The investigation of Magnetohydrodynamic nanofluid flow with Arrhenius energy activation

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Abstract. In this article, an analytically and numerically 3D nanoliquid flow by a porous rotatable disk is presented in the presence of gyrotactic microorganisms. The mathematical model in the form of partial differential system is transmuted into dimensionless form by utilizing the appropriate transformation. The homotopy analysis approach is applied to attain the analytic solution of the problem. The effect of promising parameters on velocity distribution, temperature profile, nanoparticles volume fraction and motile microorganism distribution field are evaluated through graphs and in tabular form. The existence of Brownian motion and thermophoresis impacts are more proficient for heat transfer enhancement. Further the unique features like heat absorption/generation and energy activation are also examined for the present flow problem. The obtained results are compared with the earlier investigation to check the accuracy of present model.

Keywords: nanoparticles; rotating disk; Arrhenius energy activation; bio-convection; MHD; HAM

1. Introduction

Rotating disk bio-reactors are important for various chemical engineering and medical phenomenons such as swirl-assisted pumping, chromatography, transfer of oxygen and purification. The advancement in the field of bio-engineering embraced new phenomenon such as bio-convection of microorganisms (oxy-tactic, photo-tactic, gyrotactic, magneto-tactic etc). Nanofluids show great thermal ability for example nano-particles are mixed with base fluids that show large number of applications in technological processes. Now a days, the investigators are engaged in investigating the flow of fluid across a rotatable disk. Because of its unique applications in various areas of science like engineering branches and aeronautical science such as electrical appliances, processes of crystal growth, computer storage equipments, rotatable machinery, medical instruments, vacuum cleaner instruments, gas turbine rotors and others (Hu et al. 2015, Hu 2016, Wang et al. 2017).

Firstly, Von Karman (1921) investigated the innovative work on flow of fluid through a rotatable plane. He applied momentum- integral procedure to investigate the resulting outcomes analytically. Impact of Suction/ injection in Von Karman problem was analyzed by Ackroyd (1978) and he found series solution comprising decaying parameters exponentially. Further, Miclavcic and Wang (2004) made an extension of the Von Karman problem where surface of disk accepts slip conditions. After the Von Karman’s work, many researchers examined the interesting features of rotatable disk by using different conditions. Aziz et al. (2018) considered the MHD stream of nanoliquids by pivoting plate which revolves around its own axis with fractional slip. Gholiniia et al. (2019) analyzed the Powel-Eyring nanoliquid flow through a rotatable disk. They noted that temperature distribution boosted with the increase of thermophoresis parameter and shows decline behaviour with an increase of Prandtl number. Ahmed et al. (2019) analyzed the thin film flow of Maxwell nanoliquid through a rotatable disk in the presence of a non-linear thermal radiation and magnetic parameter. They observed that film thickness shows downturn with enhancing values of the magnetic parameter.

A few more fascinating work in this direction can be seen in Hafeez et al. (2020), Zaho et al. (2020), Shah et al. (2019). A nanofluid is a fluid that consist of petty gravimetric analysis of nanometer scale i.e., less than 100 nm particles is known as nanoparticles. These nanoparticles are commonly made from oxides, carbides, carbon nanotubes and metals. Different base fluids such as glycol, ethylene, oil and water are extremely utilized in different
industrial procedures for heat transfer. Outstanding heat transport capacity is basic need in various engineering and industrial procedures such as glass fiber, production of papers, power generation and chemical procedures etc. Anyhow, the low conductivity of such type of base liquids is not plentiful to obtain the maximum thermal conductivity. The nanoparticles are used to increase the straight heat transport in various nuclear reactors, transportation, biomedicine, food and industrial fields etc. In different procedures, nanoliquids with greater thermophysical properties are utilized as potential heat transport liquids to obtain the excellent performance of heat transport. Moreover, nanofluids are utilized in engine cooling, cooling in machines, chillers, enhancement in heat transfer capacity of domestic refrigerators and freezers, heat transfer appliances, electronics cooling, space etc. Wang et al. (2019) studied the novel multi-segment mirror hybrid solar concentration photovoltaic/thermal (CPV/T) system using the spectral beam splitting technology and its composition, working principle and structural design method are introduced in this paper. The Needle optimization method is employed to design the spectral beam splitter for the CPV/T system. The Monte Carlo Ray Tracing method is used to simulate the solar concentrating process. Choi (1995) was the first who introduced a unique type of fluid called nanofluid during his research work on new cooling technologies and coolants. Analytic model for convective transfer in nanoliquids was analyzed by Buongiorno (2006). In this investigation he observed that there are seven feasible mechanisms that relate nanofluids convection by movement of nanoparticles in the base liquid. These include size of nanoparticle, Effect of magnus, inertia, Brownian motion, particle agglomeration, thermophoresis and volumetric fraction of nanoparticle. In all these above mechanisms thermophoresis and Brownian motion are very essential. The thermophoresis works across temperature gradient, in order to make, the particles movement from area of higher temperature to the area of lower temperature. And, due to Brownian motion particles tend to move from area of higher concentration to area of lower concentration. Yu et al. (2020) investigated the NO\textsubscript{2} degradation performance of nano-TiO\textsubscript{2} as a coating material for the road environment was evaluated in this research. The nano-TiO\textsubscript{2} coating materials for both road surface and roadside were prepared by using anatase nano-TiO\textsubscript{2}, activated carbon powder, silane coupling agent and deionized water. The impact of varying amounts of coating material and silane coupling agent were evaluated. Rashidi et al. (2013) have deliberated that the flow of nanofluid through a rotatable permeable disk with entropy generation and magnetic parameter. Sheikholeslami et al. (2014) noted transfer of heat and boundary layer flow of nanofluid by a rotatable disk. Guo et al. (2020) designed a high-efficiency sensing system for α-synuclein (α-Syn) oligomers was designed based on a novel one dimensional (1D)/2D structural nanohybrid (denoted as CoMnZIF® CNF) of CoMn-based zeolitic imidazolate framework nanosheets (CoMnZIF NSs) vertically grown around carbon nanofibers (CNFs). CNFs were prepared by calcining electrospun polycrylonitrile under Ar/H\textsubscript{2} atmosphere and used as the template for CoMnZIF synthesis. Sheikholeslami et al. (2020a) studies the swirl generator and four-lobed pipe in a solar collector to achieve higher performance not only in view of cooling rate but also available energy. In outputs, components of irreversibility were illustrated for different values of revolution (N), pumping power (Re (Reynolds number)) and width of tapes (D* (diameter ratio)). Four functions were scrutinized namely, Xd (exergy loss), S gen, th (thermal irreversibility), S gen, f (frictional irreversibility), Φ s (Augmented irreversibility). Turkylmazoglu (2014) discussed Magnetohydrodynamic nanofluid flow and transfer of heat through a rotatable disk. Khan et al. (2020) analyzed the Magneto rotatable flow of hybrid nanofluid along entropy generation. Li et al. (2002) explored the higher requirements for the performance of cement-based materials and the call for energy conservation and environmental protection, a wave of research on new materials has set off, and various high-performance concrete and more environmentally friendly geopolymers. With a view to solving the defects of energy consumption, environmental protection and low toughness of traditional cement-based materials, Sheikholeslami et al. (2013) investigated the analytical approach of MHD nanoliquid outflow in a semi-permeable channel. They observed that thickness of velocity boundary layer decline by increasing the value of Reynold number and it shows increase in Hartmann number to attain maximum value. Ma et al. (2019) examined the numerical investigation of MHD nanoliquid natural convection in a U-shape enclosure. Shah et al. (2019) analyzed the effect of non-linear thermal radiations on MHD nanoliquid thin film outflow through a horizontal rotatable disk. Yan et al. (2020) studied the polypyrrole-modified NaTi\textsubscript{3} (PO\textsubscript{4})\textsubscript{3} (NaTi\textsubscript{3} (PO\textsubscript{4})\textsubscript{3}) @ PPy nanocomposite for sodium energy storage via the sol-gel approach and the self-assembly procedure. The physical and electrochemical performances for the resulted NaTi\textsubscript{3} (PO\textsubscript{4})\textsubscript{3} @ PPy have been characterized in this research. Benefiting from the conductive polypyrrole coating, the NaTi\textsubscript{3} (PO\textsubscript{4})\textsubscript{3} @ PPy displays outstanding sodium storage property with the specific capacity of 95.9 mAh g\textsuperscript{-1} at 20 C after 400 cycles. Gabriela and Angel (2020) discussed the entropy generation of hybrid nanoliquid and nanoliquid outflow in thermal structure. Subhani and Nadeem (2019) examined the numerical approach of micropolar hybrid nanoliquid. Few recent investigations related to nanofluids may be viewed in Ghalandari et al. (2019), Tili et al. (2019), Nasiri et al. (2019).

Sheikholeslami et al. (2020b) investigated a clean energy storage unit to decrease the energy consumption through the building. To increase the space between the passing air and the Phase change material (PCM), instead of a smooth channel, a sinuousoidal channel has been considered for the airway passage. Furthermore, porous media and nanoparticles have been applied in this unit for the purpose of enhancing heat transfer. Considering previous research works, the simultaneous use of these three methods for augmenting the performance has not been scrutinized so far. In this study, three mechanisms for augmenting heat transfer and the interaction of these methods to augment the performance of the system have been evaluated. Based on...
comparison with experimental data, outputs are very excellent fit with maximum 9.3% deviation. Magnetohydrodynamics is also known as hydromagnetics which deals with the behavior of electrical conductive fluids and magnetic properties like liquid metals, electrolytes, salt water and plasma. The investigation of magnetohydrodynamic has a large number of applications in fields of engineering and technology. Therefore, numerous investigators represented their research work in Magnetohydrodynamics flows. Sheikholeslami and Farshad (2020) analyzed the nanoparticle turbulent transportation with install of helical tape within a solar system. Increasing radial turbulent fluctuation is main concept of utilizing the turbulator. So, install of such device generates secondary flow and produce thinner boundary layer. So, better mixing happens and consequently, cooling rate augments. With growth of secondary flow with rise of Re (Reynolds number), N (number of revolution) and NT (number of tapes). Zhao et al. (2019) accounted the Heat transport of the MHD nanofluid along permeable microtubes with electrokinetic impacts. Noghrehabadi et al. (2013) has analyzed the natural convection of nanoliquid through a vertical plate embedded in permeable media.

The process of bioconvecton represented the instability and unstructured pattern that is produced by the microorganisms which swims on the upper part of a fluid that posses lesser density. Due to up swimming, these microorganisms included motile micro-organisms such as algae remains in the upper part of the liquid layer and creates unstable top heavy density stratification Pedley et al. (1988). Xun et al. (2017) tackled gravitactic microorganisms used for bio-convection by a rotatable structure in nanoliquid. Saini and Sharma (2018) studied gravitactic microorganisms by using energy method. Chakraborty et al. (2018) analyzed the effects of external magnetic parameter on bio-convection flow which comprises of motile microorganism with convective conditions. Khan et al. (2020a, 2020b) talked about bioconvected nanofluid outflow between two stretching rotatable disks through Entropy generation. Waqas et al. (2019) provided bio-convection flow of second grade nanoliquid (modified) with motile microorganisms along a stretchable surface. It is observed that gyrotactic microorganism distribution decreases with Prandtl number and Brownian movement parameter. Recently some researcher used different methods for nonlinear modeling (Eltaher et al. 2019, Ebrahimi et al. 2019, Safaei et al. 2019, Shahsavarl et al. 2019, Bennmansour et al. 2019).

After revisiting aforementioned papers, the intention here is to study the combined effects of energy activation and motile microorganism in MHD 3D flow of nanoparticles through a porous rotating disk with chemical reaction and convective conditions. Thermophoretic dispersal process and irregular movement occurs due to existence of nanoparticles. The governing mathematical model is tackled by homotopy analysis procedure. Further the characteristics of microorganisms, mass and heat transfer and related parameters present in the equations are examined through plots and tables.

2. Mathematical formulation

We consider three-dimensional MHD nanofluid flow with heat source/sink and immersed motile microorganisms over a porous rotatable disk. Arhenius energy activation and effect of chemical reactions are also considered. Magnetic field acts in perpendicular direction. By using low Reynolds number assumption the impact of induced magnetic field is neglected. The disk is rotatable at \( z = 0 \) with constant angular velocity \( \Omega \). Impact of thermophoresis and Brownian dispersion is also present. Here \((u, v, w)\) are the velocity components in the directions of rising \((r, \phi, z)\) respectively. For present flow problem the governing PDE’s are

\[
\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0
\]

\[
u \frac{\partial u}{\partial r} - \frac{v^2}{r} + w \frac{\partial u}{\partial z} = \left( \frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{r \partial r} - \frac{\partial^2 u}{\partial z^2} \right) - \frac{\sigma B_1^2}{\rho_f} u
\]

\[
\frac{\partial v}{\partial r} + \frac{uv}{r} + w \frac{\partial v}{\partial z} = \frac{\sigma B_1^2}{\rho_f} v
\]

\[
\frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = \frac{\partial^2 w}{\partial r^2} + \frac{\partial w}{r \partial r} - \frac{\partial^2 w}{\partial z^2}
\]

\[
\frac{\partial T}{\partial r} + \frac{w}{r} \frac{\partial T}{\partial z} = a_w \left( \frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} + \frac{1}{r \partial r} \right) + \frac{Q_c}{(\rho C_p)_{f}} (T - T_w) + \frac{Q_c}{(\rho C_p)_{f}} (T - T_w)
\]

\[
\frac{\partial c}{\partial r} + \frac{w}{r} \frac{\partial c}{\partial z} = \frac{\partial^2 c}{\partial r^2} + \frac{1}{r \partial r} \left( \frac{\partial c}{\partial r} \right) + \frac{1}{r \partial r} \left( \frac{\partial c}{\partial r} \right)
\]

The appropriate boundary conditions are

\[
u = r \Omega, \quad w = 0, \quad v = r \Omega, \quad -k_2 \frac{\partial T}{\partial z} = h_2 (T_w - T), \quad -k_3 \frac{\partial c}{\partial z} = h_3 (C_w - C), \quad -k_4 \frac{\partial n}{\partial z} = h_4 (n_w - n) \quad \text{at} \quad z = 0
\]

\[
u \to 0, \quad v \to 0, \quad T \to T_w, \quad C \to C_w, \quad n \to n_w \quad \text{at} \quad z \to \infty
\]

where \(u, v, w\) denotes velocities in \(r, \phi, z\) directions respectively, \(v = \frac{\mu}{\rho_f}\) is kinematic viscosity, \(\rho_f\) is density of base fluid, \(\mu\) is dynamic viscosity, \(k^*\) is porous surface, \(\alpha_m = \frac{k}{\rho C_p}\) is thermal diffusivity, \(k\) is thermal conductivity, \((\rho C_p)_{f}\) is fluid heat capacity, \((\rho C_p)_{f}\) is effective heat capacity, \(T\) is liquid temperature, \(D_B\) the Brownian diffusivity factor, \(D_T\) the thermophoretic
coefficient, \( k \), the rate of reaction, \( C \) is concentration of nanoparticles, \( Q_i \) is heat source/sink coefficient, \( B_m^* \) is microorganism diffusion factor, \( W_i \) the maximum cell-swimming speed, \( \kappa \) is Boltzmann constant, \( C_w, C_{c}, T_w, T_{c}, n_w \) and \( n_c \) denotes the concentration, temperature and density of motile microorganisms which exist at and far away from the stretchable surface.

Following are similarity variables

\[
u = r\sqrt{T} \phi (\zeta), \quad w = -\sqrt{2B} \phi (\zeta), \quad v = r\sqrt{2B} \phi (\zeta),
\]

\[
\zeta = \frac{2\Omega}{v} \eta, \quad \theta (\zeta) = \frac{T - T_w}{T_w - T_c}, \quad \phi (\zeta) = \frac{C - C_c}{C_w - C_c}, \quad \chi (\zeta) = \frac{n - n_c}{n_w - n_c},
\]

where \( \zeta, f(\zeta), g(\zeta), \theta (\zeta), \phi (\zeta) \) and \( \chi (\zeta) \) are similarity variable, dimensionless stream functions, dimensionless temperature, dimensionless concentration and dimension-less micro-organism equation. Equation of continuity trivially holds, while by using Eq. (10) into Eqs. (2)-(9), we obtained

\[
2f'' - Af' + 2f'f'' + 2g^2 - (Ha)^2 f' + \lambda (\theta - N\theta - RbN) = 0 \tag{11}
\]

\[
2g'' - \lambda g + 2fg' - 2f'g - (Ha)^2 g = 0 \tag{12}
\]

\[
\frac{1}{\nu} \phi' + f \phi + \frac{1}{Sc Nb} \theta' - (1 + \alpha \theta) \exp \left( \frac{-E}{1 + \alpha \theta} \right) \phi = 0 \tag{13}
\]

\[
\chi'' + Lbf \chi' - Pe (\phi' (\chi + \delta) + \chi' \phi) = 0 \tag{15}
\]

Associated boundary conditions are

\[
f(0) = 0, \quad f'(0) = 1, \quad g(0) = 1, \quad \theta (0) = -B_{11} (1 - \theta (0)), \quad \phi (0) = -B_{12} (1 - \phi (0)), \quad \chi (0) = -B_{31} (1 - \chi (0)) \tag{16}
\]

\[
f' (\infty) = 0, \quad g (\infty) \rightarrow 0, \quad \theta (\infty) \rightarrow 0, \quad \phi (\infty) \rightarrow 0, \quad \chi (\infty) \rightarrow 0 \tag{17}
\]

where \( (Ha)^2 = \frac{\sigma k_r^2}{\nu T} \) is Hartman number, \( Pr = \frac{v}{\nu m} \) is Prandtl number, \( \lambda = \frac{v}{\kappa k} \) is porosity coefficient, \( Nb = \frac{\mu_k \beta (C_w - C_c)}{\mu (C_w - C_c)} \) is Brownian movement coefficient, \( \gamma = \frac{\rho C_p \nu (T_w - T_c)}{\rho (C_w - C_c)} \) is Thermophoresis coefficient, \( \tau = \frac{Q_i}{2\Omega (C_w - C_c)} \) is heat sink/source coefficient, \( Sc = \frac{v}{\nu m} \) is Schmidt number, \( Lb = \frac{a}{\nu m} \) is bio-convected Lewis number, \( Pe = \frac{bHc}{\nu} \) is bio-convected Peclet number, \( \sigma = \frac{kT^2}{\nu} \) is chemical reaction coefficient, \( E = \frac{a_k}{k T_o} \) is dimensionless energy activation, \( \delta = \frac{T_c - T_{c0}}{T_{c0}} \), \( B_{11} = \frac{B_{11}}{k_2} \frac{v}{2\Omega} \) is Biot number, \( \lambda = \frac{pB(1 - C_{c0}) (T_{w0} - T_{c0})}{\gamma (p_m - \gamma p)(C_w - C_c)} \) is mix convection coefficient, \( N_r = \frac{r_{H}^2}{\beta (1 - C_{c0}) (T_{w0} - T_{c0})} \) is Buoyancy ratio coefficient, \( R_B = \frac{r_H}{\beta (1 - C_{c0}) (T_{w0} - T_{c0})} \) is Bioconvection Rayleigh number.

The dimensionless forms of shear stress of wall, local Nusselt number, local Sherwood and motile microorganisms are

\[
\frac{N_u}{Re_T} = f'' (0), \quad \frac{Re_T}{Re_T} = g'' (0), \quad \frac{N_b}{Re_T} = -\phi (0), \quad \frac{N_m}{Re_T} = -\chi (0) \tag{18}
\]

where \( Re_T = \frac{2r(\Omega)}{\nu} \) denotes the Reynolds number.

### 3. Problem Solution

In order to find the solutions of Eqs. (11)-(15) with boundary conditions (16)-(17), we implement Homotopy analysis method. HAM methodology is useful for the solution of BVPs as it does not consider small or large parameters in case of perturbation methods. This scheme gives an easy way to confirm the solution convergence. Furthermore, it provides an easy method for the selection of base and auxiliary parameters. In this procedure, the auxiliary parameter \( h \) is commonly used to support the problem convergence. The appropriate initial guesses and linear operators are presented in the form

\[
f_0 (\eta) = 1 - e^{-\eta}, \quad g_0 (\eta) = e^{-\eta}, \quad \theta_0 (\eta) = \frac{B_{11}}{1 + B_{11}} e^{-\eta}, \quad \phi_0 (\eta) = \frac{B_{12}}{1 + B_{12}} e^{-\eta}, \quad \chi_0 (\eta) = \frac{B_{33}}{1 + B_{33}} e^{-\eta} \tag{19}
\]

\[
L_f = f'' - f', \quad L_g = g'' - g, \quad L_\theta = \theta'' - \theta, \quad L_\phi = \phi'' - \phi, \quad L_\chi = \chi'' - \chi \tag{20}
\]

As discussed in the above differential operators, contents are presented below

\[
L_f [A_1' + A_2' e^\eta + A_3' e^{-\eta}] = 0, \quad L_g [A_4' e^\eta + A_5' e^{-\eta}] = 0, \quad L_\theta [A_6' e^\eta + A_7' e^{-\eta}] = 0, \quad L_\phi [A_8' e^\eta + A_9' e^{-\eta}] = 0, \quad L_\chi [A_{10}' e^\eta + A_{11}' e^{-\eta}] = 0 \tag{21}
\]

Here \( A_j' \) \( (i = 1, 2, 3, \ldots, 11) \) indicates the arbitrary constants.

### 4. Convergence analysis

By using the homotopy analysis method when the series solutions are evaluated for the velocity, temperature, concentration and microorganism functions, the auxiliary parameters are \( h_f, h_g, h_\phi \) and \( h_\chi \). These major parameters are responsible for the solution convergence. So, related \( h \) -curves are represented in Fig. 2. For such problems the values of appropriate convergence are \(-1.1 \leq h_f \leq 0.1, -1.1 \leq h_g \leq 0.4, -1.6 \leq h_\phi \leq 0.2, -1.5 \leq h_\chi \leq 0.1, -1.6 \leq h_\chi \leq 1.0 \).
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Fig. 1 Visualize the $h$-curves of $f''(0)$, $g'(0)$, $\theta'(0)$, $\phi'(0)$ and $\chi'(0)$

Fig. 2 Visualize the influence of $\lambda$ on $f'$

Fig. 3 Visualize the influence of $\lambda$ on $g$

Fig. 4 Visualize the influence of $Ha$ on $f'$

Fig. 5 Visualize the influence of $Ha$ on $g$
Fig. 6 Visualize the influence of $Nr$ on $f'$

Fig. 7 Visualize the influence of $Rb$ on $f'$

Fig. 8 Visualize the influence of $Ha$ on $\theta$

Fig. 9 Visualize the influence of $\tau$ on $\theta$

Fig. 10 Visualize the influence of $\tau$ on $\theta$

Fig. 11 Visualize the influence of $\lambda$ on $\theta$

Fig. 12 Visualize the influence of $Nb$ on $\theta$

Fig. 13 Visualize the influence of $Nt$ on $\theta$
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Fig. 14 Visualize the influence of $Ha$ on $\phi$

Fig. 15 Visualize the influence of $E$ on $\phi$

Fig. 16 Visualize the influence of $Nt$ on $\phi$

Fig. 17 Visualize the influence of $Nb$ on $\phi$

Fig. 18 Visualize the influence of $Sc$ on $\phi$

Fig. 19 Visualize the influence of $\sigma$ on $\phi$

Fig. 20 Visualize the influence of $Lb$ on $\phi$

Fig. 21 Visualize the influence of $Pe$ on $\phi$
Table 1 Convergence of present results with Sharma and Gupta (2017)

<table>
<thead>
<tr>
<th>Order of approximation</th>
<th>$-f''(0)$</th>
<th>$-\theta'(0)$</th>
<th>$-\varphi'(0)$</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>2.57333</td>
<td>2.56547</td>
<td>0.888235</td>
</tr>
<tr>
<td>3</td>
<td>2.53480</td>
<td>2.52883</td>
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<td>5</td>
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<tr>
<td>20</td>
<td>2.53252</td>
<td>2.53208</td>
<td>0.887102</td>
</tr>
</tbody>
</table>

Table 2 Numerical values of $C_{f2}(0)$ for different values of parameters. When $Pr = Sc = Bi_2 = n = Lb = 1, Nt = \lambda = 0.2, Nb = Ha = Pe = 0.3, B_{i1} = B_{i3} = 0.4, Nr = E = \delta = \Lambda = 0.1, \tau = -0.1, \omega = \sigma = 0.01$

<table>
<thead>
<tr>
<th>$Ha$</th>
<th>$\lambda$</th>
<th>$Nr$</th>
<th>$Rb$</th>
<th>$-f''(0)$</th>
<th>$-g'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.7511</td>
<td>1.1134</td>
</tr>
<tr>
<td>1.0</td>
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<td>0.3</td>
<td>0.7</td>
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<td>1.1123</td>
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<td>0.4</td>
<td>0.5</td>
<td>0.7567</td>
<td>1.1118</td>
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<tr>
<td>0.7</td>
<td></td>
<td></td>
<td></td>
<td>0.7603</td>
<td>1.1113</td>
</tr>
</tbody>
</table>

Table 3 Numerical values of $-\theta'(0)$ for different values of parameters. When $Pr = Sc = 2, Nt = \lambda = 0.2, Nb = Ha = 0.3, B_{i1} = B_{i3} = 0.4, Bi_2 = n = Lb = 1, \tau = -0.1, E = \delta = \Lambda = 0.1, \sigma = \omega = 0.01$

<table>
<thead>
<tr>
<th>$Ha$</th>
<th>$\lambda$</th>
<th>$Nt$</th>
<th>$Nb$</th>
<th>$Sc$</th>
<th>$Pr$</th>
<th>$\tau$</th>
<th>$-\theta'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.2</td>
<td>0.2</td>
<td>0.3</td>
<td>2.0</td>
<td>2.0</td>
<td>-0.1</td>
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5. Results and discussion

The major purpose of this section is to elaborate the various interesting parameters on velocities, temperature, concentration and microorganism profiles. The results are compared with the earlier investigations of Sharma and Gupta (2017).

Figs. 2 and 3 display the influence of porosity parameter on velocity profiles. It is observed that for large values of porosity parameter $\lambda$, the velocities $f'$ and $g$ adopt decreasing trend. Physically, porosity parameter depends on the porous material permeability. In case of large porosity parameter the porous material permeability is weaker. So, the output of this weaker porous material permeability shows the decreasing behavior of $f'$ and $g$. Figs. 4 and 5 interpret that velocities $f'$ and $g$ decrease for large values of Hartman number $Ha$. Here Hartman number $Ha = 0$ denotes the flow case of hydrodynamic and $Ha \neq 0$ represents the circumstance of hydromagnetic flow. The influence of buoyancy ratio parameter $Nr$ and bioconvection Rayleigh number $Rb$ on velocity distribution $f'$ is visualized in Figs. 6 and 7. It is noticed that an increment of $Nr$ and $Rb$, a decrement in velocity distribution is observed. Greater buoyancy ratio parameter is liable for higher buoyancy force because of concentration and that is responsible for decrement in velocity distribution.

Fig. 8 demonstrates the affect of Hartman number on temperature profile $\theta$. It is noted that higher values of $Ha$, the Lorentz force increases which enhance the resistance of fluid flow as a result temperature enhance. The impact of heat source/sink parameter on temperature dispersion is delineated in Figs. 9 and 10. Here $\delta > 0$ shows heat source and $\delta < 0$ for heat sink. For large values of $\delta$, thermal layer and temperature $\theta$ increases. The impact of porosity parameter $\lambda$ on temperature dispersion is expressed in Fig. 11. It is noted that temperature profile $\theta$ increases rapidly when $\lambda$ rises. Physically, the lesser porous material permeability produces corresponding to higher $\lambda$. The lesser porous material permeability corresponds to high temperature. Fig. 12 represents variety in temperature profile for distinct values of Brownian movement $Nb$. Here

**Table 4**: Numerical values of $-\phi'(0)$ for different values of parameters. When $Pr = Sc = 2.0$, $Nt = Ha = \lambda = 0.2$, $Nb = 0.3$, $B_{i2} = n = Lb = 1$, $B_{i1} = B_{i3} = 0.4$, $\tau = -0.1$, $\omega = \delta = \sigma = 0.01$, $Nr = Rb = \delta = \Lambda = 0.1$, $E = 1.0$

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<th>$Nb$</th>
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**Table 5**: Numerical values of $-\chi'(0)$ for different values of parameters. When $Pr = Sc = 2.0$, $Nt = Ha = \lambda = 0.2$, $Nb = 0.3$, $B_{i3} = B_{i3} = 0.4$, $B_{i2} = n = Lb = Pe = Nr = 1.0$, $\omega = \delta = \sigma = 0.01$, $E = \Lambda = 0.1$, $Rb = 0.8$

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temperature profile $\theta$ is boosted directly with $Nb$. Physically, by increasing Brownian motion $Nb$ the sporadic movement of nanoparticles enhances and the promoted diffusion raises the temperature. As a result, kinetic energy is changed into heat energy which shows an increment in related layer thickness and temperature distribution $\theta$. Influence of thermophoresis parameter $Nt$ on temperature distribution is represented in Fig. 13. By increasing values of $Nt$ resulted higher temperature field and enhanced thermal layer thickness.

Fig. 14 indicated the effect of Hartman parameter $Ha$ on concentration field. By enhancing $Ha$, both the concentration layer and concentration profile are increased. Fig. 15 explains the effect of dimensionless energy activation paramerer $E$ on concentration profile. The advancement in energy activation with modified Arrhenius work, inevitably develops generative synthetic reaction to increase concentration profile. Fig. 16 explains that how thermophoresis parameter $Nt$ influence the concentration distribution. For large values of $Nt$, the thickness of related layer at the boundary for concentration profile is boosted. The impact of Brownian motion coefficient $Nb$ on concentration field is portrayed in Fig. 17. It is observed that fragile concentration field is generated by utilizing greater Brownian motion $Nb$. From Fig. 18, we noted that for higher values of Schmidt parameter $Sc$, the concentration profile declines. Physically, the Schmidt parameter $Sc$ is inversely proportional to Brownian diffusivity. Enhancing Schmidt parameter yields a lesser Brownian diffusivity. This lesser Brownian diffusivity leads to decline concentration profile. Fig. 19 shows the impact of chemical reaction coefficient $\sigma$ on concentration field. It is noted that increasing this coefficient, concentration profile and thickness of related layer at boundary decreases.

Fig. 20 represents the influence of bio-convection Lewis number $Lb$ on microorganism density profile. A rapid declining behavior is observed for microorganism density profile for higher values of $Lb$. This is because the bio-convection Lewis number $Lb$ is opposite to the diffusion procedure. The impact of Peclet number $Pe$ on motile density profile is shown in Fig. 21. For large values of $Pe$, density profile is reverted. The maximal speed of swimming cell is increased with enhancement in $Pe$. Due to maximal speed of swimming cell the density profile is reduced. To report the values of $g'(0)$ and $f''(0)$ in response to different values of $\lambda$, $Ha$, $Nr$ and $Rb$. Table 2 is constructed. For large values of $Nr$ and $Rb$ the numerical values of $f''(0)$ becomes larger and $g'(0)$ becomes smaller. Table 3 shows the numerical values of local Nusselt number for distinct values of $Ha$, $\lambda$, $Nt$, $Nb$, $Sc$, $Pr$ and $r$. We observed that local Nusselt number decays for large values of $Ha$, $\lambda$, $Nt$ and $Nb$. Table 4 indicated the numerical values of local Sherwood number for various values of $Ha$, $\lambda$, $Nt$ and $Nb$. It is noted that for higher values of $Ha$, $\lambda$ and $Nt$, Sherwood number decreases. Table 5 shows the local motile microorganism for different values of $Lb$, $Pe$, $Nr$ and $Rb$. Local motile microorganism number is increased rapidly with increasing values of $Lb$, $Pe$ and $Nr$.

6. Conclusions

In this study, the characteristics of motile microorganism distribution and MHD three dimensional viscous nanofluid flow through a porous rotatable disc with chemical reaction, Arrhenius energy activation and heat generation(absorption have been examined. Homotopy analysis procedure is implemented to derive the analytic results of physical quantities are evaluated parametrically. For increasing the Hartman number $Ha$, the velocities $f'$ and $g$ decreases while the concentration, temperature and microorganism increases. For large values of Porosity parameter $\lambda$, the velocities $f'$ and $g$ shows decreasing behavior. For higher values of buoyancy ratio coefficient $Nt$ and bioconvection Rayleigh number $Rb$, velocity distribution $f'$ decline. In the presence of thermophoresis constraint shows similar behavior for both temperature and concentration profile. Concentration distribution shows decline behavior for large values of $\sigma$ and $Sc$. By increasing the values of bio-converted pecket number $Pe$ and bio-converted Lewis number $Lb$, the motile microorganism is decline rapidly. The reported observations can be efficiently used in the improvement of transfer of heat devices as well as microbial fuel cells.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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