}^{x} g(y)dy]\, f(x)dx$$

$$= \int_{-\infty}^{+\infty} G(x)f(x)dx \qquad (5)$$

Where $F(x)$ and $F(x)$ are CDF of X and Y. What Cornell[16] suggested analytical method that was to solve the Eq(3), Eq(4) and Eq(5). To do that, performance function $g = x - y$ is given and performance index $\beta = (g - \mu_g)/\sigma_g$ is defined, that becomes $\sigma_g\, d\beta = dg$, and when then $g = 0$, upper limit of β becomes $\beta = (0 - \mu_g)/\sigma_g$, finally compounded mean and standard deviation relation are obtained standard normal probability density function of value for performance index β shown as probability of fatigue failure in Fig.3 and as follows.

$$\mu_g = \mu_x - \mu_y \qquad (6)$$

$$g = \sqrt{\sigma_x^2 + \sigma_y^2} \qquad (7)$$

$$\beta = \frac{\mu_g}{\sigma_g} = \frac{|\mu_x - \mu_y|}{\sqrt{\sigma_x^2 + \sigma_y^2}} \qquad (8)$$

Where β is performance index or reliability index and each of μ_x, μ_y, σ_x, σ_y is mean fatigue stress and mean fatigue strength, standard deviation of fatigue stress and fatigue strength. Reliability for fatigue strength and fatigue stress of composites is obtained from the relation of P_f and R, which is given by

$$p_f = P\,(X - Y \geq 0) = \Phi(-\beta) \qquad (9)$$

$$p_f = \Phi(-\beta) \qquad (10)$$

$$R = 1 - p_f = \Phi(-\beta) \qquad (11)$$

Where Φ is cumulative distribution function (CDF) of standard normal probability density function(PDF).

2.3 Reliability Analysis of FORM application

To obtain Reliability, FORM(First Order Reliability Method) for fatigue strength and stress of composites is that density function of them is transformed to standard normal density function and to find the distance from 0 point to MPP(Most Probable Point) as reliability index or index. Random variable X is transformed to X -space as X = (x_1, x_2, x_3, ... x_n) and U to U -space as U = (u_1, u_2, u_3, ... u_n) and to be standard normal distribution. Rosenblatt transform[16-22] is applied as follows.

$$F_x\,(x_i) = \Phi(U_i) \qquad (12)$$

Where $\Phi(\bullet)$ is cumulative distribution function of standard normal distribution. Transformed standard normal distribution is given by.

$$U_i = \Phi^{-1} F_x\,[(X_i)] \qquad (13)$$

$$U = \Phi^{-1}\,[F_x\,(X)] = \Phi^{-1}\left[\Phi\left(\frac{X-\mu}{\sigma}\right)\right]$$
$$= \frac{X-\mu}{\sigma} \qquad (14)$$

For random variables $X_i = \mu_i + \sigma_i\,\mu_i$ is transformed to U -space, in which performance function $g\,(u\,)$ is obtained as follows.

$$g\,(u\,) = X - Y \qquad (15)$$

$$g(u) = \mu_i + \sigma_i\mu_i - (\mu_j + \sigma_j\mu_j\,) \qquad (16)$$

When do Taylor series expansion of Eq.(16) and consider first order differential, which becomes linear solution using perturbation method. Generalized procedure is shown in Fig. 4.

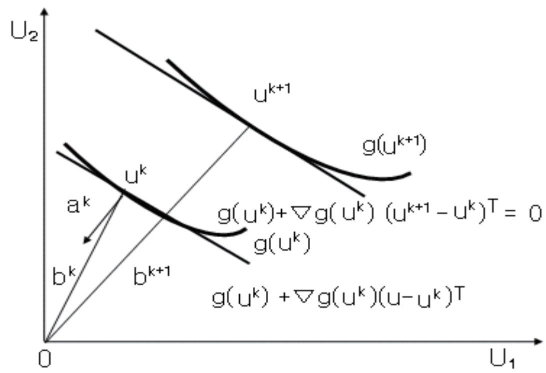

Fig. 4 MPP Search

Tangential line of standard normal distribution crossed performance function as $g(u) = 0$ and u vector minimized distance from point 0 to b is defined as follows.

$$\min \quad \| u \| \quad \text{at } U \tag{17}$$

$$\text{Constraints } g(u) = 0 \tag{18}$$

For the kth performance function linear term is considered of 1st order differential equation and arranged by Taylor series.

$$g(u) = g(u^k) + \nabla g(u^k)(u - u^k)^T \tag{19}$$

Where T is transpose Vector. In Fig. 4 MPP is located at u^{k+1} and Eq.(19) is arranged to moving point as follows.

$$g(u^{k+1}) = g(u^k) + \nabla g(u^k)(u^{k+1} - u^k)^T = 0 \tag{20}$$

$$u^k = -b^k a^k \tag{21}$$

$$u^{k+1} = -b^{k+1} a^k \tag{22}$$

Where a^k is unit Vector. Eq.(21) and Eq.(22) are submitted into Eq.(20) and then as follows.

$$g(u^k) + \nabla g((u^k)(u - u^k)^T(b^k - b^{k+1})$$

$$= g(u^k) + \| \nabla g((u^k)\| (b^k - b^{k+1})$$

$$= 0 \tag{23}$$

$$b^{k+1} = b^k + \frac{g(u^k)}{\| \nabla g((u^k)\|} \tag{24}$$

Where $\| \bullet \|$ represents for quantity or magnitude as absolute value of the vector and $| \bullet |$ is absolute value of scalar. As the results b^{k+1} put into Eq.(22) repeatedly, then MPP is obtained finally as follows.

$$u^{k+1} = -a^k \{b^k + \frac{g(u^k)}{\| \nabla g((u^k)\|}\} \tag{25}$$

After iteration of Eq. (25) convergence is achieved to any one firstly among the infinitesimal e_1, e_2, e_3, and if conversed, calculation is terminated as.

$$\| u^{k+1} - u^k \| \leq e_1 \tag{26a}$$

$$\| \nabla g(u^{k+1}) - \nabla g(u^k) \| \leq e_2 \tag{26b}$$

$$| b^{k+1} - b^k | \leq e_3 \tag{26c}$$

When infinitesimal conversed, reliability index b is obtained as failure rate $p_f = \Phi(-b)$ and reliability $R = 1 - p_f = \Phi(-b)$ is determined.

2.4 Sensitivity and Stability to Reliability

Sensitivity of reliability for composites fatigue strength is represented as failure probability (p_f) related to distribution variable (P)[21, 22]. As in Fig. 4 X-space is transformed to U-space by Rosenblatt transform[22]. Distribution variable P is function of mean fatigue strength and standard deviation. Sensitivity for reliability is defined as follows.

$$S_p = -\frac{\partial p_f}{\partial p} \tag{27}$$

$$S_p = \frac{\partial p_f}{\partial p} = -\frac{\partial \Phi(-b)}{\partial p} = -\frac{\partial \Phi(-b)}{\partial b}\frac{\partial b}{\partial p}$$

$$= -f(-b)\frac{\partial b}{\partial p} \tag{28}$$

$$\frac{\partial b}{\partial P} = \frac{\partial b}{\partial u_i^*}\frac{\partial u_i^*}{\partial b} \tag{29}$$

Where $f(-b) = \partial\Phi/\partial b$ is a compound standard normal failure density function(PDF) for fatigue of composites in U-space and u_i^* is tangent point at MMP. Eq.(29) is as follows.

$$\frac{\partial b}{\partial u_i^*} = \frac{\partial \sqrt{\sum_{j=1}^{n}(u_i^*)^2}}{\partial u_i^*} = \frac{u_i^*}{\sqrt{\sum_{j=1}^{n}(u_i^*)^2}}$$

$$= \frac{u_i^*}{b} \qquad (30)$$

$$u_i^* = \Phi^{-1}\ [F\ x_i(x_i^*)\ ] = w(p) \qquad (31)$$

Where $w(p) = \Phi^{-1}\ [F\ x_i(x_i^*)\ ]$ is function distribution variable P and derived by

$$\frac{\partial b}{\partial p} = \frac{u_i^*}{b}\ \frac{\partial w}{\partial p} \qquad (32)$$

Sensitivity is to be obtained as generalized.

$$S_p = -\ \frac{\partial p_f}{\partial p} = f(-b)\ \frac{u_i^*}{b}\ \frac{\partial w}{\partial p} \qquad (33)$$

For mean fatigue strength and standard deviation of composites.

$$S\mu_i = -\frac{\partial p_f}{\partial \mu_i} = f\ (-b)\ \frac{u_i^*}{b}\ \frac{\partial w}{\partial \mu_i} \qquad (34)$$

$$S\sigma_i = -\frac{\partial p_f}{\partial \sigma_i} = f\ (-b)\ \frac{u_i^*}{b}\ \frac{\partial w}{\partial \sigma_i} \qquad (35)$$

For normal distribution $x_i \sim N(\mu_i,\ \sigma_i\ )$ of Eq.(34), Eq.(35), using Eq.(13), Eq.(14), which is obtained by

$$w\ (\mu_i\ , \sigma_i) = \Phi^{-1}[F\ x_i(x_i^*)] = \Phi^{-1}[\Phi\ (\frac{x_i^* - \mu_i}{\sigma_i})\ ]$$

$$= \frac{x_i^* - \mu_i}{\sigma_i} \qquad (36)$$

$$\frac{\partial w}{\partial \mu_i} = -\ \frac{1}{\sigma_i} \qquad (37)$$

$$\frac{\partial w}{\partial \sigma_i} = -\ \frac{x_i^* - \mu_i}{\sigma_i^2}\ \frac{1}{\sigma_i}$$

$$= -\ \frac{u_i^*}{\sigma_i} \qquad (38)$$

$$S\mu_i = -\ \frac{\partial p_f}{\partial \mu_i} = -f\ (-b)\ \frac{u_i^*}{b\sigma_i} \quad \text{at } N_f \qquad (39)$$

$$S\sigma_i = -\ \frac{\partial p_f}{\partial \sigma_i} = -f\ (-b)\ \frac{(u_i^*)^2}{b\ \sigma_i} \quad \text{at } N_f \qquad (40)$$

Where N_f is each failure cycle of composites. Reliability sensitivity is represented as function of mean strength and standard deviation of fatigue properties of composites and calculated using Eq.(39) and Eq.(40). In case of condition of Eq.(39), $S_\mu > 0$ at $f(-b) > 0$, $b > 0$, $\sigma_i > 0$, $u_i^* > 0$ and $S_\mu < 0$, at $u_i^* < 0$. However S_σ is negative in Eq.(40). Stability of fatigue strength of composites is evaluated as follows.

$$S_{st} = -\ \frac{f(-b)}{N_f} \quad \text{at } N_f \qquad (41)$$

Where special safety factor is applied in the range of S_{sf} = 1.2, 1.5, 2.0. Fatigue strength stability of composite total safety factor, Eq.(41), S_{st} is constrained in the range of $N_f = 10^6 \sim 10^7$ and then $S_{st} = 10^{-7} \sim 10^{-10}$ is boundary condition. Where for example what 10^{-7} means at $N_f = 10^6$ represents stability on probability of failure as $1/10^7$ at the fatigue life.

3. Assessment and discussion

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3.1 Fatigue analysis of Composites

Reliability analysis procedure is shown in Fig. 5.

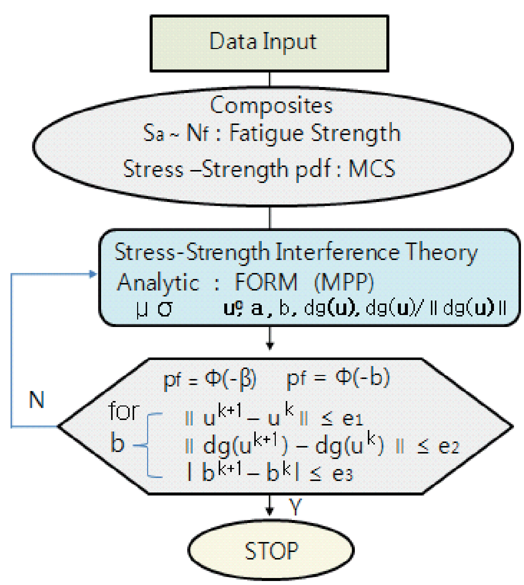

Fig. 5 Calculation process for Reliability through Analytic Method and FORM

Fatigue properties data of composites in this paper was referred Sakin et.al[13] and the data was compiled in Table 2. For the statistical data, Monte Carlo method was applied using Excel spread sheet of equation as *"RAND()*(Max-Min)+Min"* with 100 iteration more.

ASTM F311-15 has recommended for safety factor of composites for aircraft structures as limit safety factor of 1.5, temperature effect of 1.25 and special safety factor of 1.5. Total safety factor is applied S_{st} = 2.25~3.37 due to S_{sf} = 1.2, 1.5, 2.0 in this study. Composites fatigue strength data of GFRP includes angle ply composites and cross ply composites.

Table 2 Fatigue Life Properties[13]

Group	σ_f'[MPa]	C	R^2
Angle Ply (±45°) Composites	202.22 248.63 276.80 253.29	-0.0977 -0.1025 -0.1090 -0.1097	0.9955 0.9900 0.9991 0.9933
Cross Ply (0/90°) Composites	333.40 371.55 343.25 322.30	-0.0996 -0.1139 -0.1109 -0.1110	0.9878 0.9922 0.9964 0.9946

R^2: Coefficient of Determination

Fatigue data of composites as (S_a-N_f) is shown in Fig. 6 and Fig. 7. Fatigue property is reviewed as fatigue strength reduction is steep at lower cycle fatigue at $N_f = 10^3$ cycle both of them.

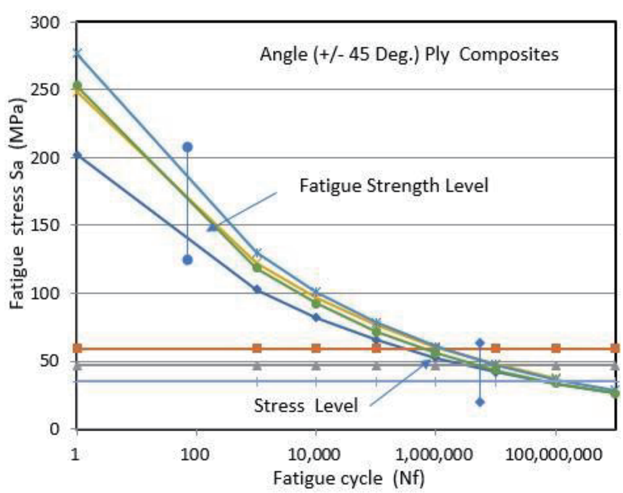

Fig. 6 Fatigue Strength of Angle Ply composites

For initial fatigue strength, fatigue strength of angle ply composite is higher than one of cross play and as long as high cycle fatigue is increased fatigue strength is shown steeply reduction both.

Fatigue strength distribution was evaluated for fatigue cycle. Mean fatigue strength and standard deviation value was calculated. The calculated data using Excel Spread sheet is shown in Table 3 and Table 4.

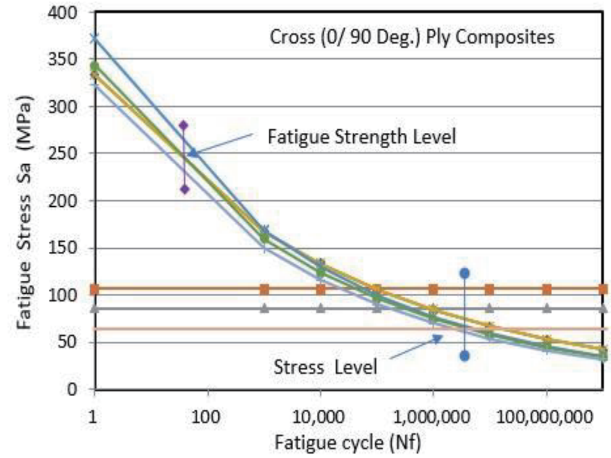

Fig. 7 Fatigue Strength of Cross Ply Composites

Table 3 Mean and Standard Deviation for Fatigue Strength and Reliability of Angle Ply Composites

Test Cycle (N_f) $X10^x$	Angle (±45°) Ply Composites				
	μ_1 [MPa]	σ_1 [MPa]	Reliability(R), (A*), (S_{sf}*)		
			1.2	1.5	2.0
0	245.28	23.47	1.00000000	1.00000000	1.00000000
2	148.01	12.33	1.00000000	1.00000000	1.00000000
3	115.97	7.61	0.99999999	1.00000000	1.00000000
4	91.08	6.43	0.99987580	0.99999998	1.00000000
5	72.59	4.07	0.96171818	0.99996033	0.99999999
6	57.94	3.20	0.39259295	0.96058596	0.99999777
7	45.11	2.01	0.01000045	0.29998869	0.98762392
8	35.31	1.53	$3.66\text{x}10^{-5}$	0.00630731	0.44301684
9	32.33	1.01	$2.26\text{x}10^{-5}$	0.00324077	0.0.2273917

A* : Analytic Method, S_{sf} * : Special Safety Factor

Reliability assessment is applied analytic method and FORM. The result is coincident with each other of ten to the minus 10~12 range. Typical results by SSIT are shown in Fig. 8 and Fig. 9.

Fatigue strength of angle ply composites as interference area at 10^2 cycle with S_{sf} =1.5 is shown in Fig. 8. Fatigue strength of cross ply composites as interference area at 10^2 cycle with S_{sf} =1.5 is shown in Fig. 9.

Table 4 Mean and Standard Deviation for Fatigue Strength and Reliability of Cross Ply Composites

Test Cycle (N_f) $X10^x$	Cross (0/90°) Ply Composites				
	μ_1 [MPa]	σ_1 [MPa]	Reliability(R), (A*), (S_{sf}*) 1.2	1.5	2.0
0	347.45	10.64	1.00000000	1.00000000	1.00000000
2	203.24	6.04	0.99999999	1.00000000	1.00000000
3	158.02	5.53	0.99998235	0.99999999	1.00000000
4	124.02	5.03	0.91249763	0.99992025	0.99999999
5	98.45	4.23	0.20804885	0.89701198	0.99999358
6	74.21	3.84	0.0.0016394	0.10020407	0.89605377
7	59.54	3.63	0.00001081	0.00212456	0.24218508
8	45.79	3.23	0.00000017	0.00000549	0.00442048
9	40.67	1.83	0.00000000	0.00000011	0..00017478

A* : Analytic Method, S_{sf} : Special Safety Factor

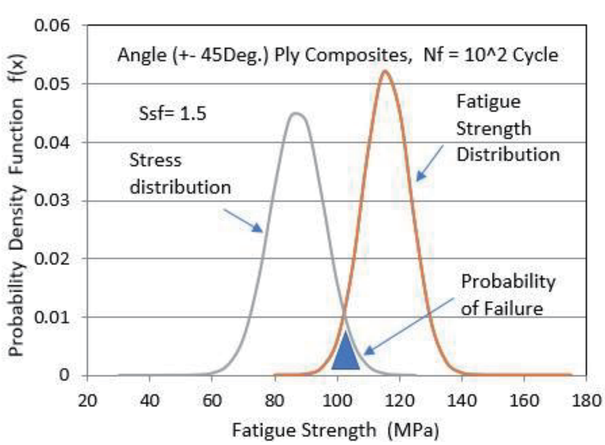

Fig. 8 Stress Strength Interference for Angle Ply Composites

Reliability of angle ply composite is higher than one of cross ply at fatigue cycle of 10^4, It shows that reliability of fatigue strength for cross ply composite at low cycle fatigue is higher than one of angle ply composites, but the effect is reversed at high cycle fatigue. This effect can be derived from the shear strength of composites. Shear strength is dominant effect at high cycle fatigue.

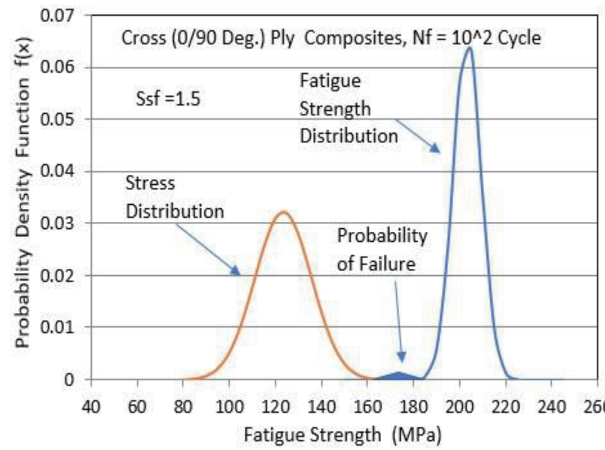

Fig. 9 Stress Strength Interference for Cross Ply Composites

3.2 Reliability of Fatigue Strength for Composites

Reliability analysis of fatigue strength is obtained using mean fatigue strength and standard deviation for each fatigue cycle from SSIT. For calculation of failure probability, $P_f =$ "$NORM.S.DIST(-b, TRUE)$" , Reliability $R = \Phi(b) =$ "$NORM.S.DIST(B, TRUE)$" in Excel spread sheet or $R = 1 - P_f$ was applied. The Reliability result of fatigue strength for special safety factor is shown in Fig. 10 and Fig.11 as well.

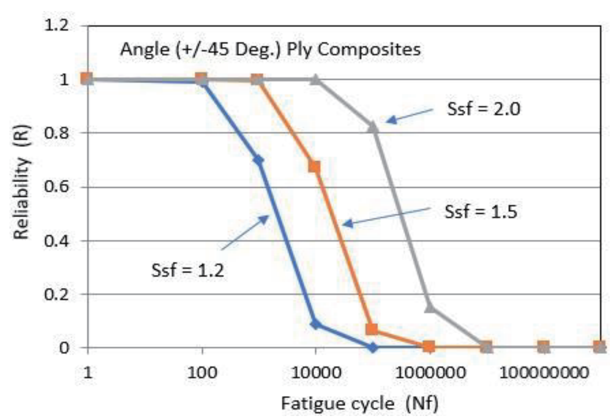

Fig. 10 Reliability of Angle Ply Composites for Special Safety factor

As special safety factor is increased reliability is going to increase, after 10^5 cycle reliability of cross ply shows lower than one of angle ply.

This result is considered as fatigue strength of angle ply composite is higher than one of cross ply composites at high cycle fatigue. It could be reviewed as this effect was generated by shear of composite fiber direction.

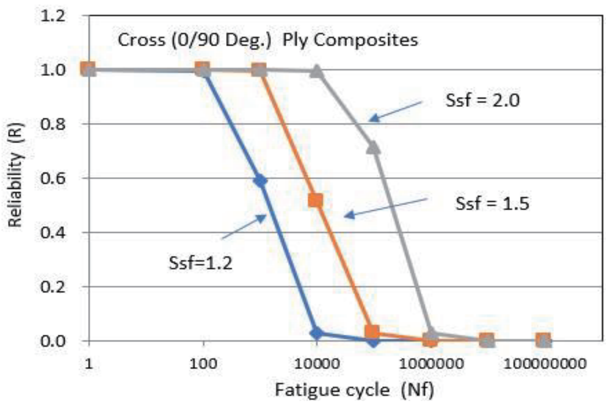

Fig. 11 Reliability of Cross Ply Composites for Special Safety factor

Reliability analysis by analytic and FORM are compared to each other in Table 5 and Table 6. Iteration is stopped when infinitesimal condition is achieved. Reliability by the two methods is coincident with good relations.

Table 5 FORM iteration results for Angle Ply Composites with S_{sf}=1.5 at 10^6 Cycle

I*	R	b^k	$g\ (u^k)$
0	0.9604889481377	0.00000000	10.108565000
1	0.9604889481377	1.756388449	1.3 x 10^{-11}
2	0.9610512273666	1.756388449	0.0446015934
A**	0.9608596489620	β = 1.757526728	

I* : Iteration No., A** : Analytic Method

Reliability of analytical method is close to approximately one of FORM at 2nd iteration. b^2 is going to approached to reliability index β closely at 2nd iteration of S_{sf} =1.5 at 10^6 Cycle in Table 5 of angle ply composite.

Reliability of analytical method is close to approximately one of FORM at 2nd iteration. b^2 is going to be approached to reliability index β closely at 2nd iteration of S_{sf} =1.5 at 10^5 Cycle in Table 5 of cross ply composites.

Table 6 FORM iteration results for Cross Ply Composites with S_{sf}=1.5 at 10^5 Cycle

I*	R	b^k	$g\ (u^k)$
0	0.8970119816523	0.000000000	12.1533639250
1	0.8970119816523	1.264707956	4.1 x 10^{-11}
2	0.8977718574674	1.264707958	0.04083481616
A**	0.8970119852711	β = 1.264707979205	

I* : Iteration No., A** : Analytic Method

3.3 Sensitivity and Stability for Reliability

Reliability analysis is implemented using Eq.(30) and Eq.(40). In those equation $f(-b)$ is as value of the function for probability density of failure in U –space.

The result is obtained by Excel Spread Sheet as "$f(-b) = NORM.S.DIST(-b, FALSE)$". Where reliability index b and u_x are applied of $u^*(u_x, u_y)$. Sensitivity calculation was done for every special safety factor S_{sf} at fatigue cycle N_f. Some of typical value of sensitivity for angle ply composite at given fatigue cycle $N_f = 10^6$ and cross ply composite at given fatigue cycle $N_f = 10^5$ is shown as follows.

$$S_\mu = -f\ (-1.7563)\ \frac{(-0.8356)}{1.7563\ x\ 3.2023} = 1.2680\ x\ 10^{-2}$$

$$S_\sigma = -f\ (-1.7563) \frac{(-0.8356)X(-0.8356)}{1.7563\ x\ 3.2023} = -1.0596X10^{-2}$$

at $N_f = 10^6$ cycle for angle ply composites

$$S_\mu = -\ f\ (-1.2647)\ \frac{(-0.5562)}{1.2647X4.2267} = 1.7930X\ 10^{-1}$$

$$S_\sigma = -\ f\ (-1.2647)\ \frac{(-0.5562)X(-0.5562)}{1.2547X4.2267} = -1.8658X10^{-2}$$

at $N_f = 10^5$ cycle for cross ply composites

Reliability sensitivity was obtained as negative value as long as standard deviation of fatigue strength was increased and positive for mean fatigue strength. Reliability cannot be effective as failure probability is increased since reduced standard deviation of fatigue strength affects to decrease failure probability and increased mean fatigue strength contributes to increase reliability. For the reliability sensitivity as special safety factor S_{sf} is increased, failure probability is decreased.

Stability was calculated using Excel Spread Sheet "$P_f = NORM.S.DIST(-b, TRUE)$" at X-space failure probability by Eq.(41) is shown in Table 7.

Table 7 Cyclic Stability as Probability of Failure between N_f=10^5 and 10^6 Cycle

Ma	S_{sf}	10^5 Cycle	10^6 Cycle
AP*	1.2	3.823 x10^{-7}	3.919 x 10^{-7}
	1.5	4.025 x 10^{-10}	3.951 x 10^{-8}
	2.0	6.510 x 10^{-17}	3.244 x 10^{-12}
CP**	1.2	NA	NA
	1.5	1.029 x 10^{-6}	NA

	2.0	6.516 x 10^{-17}	1.050 x 10^{-7}

AP* : Angle Ply Composites,
CP** : Cross Ply Composites
Ma : Materials, NA : Not-Available

Probability of failure range is concentrated in special safety factor S_{Sf}= 1.2~1.5 both angle ply composite and cross ply composites in range of failure probability of$10^{-7} \sim 10^{-10}$. NA data was deleted since mean fatigue strength is lower than fatigue stress.

4. Results

Reliability of fatigue strength of composite based on ASTM F3114-5 for composite aircraft structure standard. Fatigue test are two series as angle ply(±45°) composites and Cross ply(0/90°) composites. Fatigue strength of composite was referred the existing data. Probability density function was assumed as normal distribution for fatigue strength and stress and special safety factor of ASTM F3114-5 was applied for Reliability assessment. Reliability of fatigue strength of composite using SSIT(Stress-Strength Interference Theory) by analytical method and FORM was assessed. Sensitivity and stability for reliability was evaluated and obtained the result as follows.

1. Static strength of cross ply composite is higher than one of angle ply composites, however fatigue strength of cross play composite is steeply decreased as in high cycle fatigue. Reliability of fatigue strength for cross ply composite is superior to less than 10^3 fatigue cycle and angle ply composite is superior to beyond 10^3 fatigue cycle.
2. Sensitivity of reliability for fatigue strength of composite is contributed when mean fatigue strength is increased and mean standard deviation of fatigue strength is decreased for special safety factor of S_{Sf}= 1.2~1.5 and $N_f = 10^5 \sim 10^6$ cycle range. Stability of composite both are increased as special safety factor increased
3. This result can be applied to reliability design and fatigue design for composite structures of light weight aircraft. Regarding reliability and fatigue for composite structure or component as Helicopter Rotor Blade, this analysis procedure is available to apply it.
4. In future reliability and fatigue design approach of OPPAV(Optionally Piloted Personal Air Vehicle) will be needed to apply as primary composites structure and secondary structures for optimized weight and safety.

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