Nonlinear vibration of nanosheets subjected to electromagnetic fields and electrical current

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Abstract. Graphene Nanosheets play an important role in nanosensors due to their proper surface to volume ratio. Therefore, the main purpose of this paper is to consider the nonlinear vibration behavior of graphene nanosheets (GSs) under the influence of electromagnetic fields and electrical current create forces. Considering more realistic assumptions, new equations have been proposed to study the nonlinear vibration behavior of the GSs carrying electrical current and placed in magnetic field. For this purpose, considering the influences of the magnetic tractions created by electrical and eddy currents, new relationships for electromagnetic interaction forces with these nanosheets have been proposed. Nonlinear coupled equations are discretized by Galerkin method, and then solved via Runge–Kutta method. The effect of different parameters such as size effect, electrical current magnitude and magnetic field intensity on the vibration characteristics of GSs is investigated. The results show that the magnetic field increases the linear natural frequency, decreases the natural frequency of the GSs. Excessive increase of the magnetic field causes instability in the GSs.

Keywords: graphene nanosheets; vibration analysis; nonlinear frequency; magnetic traction; electric and eddy currents

1. Introduction

With the development of nanotechnology (Khodashenas 2015, Rezaee and Maleki 2015, Salimi et al. 2015, Kheradmandan and Barati 2017, Aydogdu et al. 2018), GSs play a very important role in actuators and sensors due to their very special properties. Graphene has become a unique material due to its extraordinary properties such as: thermal, optical conductivity and electrical, high density and excellent mechanical properties (Rezaei et al. 2020). Due to the fact that these materials can have ferromagnetic properties, the use of these plates in a wide range of applications of photonic, electronic and mechanical nanosensors as the most important nanoelectromechanical systems (NEMS), has been considered (Sia 2013, Cicek and Nadaroglu 2015, Ebrahimi et al. 2018, Mohammadian et al. 2019, Le 2020, Attar et al. 2021). One of the newly developed nanostructure is nano-plate, which has various applications in sensors (Tezerjani et al. 2017, Bendaho et al. 2019, Karami and Karami 2019) and actuators (Ponnozhi et al. 2012, Fakhrabadi et al. 2015, Lu et al. 2019). For example, self-powered mass sensor based on electromagnetic nanoplate was investigated for identification of small masses by Asadi et al. (2017). Umar et al. (2021) conducted the fabrication and characterization of CuO nanosheets based sensor device for ethanol gas sensing application. Chen et al. (2019) presented an electro-thermo-mechanical coupling system of micro/nano-scale bistable plates for piezoelectric energy-harvesting applications.

A review of the studies on vibrational behavior of ferromagnetic sheets shows that the major studies in this field are related to metric plates, and very limited studies have investigated the conductive nanosheets. Murmu and Pradhan (2009) applied the nonlocal elasticity theory to investigated the vibration behavior of GSs. Mohammadi et al. (2014) studied the free vibrations of graphene sheets under radial compressive load and temperature changes. In their study, graphene sheets were placed on the elastic substrate, and the nonlocal elasticity theory was used to model the problem. Kumar et al. (2013) investigated the thermal induced vibrations of a monolayer graphene sheet on a polymeric elastic substrate using nonlocal shell theory. The nonlinear vibrational behavior for a rectangular simply supported monolayer graphene sheet in thermal environment is proposed by Shen et al. (2010). Fazelzadeh et al. (2014) analyzed the thermomechanical vibrations of orthotropic nanosheets. Farajpour et al. (2011) studied the buckling of graphene nanosheets on Winkler-Pasternac elastic foundation using the nonlocal elasticity theory. Using the Levy solution model, Samaei et al. (2011) extracted the bending response of graphene monolayer sheets under temperature and external mechanical loads. Pradhan (2009) discussed the buckling behavior of rectangular monolayer GSs using the nonlocal elastic rectangular sheet model and the theory of high-order shear
deformation considering quantum effects. The nonlocal first-order shear deformation elasticity theory is used for buckling of a single-layered graphene sheet embedded in visco-Pasternak by Zenkour (2016). Bouadi et al. (2018) a nonlocal higher order shear deformation theory (HSDT) developed for buckling properties of single graphene sheet. Chandra et al. (2020) suggested a sandwich beam model to investigate the vibrations of bilayer GSs by considering the interlayer shear effect. Tao et al. (2020) based on the higher-order shear deformation theory investigated the nonlinear vibrations of functionally graded graphene platelets-reinforced composite in the thermal loads. Ansari and Ajori (2015) and Ansari et al. (2015, 2016) considered the effects of surface stress on post-buckling modeling, vibration behavior, and instability of a circular nanosheet. Using Kirchhoff’s thin sheet theory, Assadi (2013) analytically investigated the forced vibrations of a rectangular nanosheet by considering the surface effects. Shafiee et al. (2020) studied the small size effect on vibration response and mechanical buckling of single layered GSs based on two variable plate theory. Using the theory of nonlocal elasticity, Naderi et al. (2014) studied the vibration analysis and elastic stability of biaxially loaded GSs. Using the nonlocal elasticity theory, Barretta et al. (2019) considered the small size effect on stress distributions of homogeneous thin nanosheets. They derived the motion equations using classical thin sheet theory, assuming von Kármán’s non-linear displacement-strain relations. Ghadiri et al. (2018) analytically studied the steady-state dynamics of a rectangular GSs resting on a viscoelastic substance under thermo-mechanical- magnetic forces via the Kirchhoff plate model and nonlocal Eringen’s theory. Ajri et al. (2018) analytically analyzed the non-stationary free vibration and nonlinear dynamic behavior of the viscoelastic nano-sheets. Singh and Azam (2020) investigated vibration behavior of a nanosheet using Eringen’s nonlocal plate theory. The governing equations of nanosheet have been derived using the principle of virtual work, and the solution is obtained using the Rayleigh–Ritz method and characteristic polynomials. A multiple-scale perturbation method is employed to analyze nonlinear free vibration of nanoplate incorporating surface effects by Allahyari et al. (2020). Ajri and Seyyed Fakh Rabahi (2018) studied the nonlinear free vibration of viscoelastic nanoplates based on modified couple stress theory. In most conditions of severe environments, when the nanosheet deflection-to-thickness ratio is greater than 0.4, the nonlinearity is very important and should be given consideration. Therefore, the nonlinear free and forced vibration of nanosheet subjected to different loads has given rise to a number of studies (Shen and Huang 2007, Al-Furjan et al. 2021, Salmani et al. 2021).

Most existing studies in the literature examine the dynamic behavior of nanosheets under different types of forces by various numerical and analytical methods (Saremi et al. 2013, Vahidi Pashaki et al. 2018, Ghanadpour and Moradi 2019, Wong et al. 2019, Kachapi 2020, Mehrez et al. 2020). Many works presented in the literature include the vibration behavior of a nanosheets under electrical currents, as well as eddy currents generated by magnetic field. Furthermore, the dynamic behavior of ferromagnetic GSs located under the magnetic fields, has not been studied. Accordingly, in the present study, considering the interaction between ferromagnetic material, magnetic field and magnetic traction generated by electric currents, a new magneto-electro-mechanical coupled equations are provided to study the flexural vibration of ferromagnetic GSs carrying electric current and located in magnetic field. For this purpose, the equations of motion are derived using von Kármán’s nonlinear displacement-strain relations. The solutions obtained by the Galerkin and Runge–Kutta method are presented herein. Using these equations, the nonlinear vibration response of GSs is studied.

2. Equations of motion

According to Fig. 1, a rectangular ferromagnetic GSs with length $a$, thickness $h$ and width $b$ is investigated to carry electrical current along the axial axis ($x$-axis). Amount of the vertical magnetic field and the external force are $B_0$ and $q(t)$, respectively.

2.1 Equations of magnetic fields

Basic equations including Biot–Savart law, Ampere law and Cauchy’s relations are used for linear and angular momentum equilibrium equations to solve the dynamic problems of magnetoelastic systems (Trimarco and Maugin 2001, Chung 2007)

$$\begin{align*}
t_{ij}^{\text{E}} + \rho (f_j - v_j) + p_j^{\text{E}} &= 0, \\
e_{ijk}t_{ij}^{\text{E}} &= c_{kij}^{\text{em}}, B_{il} = 0, e_{ijk}H_{jk} = 0
\end{align*}$$

where $t_{ij}^{\text{E}}$, $p_j^{\text{E}}$, $v_j$ and $c_{kij}^{\text{em}}$ are the elastomagnetic stress tensor, electromagnetical body force, the mechanical body force, velocity vector and electromagnetic momentum, respectively. $e_{ijk}$, $B_i$ and $H_i$ are the permutation tensor, indicate density of magnetic flux and intensity of the magnetic field, respectively. In the present study, $B_l = \mu_0 (H_l + M_l)$, in which $\mu_0 = 4\pi \times 10^{-7} \frac{N}{A^2}$ and $M_l$ are vacuum permeability between the two materials and the magnetization vector, respectively.

Maxwell equations are used to calculate the electromagnetic stresses and the electromagnetic force inside the sheet. According to Maxwell’s classical relations (Arani et al. 2013)

$$\begin{align*}
&[-M_l B_l + t_{ij}^{\text{E}}] n_j = 0, \nabla \cdot B = 0 \\
&\nabla \times H = J_l, n \times [H] = 0, M = \chi_m H, H = \frac{1}{\mu_0} B
\end{align*}$$

in which $t_{ij}^{\text{E}}$, $n$, $J_l$, $\chi_m$ and $\mu_0$ are elastomagnetic stress tensor, unit vector normal to the surface, current density, magnetic field distribution vector, magnetization constant and susceptibility of the ferromagnetic medium ($\chi_m (= 10^2 - 10^3) >> 1$ for linear soft ferromagnetic materials), respectively (Maugin 2013).

Assuming the material to be conductive, polarization for the sheet is zero, and on the other hand, assuming that there
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![Diagram: Ferromagnetic GSs subjected to the magnetic field and electrical current](image)

Fig. 1 Ferromagnetic GSs subjected to the magnetic field and electrical current

is no point charge in the problem, the macroscopic electromagnetic body force, \( \mathbf{F}_{\text{EM}} \), created by the electrical current passing through the GSs subjected to the magnetic field is achieved as

\[
\mathbf{F}_{\text{EM}} = \nabla \mathbf{E} + \mathbf{J} \times \mathbf{B}
\]  

(3)

Since in this study the electromagnetic momentum is not a function of time, the relationship among momentum, electromagnetic stress components and force is given by

\[
\mathbf{F}_{\text{EM}} = \nabla \cdot \mathbf{\tau}_{\text{EM}}
\]  

(4)

Considering the Maxwell and electromagnetic stress tensor as \( \tau_{ij}^{\text{Maxwell}} \) and \( \tau_{ij}^{\text{EM}} \), the electromagnetic stress tensor is achieved as

\[
\tau_{ij}^{\text{EM}} = \tau_{ij}^{\text{Maxwell}} - \tau_{ij}^{\text{Maxwell}}
\]  

(5)

where

\[
\begin{align*}
\tau_{ij}^{\text{Maxwell}} &= \frac{\varepsilon_0 E_i E_j + \frac{B_i B_j}{\mu_0}}{} \delta_{ij} \\
\tau_{ij}^{\text{EM}} &= \frac{\varepsilon_0 E_i E_j + \frac{B_i B_j}{\mu_0}}{} \delta_{ij} - \frac{1}{2} \left[ \varepsilon_0 E_i^2 + \frac{B_i B_j}{\mu_0} - 2M_k B_k \right] \delta_{ij}
\end{align*}
\]  

(6)

where \( \varepsilon_0 \) is the vacuum permittivity.

In the ferromagnetic materials, the electricity current generally can be generated by the magnetic field. In these materials, the amount of the magnetic field due to the uniform electrical current applied in the direction of the \( x \)-axis in the rectangular plane can be obtained as

\[
B_i = \begin{cases} 
\frac{h}{\mu_0 J} & z > \frac{h}{2} \\
\frac{-h}{\mu_0 J} & z < \frac{h}{2} \\
0 & z = 0 \\
\frac{-h}{\mu_0 J} & \frac{-h}{2} < z < \frac{h}{2}
\end{cases}
\]  

(8)

Therefore, the total magnetic field intensity vector inside the GSs, which is equal to the summation of the magnetic intensity applied to the GSs, \( B_0 = B_0 \mathbf{k} \), and the magnetic intensity generated in the GSs due to the electric current, \( B_j \), is obtained as follows

\[
B = B_0 + B_j = -(\mu_0 J_0 z) j + B_0 k, \quad -\frac{h}{2} < z < \frac{h}{2}
\]  

(9)

Using Eqs. (3) and (9) the resultant magnetization vector is achieved as

\[
M = \chi_m \mathbf{H} = \chi_m \mathbf{B} = \chi_m \left[ -\mu_0 J_0 z j + B_0 k \right]
\]  

(10)

Considering the quasi static state for the problem under study, because the wavelength of electromagnetic waves is much longer than mechanical waves, it is \( \sigma \omega \times B = 0 \) and as a result, the electrical current density is obtained as

\[
J = \nabla \times \mathbf{M} + \sigma (\mathbf{w} \times \mathbf{B} + \mathbf{E}) = J_0 \left( \frac{1 + 2 \chi_m}{1 + \chi_m} \right) i
\]  

(11)

in which \( \sigma \) is the electrical conductivity tensor, \( \mathbf{E} \) is the electric field intensity vector, and since there is no single charge, its value is zero.

2.2 Electromagnetic force and torque

Supposing that the GSs is fully conductive, the magnetic forces acting on the sheet can be obtained by substituting Eqs. (9) and (11) in Eq. (3), and then integrating along the sheet. Therefore, the components of the electromagnetic force acting on the plate due to the property of magnetization and passage of the current through the magnetic field is written as

\[
\mathbf{F}_{\text{EM}} = -B_0 J_0 \frac{1 + 2 \chi_m}{1 + \chi_m} j
\]  

(12)

Since the magnetic traction vector at the bottom and the top of the GSs generates magnetic coupling, Eq. (13) is used to compute the magnetic torques

\[
t_i = t_{ij}^{\text{EM} n_j} = \begin{bmatrix} t_{xx}^{\text{EM}} & t_{xy}^{\text{EM}} & t_{xz}^{\text{EM}} \\ t_{yx}^{\text{EM}} & t_{yy}^{\text{EM}} & t_{yz}^{\text{EM}} \\ t_{zx}^{\text{EM}} & t_{zy}^{\text{EM}} & t_{zz}^{\text{EM}} \end{bmatrix} n_j = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}
\]  

(13)

in which \( n_j \) is the normal vector to the GSs, which is obtained as

\[
n = -\left( \frac{\partial \omega}{\partial x} \right) i - \left( \frac{\partial \omega}{\partial y} \right) j + \left( 1 - \frac{1}{2} \left( \frac{\partial \omega}{\partial x} \right)^2 + \left( \frac{\partial \omega}{\partial y} \right)^2 \right) k
\]  

(14)

The electromagnetic traction forces at the bottom and top of the GSs causes the magnetic couplings. By calculating the tractions generated at top and bottom of the GSs by Eq. (13), the magnetic couplings created on the ferromagnetic GSs can be calculated as

\[
C_x = \begin{cases} 
top \frac{h}{2} + bottom \frac{h}{2} & C_y = \begin{cases} top \frac{h}{2} + bottom \frac{h}{2} \\
\end{cases}
\end{cases}
\]  

(15)

2.3 Theory of nonlocal elasticity

In the nonlocal elasticity theory, the stress at any desired
point is a function of the strain field, and the relationship between strain and stress is written as

\[ t_{ij}(x) = \iiint \alpha |x' - x| \tau C_{ijkl} \epsilon_{ij}(x') dv(x') \]  

where \( t_{ij} \), \( \epsilon_{ij} \), \( C_{ijkl} \), \( |x' - x| \) are the nonlocal stress tensor, the strain tensor, and the fourth-order elasticity tensor, the Euclidean form of distance, respectively. \( \alpha \) is the nonlocal kernel function, which function as the internal characteristic size. According to Eringen’s nonlocal elasticity theory

\[ (1 - \mu \nu^2) t_{ij} = C_{ijkl} \epsilon_{ij} \]  

where \( \mu = (\sigma_0 l)^2 \) and \( \nu \) are the nonlocal parameter and the Laplace operator. Using Eq. (17), the structural relationships are expressed as

\[ \begin{bmatrix} t_x \\ t_y \\ t_{xy} \end{bmatrix} = \begin{bmatrix} E & \nu E & 0 \\ \nu E & E & 0 \\ 0 & 0 & G \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} \]  

where \( G \), \( E \) and \( \nu \) are the shear modulus, Young modulus and Poisson ratio, respectively.

### 2.4 Equations of motion

In order to derive the equations governing the flexural vibrations, the classical plate theory and the von-Kármán nonlinear theory are used. Accordingly, plate displacement fields are given in terms of mid-plane deformations as

\[ u_1(x, y, z, t) = u(x, y, t) - z \frac{\partial w(x, y, t)}{\partial x}, \]

\[ u_2(x, y, z, t) = v(x, y, t) - z \frac{\partial w(x, y, t)}{\partial y}, \]

\[ w(x, y, z, t) = w(x, y, t) \]

where, considering the coordinate system in the mid-plane of the GSs, \( u, v \) and \( w \) are the displacements components of the GSs center plane in the direction of the axes \( x, y \) and \( z \), respectively. Therefore, the displacement-strain relationships are as

\[ \epsilon_x = e_x - z \chi_x \]  

\[ \epsilon_y = e_y - z \chi_y \]  

\[ \epsilon_{xy} = e_{xy} - z \chi_{xy} \]

where

\[ e_x = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2, \]

\[ e_y = \frac{\partial v}{\partial y} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2, \]

\[ e_{xy} = \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right), \]

\[ \chi_x = \frac{\partial^2 w}{\partial x^2}, \chi_y = \frac{\partial^2 w}{\partial y^2}, \chi_{xy} = \frac{\partial^2 w}{\partial x \partial y} \]

Furthermore, the stress tensor is obtained as

\[ t_{ij} = t_{ij}^M + t_{ij}^E \]  

where \( t_{ij}^E \) and \( t_{ij}^M \) are the electromagnetic stress tensor and mechanical stress tensor, respectively. By assuming the studied sheet to be isotropic and elastic (Maugin 2013)

\[ t_{ij}^M = C_{ijkl} \epsilon_{ij} = \lambda \epsilon_{kk}^M + 2\mu \epsilon_{ij} \]  

where \( G = \frac{E}{2(1+\nu)} \) and \( \lambda = \frac{E\nu}{(1-2\nu)(1+\nu)} \) are the Lamé constants. By using the electromagnetic and mechanical stress from Eq. (5), the total stress tensor is expressed as

\[ t_{ij} = C_{ijkl} (\epsilon_{ij}^M + \epsilon_{ij}^E) - B_i M_j \]

where \( \epsilon_{ij}^M \) and \( \epsilon_{ij}^E \) are mechanical and electromagnetic strain tensors, respectively. Electromagnetic strains are small compared to elastic strains and can be neglected, therefore

\[ \epsilon_{ij} = \epsilon_{ij}^M = \frac{1 + \nu}{E} t_{ij}^M - \frac{\nu}{E} t_{kk}^M \delta_{ij} \]

Considering an element of the ferromagnetic sheet, the resultant shear forces and bending moments can be obtained as follows

\[ \begin{bmatrix} N_{xxy}, N_{yxy}, N_{xyy} \end{bmatrix} = \int \frac{\alpha}{z} (t_{xx}, t_{yy}, t_{xy}) dz, \]

\[ \begin{bmatrix} M_{xx}, M_{yxy}, M_{xyy} \end{bmatrix} = \int \frac{\alpha}{z} (t_{xx}, t_{yy}, t_{xy}) dz, \]

Considering the small size effect and the theory of nonlocal elasticity

\[ (1 - \mu \nu^2) \begin{bmatrix} N_{xx} \\ N_{xy} \\ N_{yy} \end{bmatrix} = K \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 - \nu \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial x} + \frac{1}{2} (\frac{\partial w}{\partial x})^2 \\ \frac{\partial v}{\partial y} + \frac{1}{2} (\frac{\partial w}{\partial y})^2 \\ \frac{1}{2} (\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y}) \end{bmatrix} \]

\[ (1 - \mu \nu^2) \begin{bmatrix} M_{xx} \\ M_{xy} \\ M_{yy} \end{bmatrix} = -D \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 - \nu & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ \frac{\partial^2 w}{\partial x \partial y} \end{bmatrix} \]

where \( K = \frac{Eh}{(1-\nu^2)} \) and \( D = \frac{Eh^3}{12(1-\nu^2)} \).

Using the Hamilton’s principle, the equations of motion can be expressed as

\[ \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} + t_x^{EM} = m_b \frac{\partial^2 u}{\partial t^2} \]

\[ \frac{\partial N_{xy}}{\partial y} + \frac{\partial N_{yy}}{\partial x} + (F_y^{EM} + t_y^{EM}) = m_b \frac{\partial^2 v}{\partial t^2} \]
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\[ \frac{\partial^2 M_{xx}}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} + \frac{\partial}{\partial x} \left( N_{xx} \frac{\partial w}{\partial x} + N_{xy} \frac{\partial w}{\partial y} \right) + \frac{\partial}{\partial y} \left( N_{xy} \frac{\partial w}{\partial x} + N_{yy} \frac{\partial w}{\partial y} \right) + t_z^E M \frac{\partial^2 w}{\partial t^2} - m_2 \left( \frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{\partial^4 w}{\partial y^2 \partial t^2} \right) \]

where \( m_0 = \int_0^L \int_0^\infty \rho \, dz \) and \( m_2 = \int_0^L \int_0^\infty \left( \frac{\partial C_y}{\partial x} + \frac{\partial C_x}{\partial y} \right) \, dz \) are the magnetic coupling forces created by magnetic tractions, respectively, and \( t_z^E M \) is the forces created by the magnetic tractions. It should be noted that this force component is created due to the presence of magnetic field and electrical current in the GSs, and this term has been not presented in the models reported in previous studies in this field. Using Eqs. (13), (15) and (27), the electromagnetic and electrical tractions, respectively, and the nonlinear differential equations governing the time part \( \frac{\partial}{\partial x} \left( N_{xx} \frac{\partial w}{\partial x} + N_{xy} \frac{\partial w}{\partial y} \right) \) of the deflection for the ferromagnetic nanosheet, which is a second-order differential equation, is obtained as follows

\[ K(1 - \nu) \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{1}{2} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y \partial x} + \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial y^2} + \frac{\partial w}{\partial y} \frac{\partial^2 w}{\partial x^2} \right) \]

\[ + (1 - \mu \nu^2) t_z^E M = 0 \]

\[ \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial y^2} + \frac{\partial w}{\partial y} \frac{\partial^2 w}{\partial x^2} \right) \]

\[ + (1 - \mu \nu^2) (F_y^E + t_y^E M) = 0 \]

\[ D \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = (1 - \mu \nu^2) \left( \frac{\partial C_y}{\partial x} + \frac{\partial C_x}{\partial y} \right) + \frac{\partial^2 w}{\partial x^2} \frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial y^2} \frac{\partial w}{\partial x} \right) \]

\[ + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} - (1 - \mu \nu^2) t_z^E M \frac{\partial w}{\partial x} \]

\[ + (1 - \mu \nu^2) m_2 \left( \frac{\partial w}{\partial x^2} + \frac{\partial w}{\partial y^2} \right) \]

3. Solving the governing equations

In order to solve the differential governing equations which are nonlinear coupled equations, the Galerkin decomposition technique is established. Considering the simply supported boundary conditions, the displacement fields are expressed as (Amabili 2006)

\[ u(x, y, t) = U(t) \cos(\alpha x) \sin(\beta y) \]
\[ v(x, y, t) = V(t) \sin(\alpha x) \cos(\beta y) \]
\[ w(x, y, t) = W(t) \sin(\alpha x) \sin(\beta y) \]

where the functions \( U(t) \), \( V(t) \) and \( W(t) \) are generalized coordinates. By applying the Galerkin method, \( u(t) \) and \( v(t) \) are obtained from Eqs. (40) and (41) in terms of \( W(t) \). By substituting this function in Eq. (42), the nonlinear differential equation governing the time part of the deflection for the ferromagnetic nanosheet, which is a second-order differential equation, is obtained as follows

\[ \frac{\partial^2 w}{\partial t^2} + a_1 W(t) + a_2 W(t)^2 + a_3 W(t)^3 = \lambda \sin(\omega t) \]

where the coefficients \( a_i \) are the constants from the stiffness matrices components and the shear geometric derivatives. The spectrum analysis is conducted by short-time Fourier transform (STFT). The short-time Fourier transform allows to perform time-frequency analysis and obtaining the frequencies of nonlinear systems. Despite the FFT method, STFT method breaks up the signal in time domain to a number of signals of shorter duration, then transforms each signal to frequency domain (Allen and Rabiner 1977). For this purpose, after obtaining the numerical solution of Eq. (44) by Runge–Kutta methods, the time response of the system is extracted, and by applying the STFT, the spectrum response of the system is obtained in the frequency domain. Since, the frequency
curve peaks represent the frequencies in the time response, it can be used to extract the nonlinear frequency values of the system for various parameters. Then, by using the numerical solution of Eq. (44), the effect of various parameters on the dynamic response and vibration characteristics of electrically conductive sheets exposed to magnetic field are studied.

4. Numerical results

In this section, the vibrational behavior of GSs carrying electric current and located in magnetic field is investigated. First, the results are validated, and then the effect of different parameters on linear and nonlinear natural frequencies is studied. Considering linear terms and eliminating nonlinear terms from the GSs equations of motion, the linear natural frequencies of the system can be obtained. In Table 1, the obtained natural frequencies of local and nonlocal models for square GSs are compared with the reference results (Wang et al. 2015). According to this table, it can be seen that the results obtained from this study are completely consistent with the reference results (Wang et al. 2015), and this shows the high accuracy of the current method. Also, in Table 2, the natural frequency of a simply supported monolayer GSs (\( E = 1.02 \) TPa, \( h = 0.33 \) nm, \( \rho = 2300 \) kg/m\(^3\), \( a = b = 10 \) nm) have been compared with the exact solution presented in Refs. (Pradhan and Phadikar 2009, Kitipornchai et al. 2005). As it can be seen, the numerical results obtained from this study have acceptable consistency with the solutions obtained from the exact solution.

In the next, effect of various parameters on the linear and nonlinear natural frequencies of the system is investigated. The numerical values used to extract the results are given in Table 3. Also, dimensionless parameters, including dimensionless linear natural frequency \( \Omega_L \), dimensionless nonlinear natural frequency \( \Omega_{N_L} \), dimensionless magnetic field intensity \( \beta_B \), and dimensionless electric current \( \beta_f \) are defined as follows

\[
\Omega_L = \frac{\omega^2}{\pi^2} \sqrt{\frac{m_0}{D}} \beta_B, \quad \beta_B = \frac{B}{\sqrt{\frac{\varepsilon_0 D}{\mu}}}, \quad \beta_f = \sqrt{\frac{\rho \omega^2}{D}} \beta_f
\]  

(45)

Fig. 2 depicts the effect of the electrical current and magnetic field intensity on dimensionless linear natural frequency of the GSs for different values of \( \mu \). Based on the results, it is observed that the magnetic field increases the linear natural frequency of the GSs, and the amount of this increase intensifies with increasing the magnetic field. As can be seen, for values of \( \beta_B \) less than 18, the magnetic fields have little effect on increasing the natural frequency, and in these areas the effect of the magnetic field can be ignored. In contrast, as \( \beta_B \) increases, the natural frequency of the system increases sharply. As an instance, for \( \beta_B = 40 \) natural frequency increases by about 5 times the GSs frequency in the absence of magnetic field. In the case of electric current, the same behavior is observed, with the difference that the effect of electric current on the increase of natural frequency is relatively less than the effect of magnetic field.

Table 1 Comparison of frequency ratios for square nanosheets

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>( \frac{\omega_{non}}{\omega_{loc}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref. (Wang et al. 2015)</td>
<td>Present results</td>
</tr>
<tr>
<td>0</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.5</td>
<td>0.9762</td>
</tr>
<tr>
<td>1.0</td>
<td>0.9139</td>
</tr>
<tr>
<td>1.5</td>
<td>0.8321</td>
</tr>
</tbody>
</table>

Table 2 Comparison of the natural frequencies of the simply supported GSs with exact solution

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Present result</th>
<th>Exact(^a)</th>
<th>Present result</th>
<th>Exact(^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m ) (nm)</td>
<td>10</td>
<td>( v )</td>
<td>0.186</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b ) (nm)</td>
<td>10</td>
<td>( \mu ) (nm)</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( h ) (nm)</td>
<td>1.5</td>
<td>( \mu_0 ) (H/m)</td>
<td>( 4\pi \times 10^{-7} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho ) (kg/m(^3))</td>
<td>1062.5</td>
<td>( \chi = )</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E ) (TPa)</td>
<td>1.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Fig. 2 Influence of electrical current and magnetic field intensity on dimensionless linear natural frequency of the GSs for different values of \( \mu \)](image-url)
In the following, the behavior of nonlinear vibrations of the simply supported GSs is investigated. The dynamic response along with the frequency response curves and the phase curve of the GSs for different values of $\beta_B$ (i.e., 0.01, 0.02 and 0.023) are shown in Fig. 4. As shown, for very small values of $\beta_B$, the behavior of the system is linear, and the oscillations frequency is approximately equal to the normal frequency of the linear system. In contrast, with increasing $\beta_B$, the coefficient of nonlinear terms becomes larger, and the nonlinear behavior of the system is amplified, which is clearly visible due to the creation of second- and third-order super-harmonics in the frequency function of Fig. 4(b). Fig. 4(b) shows that for $\beta_B = 0.02$ and $\beta_B = 0.023$, the resonance frequency of the system decreases by about 12% and 52%, respectively, compared to the normal frequency of the GSs in the absence of magnetic field. Considering that the magnetic field reduces the natural frequency, it can be said that the magnetic field causes the softening behavior, and as a result, excessive increase of the magnetic field can cause instability in the system. This can be seen in Fig. 5, where the dynamic response of the system for $\beta_B = 0.025$ is demonstrated. As shown, for the critical $\beta_B = 0.025$, the amplitude of the system vibrations increases over time, and tends towards very large values. Due to the fact that in such conditions, the motion of the system is oscillating, the intensity of the critical magnetic field will cause dynamic instability.

Fig. 3 Effect of aspect ratio on natural frequency of the simply supported GSs

Fig. 4 Dynamic response, frequency response curve and phase curve of monolayer GSs for different values of $\beta_B$ (a) = 0.01; (b) 0.02; and (c) 0.023
general, the equivalent stiffness of the structure can be simulated using a linear spring and a local parameter 0.8 nm increases. Based on this, it can be stated that the equivalent stiffness of the structure by increasing the equivalent stiffness of the structure. As an example, for \( \beta_J = 0 \), an increase of \( \mu \) from zero to 2, increases \( \beta_F \) by about 6%. In addition, it is observed that the electrical current also reduces the intensity of the critical magnetic field. Increasing \( \beta_J \) from zero to 1 decreases the critical \( \beta_F \) by about 34%.

In order to investigate the behavior of electro-magnetic coupling of GSs, which carries electric current, and is exposed to magnetic field and external harmonic force, the frequency response of the system in steady state has been exposed to magnetic field and external harmonic force, the coupling of GSs, which carries electric current, and is exposed to magnetic field also increases. Based on this, it can be stated that the nonlocal parameter increases the stability range of the GSs by increasing the equivalent stiffness of the structure. As an example, for \( \beta_J = 0 \), an increase of \( \mu \) from zero to 2, increases \( \beta_F \) by about 6%. In addition, it is observed that the electrical current also reduces the intensity of the critical magnetic field. Increasing \( \beta_J \) from zero to 1 decreases the critical \( \beta_F \) by about 34%.

In order to investigate the behavior of electro-magnetic coupling of GSs, which carries electric current, and is exposed to magnetic field and external harmonic force, the frequency response of the system in steady state has been extracted. Fig. 7 shows the maximum amplitude of GSs oscillations in terms of external harmonic frequency for different values of \( \beta_F \).
oscillations in terms of external harmonic force frequency for different values of $\beta_R$. As the results show, due to the nonlinear behavior in the system in certain areas of the excitation frequency, the jump phenomenon occurs. In the absence of a magnetic field, it occurs with increasing frequency of excitation of resonant regions in the system. Resonances are created in the vicinity of 9 and 11, and when this parameter reaches to 10.05, the jump phenomenon occurs, and the behavior of the system becomes unstable. Another conclusion that can be seen from Fig. 7 is that the magnetic field has significant effect on the frequency-amplitude curve of these systems. By increasing the parameter $\beta_R$, the jump phenomenon is removed from the system behavior. For $\beta_R = 5$, the jump phenomenon is observed in the region of $\omega = 10.6$ and, with increasing the value of this parameter, the maximum amplitude of fluctuations is sharply reduced, and resonant regions are not observed in the dynamic behavior of the system.

5. Conclusions

In the present work, the vibration behavior of simply supported graphene nanosheets was studied by considering the interactions between the displacement fields and magnetic field created forces. Considering the influences of the magnetic tractions created by electrical and eddy currents, new mathematical relationships were presented for the interactions between electromagnetic fields with ferromagnetic materials. The nonlinear governing motion equations were extracted using the nonlocal classical plate theory considering von Kármán nonlocal theory, and then discretized using the Galerkin method. After solving the equations numerically, effect of different parameters on magnetic field cannot be analyzed by linear theories. In order to achieve acceptable results, the effects of geometric nonlinearity should be considered in the equations.

References


Saremi, M., Saremi, M., Niazi, H. and Goharrizi, A.Y. (2013),...


