On Inflated Achievable Sum Rate of 3-User Low-Correlated SC NOMA

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Abstract

In the Internet of Thing (IoT) framework, massive machine-type communications (MMTC) have required large spectral efficiency. For this, non-orthogonal multiple access (NOMA) has emerged as an efficient solution. Recently, a non-successive interference cancellation (SIC) NOMA scheme has been implemented without loss. This lossless NOMA without SIC is achieved via correlated superposition coding (SC), in contrast to conventional independent SC. However, conventional minimum high-correlated SC for only 2-user NOMA schemes was investigated in the lossless 2-user non-SIC NOMA implementation. Thus, this paper investigates a 3-user low-correlated SC scheme, especially for an inflated achievable sum rate, with a design of 3-user low-correlated SC.

First, we design the 3-user low-correlated SC scheme by taking the minimum sum rate between 3–user SIC NOMA and 3-user non-SIC NOMA, both with correlated SC. Then, simulations demonstrate that the low correlation in the direction of the first user’s power allocation inflates the sum rate in the same direction, compared to that of conventional minimum high-correlated SC NOMA, and such inflation due to low correlation is also observed similarly, in the direction of the second user’s power allocation. Moreover, we also show that the two low correlations of the first and second users inflates doubly in the both directions of the first and second users’ power allocations.

As a result, the proposed 3-user low-correlated SC could be considered as a promising scheme, with the inflated sum rate in the future fifth-generation (5G) NOMA networks.

Keywords: NOMA, 5G, Correlation Coefficient, Superposition coding, Successive interference cancellation, Power allocation.

1. Introduction

The Internet of Thing (IoT) framework has required a massive connectivity, such as massive machine-type communications (MMTC) [1], [2]. For this spectral efficiency, non-orthogonal multiple access (NOMA) has been considered as an appropriate multiple access scheme [3], [4]. Higher spectral efficiency can be a superiority of NOMA, compared with existing orthogonal multiple access (OMA) in 4G cellular communications [5-7]. The superiority of NOMA was optimized [8]. Cooperative NOMA was studied for full-duplex relaying [9]. Underwater visible light communication for NOMA was investigated [10]. A power-outage tradeoff for NOMA was studied [11]. The bit-error rate for NOMA were investigated [12], whereas
local oscillator imperfection in NOMA was considered [13]. The power splitting for correlated superposition coding (SC) NOMA was investigated [14]. For $M$-user NOMA systems, power allocation was studied for first and second strongest channel gain users [15]. In addition, the achievable data rate for NOMA with the asymmetric binary pulse amplitude modulation (2PAM) was studied [16]. Recently, the NOMA schemes were investigated for correlated information sources [17]. Negatively-correlated information sources were investigated in [18]. A rate-lossless non-successive interference cancellation (SIC) NOMA scheme was studied for asymmetric 2PAM [19].

Recently, a non-SIC NOMA scheme has been implemented without loss [20]. Such lossless non-SIC NOMA is achieved via correlated SC, in contrast to conventional independent SC. Note that correlated SC uses superimposed signals which are correlated, whereas in independent SC, the signals are superimposed independently. However, the conventional minimum high-correlated SC scheme for only 2-user NOMA was investigated in the lossless 2-user non-SIC NOMA implementation. Thus, this paper investigates a 3-user low-correlated SC scheme, especially for an inflated achievable sum rate, with a design of 3-user low-correlated SC.

First, we design the 3-user low-correlated SC scheme by taking the minimum sum rate between 3–user SIC NOMA and 3-user non-SIC NOMA, both with correlated SC. Then, simulations demonstrate that the low correlation in the direction of the first user’s power allocation inflates the sum rate in the same direction, compared to that of conventional minimum high-correlated SC NOMA, and such inflation due to low correlation is also observed similarly, in the direction of the second user’s power allocation. Furthermore, we also show that the two low correlations of the first and second users inflates doubly in the both directions of the first and second users’ power allocations.

The remainder of this paper is organized as follows. In Section 2, the system and channel model are described. The achievable sum rate of 3-user low-correlated SC NOMA is derived in Section 3. The numerical results are presented and discussed in Section 4. Finally, the conclusions are presented in Section 5.

The main contributions of this paper are summarized as follows:

- We propose a 3-user low-correlated SC NOMA scheme, in contrast to a conventional 3-user minimum high-correlated SC NOMA scheme.
- Then, we design the 3-user low-correlated SC scheme by taking the minimum sum rate between 3–user SIC NOMA and 3-user non-SIC NOMA, both with correlated SC.
- It is shown that the low correlation in the direction of the first user’s power allocation inflates the sum rate in the same direction, compared to that of conventional minimum high-correlated SC NOMA.
- And such inflation due to low correlation is also observed similarly, in the direction of the second user’s power allocation.
- Moreover, we also show that the two low correlations of the first and second users inflates doubly in the both directions of the first and second users’ power allocations.

2. System and Channel Model

In a cellular downlink NOMA network with one base station and three users, the complex channel coefficient between the $m$th user and base station is denoted by $h_m, m = 1, 2, 3$, and the channels are sorted as
| $|h_1| \geq |h_2| \geq |h_3|$.

The base station can send the superimposed signal $x = \sqrt{P_1 s_1} + \sqrt{P_2 s_2} + \sqrt{P_3 s_3}$, where $P$ is the average power of $x$, $s_m$ is the signal for the $m$th user with the average unit power, and $\alpha_m$ is the power allocation coefficient, with $\alpha_1 + \alpha_2 + \alpha_3 = 1$. The received signal $r_m$ at the $m$th user is expressed as follows:

$$r_m = h_m x + n_m, \quad (1)$$

where $n_m \sim CN(0, \sigma^2)$ is a complex additive white Gaussian noise (AWGN). On the other hand, the base station can also send the correlated superimposed signal $z = \sqrt{P_A \beta_1 c_1} + \sqrt{P_A \beta_2 c_2} + \sqrt{P_A \beta_3 c_3}$, where the average allocated total power $P_A$ of $c_1$ and $c_2$ is given by

$$P_A = \frac{P}{\sum_{i=1}^{3} \sum_{j=1}^{3} \rho_{ij} \beta_i \beta_j}, \quad (2)$$

with the correlation coefficient of the messages’ signals $\rho_{1,2} = E[c_1 c_2^*]$, and $c_m$ is the signal for the $m$th user with the average unit power. $\beta_m$ is the power allocation coefficient, with $\beta_1 + \beta_2 + \beta_3 = 1$. The received signal $y_m$ at the $m$th user is expressed as follows:

$$y_m = h_m z + n_m. \quad (3)$$

### 3. Derivation of Achievable Sum Rate of Proposed 3-User Low-Correlated SC NOMA

In this section, in order to derive the achievable sum rate of the 3-user low-correlated SC NOMA scheme, we first present the achievable sum rate of the existing NOMA schemes [21]: The conditional achievable sum rate given $|h_1|$, $|h_2|$, and $|h_3|$ in conventional 3-user correlated SC/SIC NOMA schemes is expressed as:

$$R_{\text{sum},|h_1|,|h_2|,|h_3|} = R_{1}^{(\text{CSC/SIC})} + R_{2}^{(\text{CSC/SIC})} + R_{3}^{(\text{CSC/SIC})}, \quad (4)$$

where

$$R_{i}^{(\text{CSC/SIC})} = \log_2 \left( 1 + \frac{|h_i|^2 P_i \beta_i (1 - \rho_{i2,3}^2)}{\sigma^2} \right), \quad (5)$$
\[
R_{2|h_2|}^{(CSC/SIC)} = \log_2 \left( \frac{\|h_2\|^2 P_A \left( \beta_1 (1 - \rho_{1,3}^2) + \beta_2 (1 - \rho_{2,3}^2) \right) + \sigma^2}{\|h_2\|^2 P_A \beta_1 (1 - \rho_{1,3}^2) + \sigma^2} \right),
\]

(6)

and

\[
R_{3|h_3|}^{(CSC/SIC)} = \log_2 \left( \frac{\|h_3\|^2 P_A + \sigma^2}{\|h_3\|^2 P_A \left( \beta_1 (1 - \rho_{1,3}^2) + \beta_2 (1 - \rho_{2,3}^2) \right) + \sigma^2} \right),
\]

(7)

with

\[
\rho_{1,2,3}^2 = \frac{(\rho_{1,2} - \rho_{1,3})^2 + 2\rho_{1,2}\rho_{3,1}(1 - \rho_{2,3})}{1 - \rho_{2,3}^2}.
\]

(8)

And for 3-user correlated SC/non-SIC NOMA schemes, the conditional achievable sum rate given \( |h_1|, |h_2|, \) and \( |h_3| \) is expressed as:

\[
R_{\text{sum} |h_1| |h_2| |h_3|}^{(CSC/non-SIC)} = R_{1|h_1|}^{(CSC/non-SIC)} + R_{2|h_2|}^{(CSC/non-SIC)} + R_{3|h_3|}^{(CSC/non-SIC)},
\]

(9)

where

\[
R_{1|h_1|}^{(CSC/non-SIC)} = \log_2 \left( \frac{\|h_1\|^2 P + \sigma^2}{\|h_1\|^2 P \beta_2 (1 - \rho_{2,3}^2) + \beta_3 (1 - \rho_{3,1}^2) + \sigma^2} \right),
\]

(10)

\[
R_{2|h_2|}^{(CSC/non-SIC)} = \log_2 \left( \frac{\|h_2\|^2 P + \sigma^2}{\|h_2\|^2 P \beta_1 (1 - \rho_{1,3}^2) + \beta_3 (1 - \rho_{3,2}^2) + \sigma^2} \right),
\]

(11)
on Inflated Achievable Sum Rate of 3-User Low-Correlated SC NOMA

\[ R_3^{(CSC/\text{non-SIC})} = \log_2 \left( \frac{|h_3|^2 P + \sigma^2}{|h_3|^2 P_A \left( \beta_1 (1 - \rho_{1,3}) + \beta_2 (1 - \rho_{2,3}) \right) + \sigma^2 + 2 \sqrt{\beta_1 \beta_2 (\rho_{1,2} - \rho_{1,3} \rho_{3,2})}} \right). \]  

(12)

Then, for the conventional minimum high-correlated SC/non-SIC NOMA scheme, the conditional achievable sum rate given \(|h_1|, |h_2|, \text{ and } |h_3|\) is calculated, based on the following conditions [21]:

\[ R_1^{(CSC/\text{non-SIC})} = R_1^{(CSC/\text{SIC})}, \]  

(13)

\[ R_2^{(CSC/\text{non-SIC})} = R_2^{(CSC/\text{SIC})}, \]  

(14)

and

\[ R_3^{(CSC/\text{non-SIC})} = R_3^{(CSC/\text{SIC})}. \]  

(15)

Note that in the conventional correlated SC/non-SIC NOMA scheme, the minimum and high values of the correlation coefficients are used, because the lower value has the discontinuity in the power allocation range to be achieved, and the higher value reduces such power allocation range.

Now, on the contrary, we define the conditional achievable sum rate given \(|h_1|, |h_2|, \text{ and } |h_3|\) for the 3-user low-correlated SC/non-SIC NOMA scheme, based on the following condition:

\[ \min \left\{ R_{\sum}^{(CSC/\text{SIC})}, R_{\sum}^{(CSC/\text{SIC})}, R_{\sum}^{(CSC/\text{SIC})} \right\}. \]  

(16)

Since \[ R_3^{(CSC/\text{non-SIC})} = R_3^{(CSC/\text{SIC})}, \] we simplify the above-mentioned condition more as follows:

\[ \min \left\{ R_1^{(CSC/\text{non-SIC})}, R_2^{(CSC/\text{non-SIC})}, R_3^{(CSC/\text{SIC})}, R_3^{(CSC/\text{SIC})} \right\}. \]  

(17)

It should be noted that our design guarantees \[ \min \left\{ R_{\sum}^{(CSC/\text{SIC})}, R_{\sum}^{(CSC/\text{SIC})}, R_{\sum}^{(CSC/\text{SIC})} \right\} \leq R_{\sum}^{(CSC/\text{SIC})}, \] which is a necessary condition for the conventional minimum high-correlated SC/non-SIC NOMA scheme.

4. Numerical Results and Discussions

In this section, we investigate the effects of the proposed 3-user low-correlated SC schemes, by comparing...
of the proposed low-correlated 3-user SC scheme to $R_{\text{sum}}^{(\text{CSC/non-SIC})}$ of the conventional minimum high-correlated 3-user SC scheme. For this, the conditionals are assumed to be $|h_1| = \sqrt{2}$, $|h_2| = 1$, and $|h_3| = 0.1$, and the average total transmitted signal-to-noise power ratio (SNR) is $P/\sigma^2 = 50$. Then, for the conventional minimum high-correlated 3-user SC scheme, $\rho_{1,2} = \rho_{1,3} = \rho_{2,3} = \sqrt{0.50}$, whereas for the proposed low-correlated 3-user SC scheme, $\rho_{1,2} = \sqrt{0.50}$, $\rho_{1,3} = \sqrt{0.35}$, and $\rho_{2,3} = \sqrt{0.50}$. Note that we investigate the lower value $\rho_{1,3} = \sqrt{0.35} < \sqrt{0.5}$ for the correlation coefficient $\rho_{1,3}$, based on the observation that as the correlation coefficient $\rho_{1,3}$ decreases, the discontinuity of the power allocation range to be achieved becomes broader, whereas as the correlation coefficient $\rho_{1,3}$ increases, the power allocation range to be achieved becomes narrower.

As shown in Fig. 1, for the low correlation, i.e., $\rho_{1,3} = \sqrt{0.35} < \sqrt{0.50}$, the sum rate $R_{\text{sum}}^{(\text{CSC/non-SIC})}$ of the proposed low-correlated 3-user SC scheme is inflated in the direction of the axis $\alpha_1$, with respect to that of the conventional minimum high-correlated 3-user SC scheme. Notably, the axis $\alpha_1$ is related with the low correlation $\rho_{1,3}$. In addition, we observe that the sum rate $R_{\text{sum}}^{(\text{CSC/non-SIC})}$ of the proposed low-correlated 3-user SC scheme is deflated to a small amount, compared to that of the conventional minimum high-correlated 3-user SC scheme, over the power allocation ranges of the smaller sum rates.

Second, in order to investigate the impact of another correlation coefficient $\rho_{2,3}$ on the sum rate, we depict $R_{\text{sum}}^{(\text{CSC/non-SIC})}$ of the proposed low-correlated 3-user SC scheme with $\rho_{1,2} = \sqrt{0.50}$, $\rho_{1,3} = \sqrt{0.50}$, and $\rho_{2,3} = \sqrt{0.35}$ in Fig. 2.

![Figure 1. Comparison of achievable sum rates of conventional minimum high-correlated 3-user SC scheme and proposed low-correlated 3-user SC scheme, ($\rho_{1,3} = \sqrt{0.35}$).](image)
Figure 2. Comparison of achievable sum rates of conventional minimum high-correlated 3-user SC scheme and proposed low-correlated 3-user SC scheme, ($\rho_{2,3} = \sqrt{0.35}$).

As shown in Fig. 2, in this time, the sum rate $R_{\text{sum}}^{(\text{CSC/non-SIC})}$ of the proposed low-correlated 3-user SC scheme is inflated in the direction of the axis $\alpha_2$, with respect to that of the conventional minimum high-correlated 3-user SC scheme. Thus, we conjecture that the low correlation inflates the sum rate, although the sum rate deflates, over the power allocation ranges of the smaller sum rates.

Third, in Fig. 3, to investigate the compound impact of the two correlation coefficients $\rho_{1,3}$ and $\rho_{2,3}$, we depict $R_{\text{sum}}^{(\text{CSC/non-SIC})}$ of the proposed low-correlated 3-user SC scheme with $\rho_{1,2} = \sqrt{0.50}$, $\rho_{1,3} = \sqrt{0.35}$, and $\rho_{2,3} = \sqrt{0.35}$.

Figure 3. Comparison of achievable sum rates of conventional minimum high-correlated 3-user SC scheme and proposed low-correlated 3-user SC scheme, ($\rho_{1,3} = \sqrt{0.35}$, $\rho_{2,3} = \sqrt{0.35}$).
As shown in Fig. 3, with the two low correlations \( \rho_{1,3} = \sqrt{0.35} \), and \( \rho_{2,3} = \sqrt{0.35} \), the sum rate \( \mathcal{R}_{\text{sum}}^{(\text{CSC/non-SIC})} \) of the proposed low-correlated 3-user SC scheme is inflated doubly in the directions of the axis \( \alpha_1 \) and the axis \( \alpha_2 \), with respect to that of the conventional minimum high-correlated 3-user SC scheme. Therefore, we conclude that whether \( \rho_{1,3} \) or \( \rho_{2,3} \), the low correlations inflate effectively the sum rate, even though the sum rate deflates, over the power allocation ranges of the smaller sum rates.

5. Conclusion

In this paper, we investigated a 3-user low-correlated SC scheme, especially for an inflated achievable sum rate, with a design of 3-user low-correlated SC.

First, we designed the 3-user low-correlated SC scheme by taking the minimum sum rate between 3-user SIC NOMA and 3-user non-SIC NOMA, both with correlated SC. Then, simulations demonstrated that the low correlation in the direction of the first user’s power allocation inflates the sum rate in the same direction, compared to that of conventional minimum high-correlated SC NOMA, and such inflation due to low correlation is also observed similarly, in the direction of the second user’s power allocation. Moreover, we also showed that the two low correlations of the first and second users inflates doubly in the both directions of the first and second users’ power allocations.

As a result, the proposed 3-user low-correlated SC scheme could be a promising SC scheme, with the inflated sum rate in the future fifth-generation (5G) NOMA networks.

References

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