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# Understanding of Mathematics Terms with Lexical Ambiguity

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The purpose of this study is to explore how mathematics educators understand the terms having lexical ambiguity. Five terms with lexical ambiguity, *leave*, *times*, *high*, *continuous*, and *convergent* were selected based on literature review and recommendations of college calculus instructors. The participants consisted of four mathematics educators at a large Midwestern university. The qualitative data were collected from open-ended items in the survey. As a result of analysis, I provided participants' sentences with five terms showing their understanding of each term. The data analysis revealed that mathematics educators were not able to separate the meanings of the words such as *leave* and *high* when these words are frequently used in daily life, and the meanings in mathematics context are similar with that in daily context. Lexical ambiguity shown by mathematics educators can help mathematics teachers to understand the terms with lexical ambiguity and improve their instructions when those terms should be found in students' conversations.

*Keywords*: lexical ambiguity, mathematical term, leave, times, high, continuous, convergent.
MESC Classification: 97C50, 97E40
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## I. INTRODUCTION

Learning terms is unavoidable in learning mathematics. Language is considered essential in learning mathematical concepts, and without any language, it is impossible to learn mathematics and communicate mathematical ideas with others (Fulmer et al., 2020). Because of the importance of language in learning mathematics, Lemke (1990) argued that understanding how specific language is used in educational and disciplinary contexts is one of students' important goals of learning. He also discussed that most students could not reach at this educational purpose in science education (Lemke, 1990). The situation in

mathematics education is not different with Lemke's (1990) argument about science education. Thus, educators and teachers should focus on teaching and using mathematical terms. In particular, there are many mathematical terms that can obstruct students' learning because their mathematical meanings are similar with or clearly distinguished from daily meaning. I refer such terms to words having *lexical ambiguity*.

In these days, the main philosophical position of learning is considering learning as social activity. According to Windschitl (2002), students participate in social classroom activities with tools such as graphs, pictures, real objects, and languages to learn mathematics. Thus, with the idea that language is very important in a mathematics classroom because social interaction cannot occur without language (Windschitl, 2002). However, previous studies (Fulmer et al., 2020) have shown that most teachers and students did not realize the significance of roles of languages. Therefore, students would have difficulties in using some terms if these terms have the same forms with, but different meaning from ones, which students usually use in daily life. To build students' understanding of mathematics terms from familiar meanings to new mathematical ones, they require to reveal the relationship between mathematical meaning and daily meaning. Hence, research on lexical ambiguity is necessary, and it can provide the guideline for teacher's using words with lexical ambiguity.

## II. LITERATURE REVIEW

Many studies in mathematics education have observed lexical ambiguity itself as well as its effect on students' learning. These studies have considered lexical ambiguity as obstacle to learning mathematics. For this reason, previous studies defined what lexical ambiguity is as well as they found words having lexical ambiguity in actual mathematics classrooms. Fundamental research for lexical ambiguity done by Durkin and Shire (1991) provided their classification of terms with some descriptions (see Table 1). As shown in Table 1, they considered pronunciation, contextual meaning, and shapes of words as what can make ambiguity. Particularly, the two categories – homonymy and polysemy – were underlined in this study because these categories are related with the meaning of terms. Terms in the category of homonymy share the same spelling with different meanings. Compared with homonymy, words in polysemy have different but related meanings between mathematics and everyday uses. If two different terms have the same pronunciation, the terms are defined as the words in homophony (Durkin & Shire, 1991).

In addition, Barwell (2005) examined the effect of lexical ambiguity on learning mathematics. In contrast to the opinion in which lexical ambiguity is problematic, he argued that understanding separate meanings of one word might be unhelpful in learning

mathematics. Moreover, difficulties in learning these words seem to depend on the types of lexical ambiguity. In statistics education, Kaplan et al. (2009) examined meanings of some statistical terms identified as words having lexical ambiguity. As a part of a large research project, their study also had consistent findings with Barwell's (2005) research. In addition, Kaplan et al. (2009) suggested careful use of terms with lexical ambiguity.

In addition to examine lexical ambiguity, it is required that teachers understand what words have lexical ambiguity and how they handle the problems caused by lexical ambiguity. According to Forman (2003), teachers should utilize "revoicing" that is one of teacher's strategies to orchestrate small group discussions. If teachers have lack of understanding about lexical ambiguity in their language use, it could be impossible to do "revoicing" in their instructions. Therefore, studying about teachers' understanding lexical ambiguity is valuable so that students engage in mathematical activities successfully.

The purpose of this study is to reveal lexical ambiguity, which mathematics educators at a Midwestern university have. I expected that the findings can give some implications for using mathematical terms with lexical ambiguity. Direct observation on how mathematics educators define some words will help teachers encourage students to use various terms even with lexical ambiguity. Revealing educators' understanding meaning of some words with lexical ambiguity would provide recommendations in using specific words.

**Table 1.** Categories by Durkin and Shire (1991)

| Category | Description |
|---|---|
| Homonymy | The property of some terms that share the same shape but have distinct meanings. |
| Polysemy | The property of some terms that have two or more different but related meaning |
| Homophony | The phenomenon where two different words have the same pronunciation |
| Shifts of application | Occasions where the same meaning can be considered from different attitude. |

There are a variety of terms known to have lexical ambiguity, but it is required to specify a list of words to be scrutinized. Based on Barwell's (2005) research and experience of researchers, the five words – *leave*, *times*, *high*, *continuous*, and *convergent* – were chosen. *Leave*, *times*, and *high* were selected from the Barwell's (2005) research, *continuous* and *convergent* were suggested by some college calculus instructors. Another

rationale for this selection was to cover wide levels of mathematics. *Leave*, *times* and *high* are usually found in all levels of schools, and *continuous* and *convergent* mainly appear in college mathematics. In addition, these words are categorized into the polysemy, which means that these words have two or more different but related meanings (Durkin, 1991). Lastly, the questions guiding this study are following: when the five words with lexical ambiguity are provided – strongly related common meaning and mathematical meaning – what mathematics educators' definitions are. I aimed to examine how mathematics educators distinguish the daily and mathematical meaning of the given terms.

## III. METHOD

### 1. SURVEY

A survey was designed and distributed to collect qualitative data, which can inform how mathematics educators understood both daily meanings and mathematical meanings of the five selected English words: *leave*, *time(s)*, *high*, *continuous*, and *convergent*. Although Barwell (1991) utilized closed-ended items in his research, I used open-ended items by following the approaches of Kaplan et al. (2009). The survey tool used by Kaplan et al. (2009) was easily applicable as well as analytical. For example, the item used in the work of Kaplan et al. (2009) was "Define the word, 'sample'." However, providing solely one word to participants can lead to their confusion because they cannot select specific meanings among various meanings, which can be induced in contexts. Therefore, each item shows a sentence, which participants can infer meanings of each term. Five sets of two sentences were designed and presented to respondents in order to examine daily meaning and mathematical meanings. Furthermore, each item in the survey requested the participants to make a sentence with the same meaning of the word with that in the given sentence. The sentences that participants created enabled me to collect additional evidence to reveal the clearer meaning if the written meaning by a participant is obscure.

### 2. PARTICPANTS

Four mathematics educators, graduate students in mathematics education, participated in the study. All participants were teaching assistants of mathematics content courses at a Midwestern university. The sample included two Caucasian, one Korean, and one Indonesian Americans. Regardless of how many experiences of teaching mathematics

they have, these educators might have more chances to meet these words having lexical ambiguity. This selection of the participants can provide indirect evidence of lexical ambiguity that teachers can have, including detailed daily meanings and mathematical meanings.

### 3. DATA ANALYSIS

An analysis tool for qualitative data was adopted from the study of Kaplan et al. (2009). Data analysis began with confirmation of the daily meanings based on Oxford English Online Dictionary. In addition, the mathematical meaning of each word was referred to mathematics textbooks and opinions of some teachers. Based on searching the documents, various meanings of each term were collected. The responses were coded with synonyms aligned with the used meanings. Through the first round of the analysis, the collected meanings were modified when some participants wrote too specific meanings or unexpected meanings, which were not found in the document review. Then, another round of analysis was conducted in order to confirm that the codes were valid and reliable to understand lexical ambiguity. Finally, categories of daily meaning and mathematical meaning were compared to find relationship among these meanings derived from the modified framework.

## IV. RESULTS

The analysis results will provide categories of meanings according to the final framework. I will provide the examples of significant finding. In addition, the relationship between everyday meaning and mathematical meaning will be explored per term.

### 1. LEAVE

*Leave* is used to express to have a remainder as well as to depart from ("Leave", 1902). As shown in the survey (see Appendix), the daily meaning of *leave* in the given sentence is "going away from." The mathematical meaning in the sentence "five takes away two leaves three" is aligned to *equal*. Table 2 shows all of the answers that participants gave. As seen in Table 2, the participants provided the same answers that can be represented by the word *depart*. However, only one whose first language is English mentioned *equal* among the responses of mathematical meaning. Others' answers can be understood as *remain*. Moreover, the participants except for one Caucasian American seemed to

consider the mathematical meaning of *leave* as one of its daily meanings. One example from the item making a sentence with the same meaning is "brother wanted sister to leave him alone" although the given sentence is for mathematical use.

**Table 2.** Sentences Including *leave* Created by the Participants

| The sentence in mathematical context | The sentence in daily context |
|---|---|
| If I give you four cookies and I take three that leaves five for whoever wants them. | I have to leave the party at 10:00 tonight. |
| We have five minutes left for finishing your work. | I will leave this town in a couple of days later. |
| I have $10 and spent $6 for lunch, and that leaves me with $5 only. | With a heavy heart, Mary has to leave her baby with the babysitter. |
| Brother wanted sister to leave him alone. | She was excited to leave for her Hawaii vacation. |

## 2. TIME(S)

*Time(s)* also has a number of meanings. In this survey sentence, the daily meaning of *time(s)* is "any one of the occasions on which something is done or happens" by Oxford English Online Dictionary ("Time", 2012). In mathematical context, *time(s)* means multiplication ("Times", 2012). Every participant provided the exact mathematical meaning of *time(s)*. In the case of the daily meaning, only one participant whose first language is not English gave the answer as the same as the mathematical meaning. The sentence generated by this participant was, "I went to Hawkeyes games 5 times this semester", which may be similar with the sentence given in the survey.

## 3. HIGH

The term *high* has the meaning of extending far upward ("High", 1898). In mathematics, there is no difference between large number and high number. For this reason, all participants answered that *high* means *large* for mathematical use. In the case of the daily meaning, one participant wrote "above average" and another gave "a relatively large." The other two mentioned a kind of *tall*. Compared with the meaning given by participants, the sentences had various meanings of *high*. For example, one of the participants wrote the following sentence, "I have a high expectation of you" as use of

the daily meaning." In addition to this sentence, "the lightning strikes at a high frequency", "your blood pressures are so high" were found in the responses for mathematical use of *high*.

**Table 3.** Sentences Including *high* Created by the Participants

| The sentence in mathematical context | The sentence in daily context |
|---|---|
| If I am playing cards, 9 might be considered a high number. | The airplane was flying very high in the sky. |
| Your blood pressure is so high. You should be very careful. | Do you think this tree is as high as that three when it is grown up? |
| The lightning strikes at a high frequency. | I have a high expectation of you. |
| The concert had a high attendance. | That overpass has a high clearance. |

## 4. CONTINOUS

While continuous function in mathematics has a very complicated definition, requiring the notation of infinity, *continuous* usually means extending without interruption of substance or unbroken connection in everyday life ("Continuous", 1893). In this survey, all participants suggested the meaning of *unbroken* or *connected* for the item of mathematics meaning instead of the strict mathematical definition. In the everyday meanings of *continuou*s, the idea of *endless* was found. Examples of participants' sentences with the mathematical term were "time is a continuous variable" and "when do we assume the data can be continuous?" In the case of the daily meaning, an instance was "continuous improvement is very important in this company".

## 5. CONVERGENT

In mathematics, *convergent* is defined as *not divergent*, but *divergent* has a complex definition. One participant gave the exact answer for the mathematical meaning, "not divergent". However, the others provided a sense of *limitation* or *finite* such as "finite limit" and "has a finite amount". In daily situation, *convergent* means one of followings: inclining toward each other or toward a common point of meeting; tending to meet in a point or focus ("Convergent", 1893). All responders provided the answers similar with

the dictionary definition, including meaning of *agreement*. Considering the sentences participants made in mathematical context, the same meaning was used as the meaning in the given sentences such as "this series of real numbers is convergent" and "After solving a problem, I found that the series was convergent because all of the number is the series added up to 1."

## V. DISCUSSION

As seen in the results, most participants considered *leave* as *remain*, not *equal*. The meaning aligned with *remain* is strongly related with the daily meaning. The meaning used in the survey items had another meaning, that is, *depart from*. According to Durkin's (1991) criteria, *leave* in the survey is classified into homonymy by the dictionary definition because the two meanings *depart from* and *equal* have no similarities in their meanings. However, based on the responses, in this study this word can be classified into polysemy because *remain* given mathematical meaning by the participants can be used in daily life. This result indicates that the daily meaning could have significant effects on using *leave* in mathematical contexts although students experience using one of daily meanings before coming to mathematics classrooms and the given meaning had no relation with the mathematical meaning. If a teacher, who frequently uses *leave* in classroom, believes that this effect is problematic for students' learning, the teacher should use this word frequently, but carefully. Then, it can be helpful to discuss the different meanings of *leave* with students explicitly. Otherwise, it is probably beneficial to find way to use this effect for better understanding of the term.

One more interesting finding in this study is participant' sentences which included the daily meanings of the terms even though that the participant was asked to use mathematical meanings to create sentences. This finding implies that the participant might not recognize the mathematical purpose in the given sentence. It is very interesting because that given sentences had the words, which represent the numbers. Therefore, it is possible that teachers can make students confused because the purpose of teacher's language is uncertain to students.

In the case of *time(s)*, all participants clearly differentiated both of the daily meaning and the mathematical meaning. In contrast to *time(s)*, the participants had difficulties to explain their own meanings of *high*. Particularly, a half of the participants provided the words like "high frequency" and "high pressure" when they were asked to use the same meaning with *high* in mathematical context. High frequency and pressure indicate the specific outcomes of measurement usually expressed by high number. Therefore, some participants showed inaccurate understanding about the mathematical meaning of *high*.

Therefore, teachers need to recognize their own meaning of *high* because *high* has a specific form of lexical ambiguity.

As mentioned above, *continuous* and *convergent* appear in more advanced mathematics compared to other terms. Thus, it is possible to have little chance to use these words because they may be more academic than other words in this study. For this reason, the participants seemed less familiar with *continuous* and *convergent* in daily context. Due to this characteristic of these words, the participants knew mathematical meanings correctly, but the meanings cannot be express in intimate way. Thus, participants' ideas were evidently observed in the sentences that they generated. The representative example was "this series of real numbers is convergent."

There were no remarkable differences in the responses of the participants. However, the participants whose first language was not English sometimes showed unexpected answers. For example, one participant mentioned *endless* for the daily meaning of *continuous*. Moreover, one non-American participant mentioned that *time(s)* had the same meaning with *multiply* in both of daily and mathematical meanings. This might be because people, whose first language was not English, sometimes use their mother tongue to understand English words. This means that there are some invisible relationships between two languages in their minds. This relation also can cause difficulties in learning mathematics terms with lexical ambiguity.

## VI. CONCLUSIONS

This study examined lexical ambiguity of the five words. Although all participants in this study were graduate students in mathematics education, they seemed to have confusion with the meanings of the terms. The participants were not able to identify contextual meanings of some words used frequently in daily life. In this case, particularly when those words are polysemy not homonymy in Durkin's (1991) categorization, it is necessary to revisit Barwell's (2005) arguments: difficulties in learning mathematics rely on the types of lexical ambiguity. As shown in discussion, *leave* and *high* can be good examples. Furthermore, if some words appear in advanced mathematics, it would be possible that those terms are more academic and rarely used in everyday lives. Thus, the participants were able to easily recognize the difference of the meanings.

However, this study has some limitations. A small number of mathematics educators participated, and the location of the study is restricted into one university and one graduate major. These questioned to what extent the findings can be generalized. Therefore, additional research with more and various participants in other areas should be conducted. Because this study examined only mathematics educators at the university,

further studies should scrutinize teachers' language in their classroom conversations.

The findings in this study suggested some implications for teacher educators. Teacher education programs have highlighted interactions among students as well as between a teacher and students for better learning mathematics. However, pre-service teachers should recognize what cause students' difficulties to understand mathematics concepts through the interactions in a classroom. For this reason, lexical ambiguity is what pre-service teachers should know not only to understand mathematics concepts by themselves, but also to facilitate classroom discussions appropriately. Thus, teacher educators need to focus on lexical ambiguity explicitly in their programs.

Consequently, lexical ambiguity was observed in this study again. Furthermore, I suggest that it is necessary to help teachers understand the significant of using language and the shape of lexical ambiguity. According to Simon's (1995) model "Mathematics teaching cycle," various meanings of mathematical words can be found in student's previous knowledge (Simon, 1995). And students would be confused in learning or social activities if meanings of each person in activities are different from each other. Therefore, this study can contribute to improving not only better social activities in mathematics classroom but also teachers' understanding of students' previous knowledge. Finally, it is expected to find decreasing difficulties and increasing the efficiency in learning mathematics if further research is conducted.

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# APPENDIX

## Questionnaire for Using Words

I would deeply appreciate your taking time to complete this survey. The purpose this survey is investigating how you think about some words used in mathematics class. Your answers will be used only for the research. Your cooperation will be very thankful as an important source of my research.

**I. Personal Information**

Age: _____

Major Department: _____

The first Languages: _____

**II. Please write the meaning of a bolded word used in each sentence. There are 5 words and 10 sentences. And please make another sentence including that word with the same meaning you think**.

1. *Leave*

a. He would not be allowed to **leave** the city.

Meaning: _____

_____

Sentence: _____

_____

b. Five takes away two **leaves** three.

Meaning: _____

_____

Sentence: _____

_____

2. *Times*

a. I visit the U.S. two **times** a year.

Meaning: _____

_____

Sentence: _____

_____

b. Five **times** two is ten.

Meaning: _____

_____

Sentence: _____

_____

3. *High*

a. They have a **high** wall.

Meaning: _____

_____

Sentence: _____

_____

b. They have a **high** number.

Meaning: _____

_____

Sentence: _____

_____

4. *Continuous*

a. There is a **continuous** stream of cars on the street.

Meaning: _____

_____

Sentence: _____

_____

b. There is a **continuous** function from a set A to a set B.

Meaning: _____

_____

Sentence: _____

_____

5. *Convergent*

a. We need to make **convergent** opinion.

Meaning: _____

_____

Sentence: _____

_____

b. We need to make a **convergent** series of rational numbers.

Meaning: _____

_____

Sentence: _____

_____