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CERTAIN SUBCLASSES OF ANALYTIC FUNCTIONS ASSOCIATED WITH MILLER-ROSS-TYPE POISSON DISTRIBUTION SERIES

Bilal ŞEKER, Sevtap SÜMER EKER*, and Bilal ÇEKİÇ

Abstract. The purpose of the present paper is to obtain some sufficient conditions for analytic functions, whose coefficients are probabilities of the Miller-Ross type-Poisson distribution series, to belong to classes $\mathcal{G}(\lambda,\delta)$ and $\mathcal{K}(\lambda,\delta)$.

1. Introduction

Let $\mathcal{A}$ stand for the standard class of analytic functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \qquad z \in \mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}, \tag{1}$$

and let $\mathcal{S}$ the class of functions in $\mathcal{A}$ which are univalent in $\mathbb{U}$ (see [7]).

A function $f \in \mathcal{A}$ be of the form (1) is said to be in the class $\mathcal{G}(\lambda,\delta)$ if it satisfies the following condition

$$\operatorname{Re}\left(\frac{zf'(z) + \lambda z^2 f''(z)}{f(z)}\right) > \delta,$$

where $0 \le \lambda < 1$, $0 \le \delta < 1$ and $z \in \mathbb{U}$.

A function $f \in \mathcal{A}$ be of the form (1) is said to be in the class $\mathcal{K}(\lambda,\delta)$ if it satisfies the following condition

$$\operatorname{Re}\left(\frac{(zf'(z) + \lambda z^2 f''(z))'}{f'(z)}\right) > \delta,$$

where $0 \le \lambda < 1$, $0 \le \delta < 1$ and $z \in \mathbb{U}$. A function $f \in \mathcal{A}$ be of the form (1) is said to be in the class $\mathcal{R}^{\tau}(A,B)$ if it satisfies the following condition

$$\left|\frac{f'(z) - 1}{(A-B)\tau - B(f'(z) - 1)}\right| < 1,$$

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*Corresponding author

where $-1 \leq B < A \leq 1$, $\tau \in \mathbb{C}\backslash\{0\}$ and $z \in \mathbb{U}$.

Kenneth S. Miller and Bertram Ross proposed the special function, which is called the Miller-Ross function defined as

$$\mathbb{E}_{\nu,c}(z) = z^{\nu} e^{cz} \gamma^{*}(\nu, cz),$$

where γ^{*} is the incomplete gamma function (p.314, [11]). Using the properties of the incomplete gamma functions the Miller-Ross function can easily be written as

$$\mathbb{E}_{\nu,c}(z) = z^{\nu} \sum_{n=0}^{\infty} \frac{(cz)^n}{\Gamma(n+\nu+1)} \qquad z, c, \nu \in \mathbb{C}. \tag{2}$$

In recent years a large literature has evolved on the use of distribution series such as Poisson, Pascal, Borel etc., in geometric function theory. Many researchers have examined some important features in the field of geometric function theory, such as coefficient estimates, inclusion relations, and conditions of being in some known classes, using different probability distributions, see for example [2, 3, 5, 8, 9, 13, 14, 15, 16, 18, 20, 21, 28].

We now recall that a discrete random variable X whose probability mass function is given by

$$P\left[X = i\right] = \frac{e^{-m} m^i}{i!}, \qquad i = 0, 1, 2, ..., \quad m > 0,$$

is said to have a Poisson distribution with parameter m.

Recently, Porwal and Dixit [17] introduce Mittag–Leffler-type Poisson distribution and obtain moments, moment generating function. Bajpai [4] introduced Mittag–Leffler-type Poisson distribution series. After that Murugusundaramoorthy and El-Deeb [12] studied the Mittag-Leffler type Borel distribution. Lately Srivastava et al. [26] introduced the Miller-Ross-type Poisson distribution which is a two parameter Mittag-Leffler type Poisson distribution and obtained moments, moment generating function. Motivated by results on connections between various subclasses of analytic univalent functions using special functions and distribution series [1, 10, 17, 19, 23, 24, 25, 22] we obtain some sufficient conditions for the Miller–Ross-type Poisson distribution series to be in classes $\mathcal{G}(\lambda, \delta)$ and $\mathcal{K}(\lambda, \delta)$. First we recall the definition of the Miller–Ross-type distribution.

The probability mass function of the Miller–Ross-type Poisson distribution is given by

$$P_{\nu,c}(m; k) = \frac{m^{\nu} (cm)^k}{\mathbb{E}_{\nu,c}(m) \Gamma(k+\nu+1)}, \qquad k = 0, 1, 2, ... \tag{3}$$

where $\nu > -1$, $c > 0$ and $\mathbb{E}_{\nu,c}(z)$ is Miller-Ross function given in (2).

The Miller–Ross-type Poisson distribution series, is defined by

$$\mathbb{K}_{\nu,c}^{m}(z) = z + \sum_{n=2}^{\infty} \frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)\mathbb{E}_{\nu,c}(m)} z^n, \qquad z \in \mathbb{U}. \tag{4}$$

(see [26], see also [22]). Furthermore, by using the convolution (or Hadamard product), we define

$$\begin{aligned}\mathbb{K}_{\nu,c}^{m} f(z) &= \mathbb{K}_{\nu,c}^{m}(z) * f(z) \\ &= z + \sum_{n=2}^{\infty} \frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)\mathbb{E}_{\nu,c}(m)} a_n z^n.\end{aligned}$$

To establish our main results, we will require the following Lemmas:

Lemma 1[6] If $f \in \mathcal{R}^{\tau}(A,B)$ is of the form (1) then

$$|a_n| \leq (A-B)\frac{|\tau|}{n}, \qquad n \in \mathbb{N}\backslash\{1\}. \tag{5}$$

The bound given in (5) is sharp for the function

$$f(z) = \int_0^z \left(1 + \frac{(A-B)|\tau| z^{n-1}}{1+Bz^{n-1}}\right) dz, \qquad n \in \mathbb{N}\backslash\{1\},\ z \in \mathbb{U}.$$

Lemma 2[27] A function $f \in \mathcal{A}$ belongs to the class $\mathcal{G}(\lambda,\delta)$, if

$$\sum_{n=2}^{\infty} \left[n + \lambda n(n-1) - \delta\right] |a_n| \leq 1 - \delta.$$

Lemma 3[20] A function $f \in \mathcal{A}$ belongs to the class $\mathcal{K}(\lambda,\delta)$, if

$$\sum_{n=2}^{\infty} n\left[n + \lambda n(n-1) - \delta\right] |a_n| \leq 1 - \delta.$$

2. Main Results

Theorem 2.1. *Let $m, c > 0$ and $\nu > -1$. If the condition*

$$\begin{aligned}\frac{1}{\mathbb{E}_{\nu,c}(m)} \Bigg\{ &\lambda\left(\frac{1}{m^2}\mathbb{E}_{\nu-2,c}(m) - \frac{m^{\nu}}{\Gamma(\nu-1)}\right) \\ &+ (2\lambda - 2\lambda\nu + 1)\left(\frac{1}{m}\mathbb{E}_{\nu-1,c}(m) - \frac{m^{\nu}}{\Gamma(\nu)}\right) \\ &+ \left[(1-\lambda\nu)(1-\nu) - \delta\right]\left(\mathbb{E}_{\nu,c}(m) - \frac{m^{\nu}}{\Gamma(\nu+1)}\right)\Bigg\} \leq 1 - \delta\end{aligned}$$

is satisfied, then $\mathbb{K}_{\nu,c}^{m} \in \mathcal{G}(\lambda,\delta)$.

Proof. By (4) and by Lemma 1, it suffices to show that

$$\sum_{n=2}^{\infty}[n+\lambda n(n-1)-\delta]\left|\frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)\mathbb{E}_{\nu,c}(m)}\right| \leq 1-\delta.$$

Now,

$$\sum_{n=2}^{\infty}[n+\lambda n(n-1)-\delta]\left|\frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)\mathbb{E}_{\nu,c}(m)}\right|$$

$$=\frac{1}{\mathbb{E}_{\nu,c}(m)}\sum_{n=1}^{\infty}[(\lambda(n+1)n+n+1-\delta]\,\frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu+1)}$$

$$=\frac{1}{\mathbb{E}_{\nu,c}(m)}\sum_{n=1}^{\infty}\left[(\lambda n^2+(1+\lambda)n+1-\delta\right]\frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu+1)}$$

$$=\frac{1}{\mathbb{E}_{\nu,c}(m)}\left\{\sum_{n=1}^{\infty}\lambda\left[(\nu+n)(\nu+n-1)+(\nu+n)(1-2\nu)+\nu^2\right]\frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu+1)}\right.$$
$$\left.+\sum_{n=1}^{\infty}[(\nu+n)(1+\lambda)-\nu(1+\lambda)+1-\delta]\,\frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu+1)}\right\}$$

$$=\frac{1}{\mathbb{E}_{\nu,c}(m)}\left\{\lambda\sum_{n=1}^{\infty}\frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu-1)}+[2\lambda(1-\nu)+1]\sum_{n=1}^{\infty}\frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu)}\right.$$
$$\left.+[(1-\lambda\nu)(1-\nu)-\delta]\sum_{n=1}^{\infty}\frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu+1)}\right\}$$

$$=\frac{1}{\mathbb{E}_{\nu,c}(m)}\left\{\lambda\left(\frac{1}{m^2}\mathbb{E}_{\nu-2,c}(m)-\frac{m^{\nu}}{\Gamma(\nu-1)}\right)\right.$$
$$+(2\lambda-2\lambda\nu+1)\left(\frac{1}{m}\mathbb{E}_{\nu-1,c}(m)-\frac{m^{\nu}}{\Gamma(\nu)}\right)$$
$$\left.+[(1-\lambda\nu)(1-\nu)-\delta]\left(\mathbb{E}_{\nu,c}(m)-\frac{m^{\nu}}{\Gamma(\nu+1)}\right)\right\}$$

$$\leq 1-\delta$$

by the given hypothesis. This completes the proof of Theorem 2.1. □

Theorem 2.2. *Let $m, c > 0$ and $\nu > -1$. If the condition*

$$\begin{aligned}\frac{1}{\mathbb{E}_{\nu,c}(m)}\Bigg\{&\lambda\left(\frac{1}{m^3}\mathbb{E}_{\nu-3,c}(m)-\frac{m^\nu}{\Gamma(\nu-2)}\right)\\ &+\big(3\nu+2\lambda-2\big)\left(\frac{1}{m^2}\mathbb{E}_{\nu-2,c}(m)-\frac{m^\nu}{\Gamma(\nu-1)}\right)\\ &+\big[4-\delta+3\nu^2-5\nu-\lambda(4\nu-3)\big]\left(\frac{1}{m}\mathbb{E}_{\nu-1,c}(m)-\frac{m^\nu}{\Gamma(\nu)}\right)\\ &+\big[(\nu-1)(\delta-\nu^2)+(1-2\nu)(1-\lambda\nu)\big]\left(\mathbb{E}_{\nu,c}(m)-\frac{m^\nu}{\Gamma(\nu+1)}\right)\Bigg\}\le 1-\delta\end{aligned}$$

is satisfied, then $\mathbb{K}^m_{\nu,c}\in\mathcal{K}(\lambda,\delta)$.

Proof. By (4) and by Lemma 1, it suffices to show that

$$\sum_{n=2}^{\infty} n\,[n+\lambda n(n-1)-\delta]\,\frac{m^\nu(cm)^{n-1}}{\Gamma(n+\nu)}\frac{1}{\mathbb{E}_{\nu,c}(m)}\le 1-\delta.$$

Now,

$$\begin{aligned}&\sum_{n=2}^{\infty} n\big[n+\lambda n(n-1)-\delta\big]\frac{m^\nu(cm)^{n-1}}{\Gamma(n+\nu)}\frac{1}{\mathbb{E}_{\nu,c}(m)}\\ &=\frac{1}{\mathbb{E}_{\nu,c}(m)}\sum_{n=1}^{\infty}(n+1)\big[(n+1)+\lambda(n+1)n-\delta\big]\frac{m^\nu(cm)^n}{\Gamma(n+\nu+1)}\\ &=\frac{1}{\mathbb{E}_{\nu,c}(m)}\sum_{n=1}^{\infty}\big[\lambda n^3+(1+2\lambda)n^2+(2-\delta+\lambda)n+1-\delta\big]\frac{m^\nu(cm)^n}{\Gamma(n+\nu+1)}\\ &=\frac{1}{\mathbb{E}_{\nu,c}(m)}\Bigg\{\sum_{n=1}^{\infty}\lambda(\nu+n)(\nu+n-1)(\nu+n-2)\frac{m^\nu(cm)^n}{\Gamma(n+\nu+1)}\\ &\qquad+\sum_{n=1}^{\infty}3\lambda(\nu-1)(\nu+n)(\nu+n-1)\frac{m^\nu(cm)^n}{\Gamma(n+\nu+1)}\\ &\qquad+\sum_{n=1}^{\infty}\lambda\big[(3\nu^2-3\nu+1)(\nu+n)-\nu^3\big]\frac{m^\nu(cm)^n}{\Gamma(n+\nu+1)}\\ &\qquad+(1+2\lambda)\sum_{n=1}^{\infty}\big[(\nu+n)(\nu+n-1)+(1-2\nu)(\nu+n)+\nu^2\big]\frac{m^\nu(cm)^n}{\Gamma(n+\nu+1)}\\ &\qquad+\sum_{n=1}^{\infty}\big[(2-\delta+\lambda)(\nu+n)-(2-\delta+\lambda)\nu+(1-\delta)\big]\frac{m^\nu(cm)^n}{\Gamma(n+\nu+1)}\Bigg\}\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{\mathbb{E}_{\nu,c}(m)} \Bigg\{ \lambda \left(\frac{1}{m^3} \mathbb{E}_{\nu-3,c}(m) - \frac{m^\nu}{\Gamma(\nu-2)} \right) \\
&\quad + \big(3\nu + 2\lambda - 2\big) \left(\frac{1}{m^2} \mathbb{E}_{\nu-2,c}(m) - \frac{m^\nu}{\Gamma(\nu-1)} \right) \\
&\quad + \big[4 - \delta + 3\nu^2 - 5\nu - \lambda(4\nu - 3)\big] \left(\frac{1}{m} \mathbb{E}_{\nu-1,c}(m) - \frac{m^\nu}{\Gamma(\nu)} \right) \\
&\quad + \big[(\nu-1)(\delta - \nu^2) + (1-2\nu)(1-\lambda\nu)\big] \left(\mathbb{E}_{\nu,c}(m) - \frac{m^\nu}{\Gamma(\nu+1)} \right) \Bigg\} \\
&\leq 1 - \delta
\end{aligned}
$$

by the given hypothesis. This completes the proof of Theorem 2.2. □

Theorem 2.3. *Let $m, c > 0$, $\nu > -1$ and $f \in \mathcal{R}^\tau(A, B)$. If the condition*

$$
\begin{aligned}
&\frac{(A-B)}{|\tau| \mathbb{E}_{\nu,c}(m)} \left\{ \lambda \left(\frac{1}{m^2} \mathbb{E}_{\nu-2,c}(m) - \frac{m^\nu}{\Gamma(\nu-1)} \right) \right. \\
&\qquad + (2\lambda - 2\lambda\nu + 1) \left(\frac{1}{m} \mathbb{E}_{\nu-1,c}(m) - \frac{m^\nu}{\Gamma(\nu)} \right) \\
&\qquad \left. + [(1-\lambda\nu)(1-\nu) - \delta] \left(\mathbb{E}_{\nu,c}(m) - \frac{m^\nu}{\Gamma(\nu+1)} \right) \right\} \leq 1 - \delta
\end{aligned}
$$

is satisfied then $\mathbb{K}^m_{\nu,c} f \in \mathcal{K}(\lambda, \delta)$.

Proof. Since

$$
\mathbb{K}^m_{\nu,c} f(z) = z + \sum_{n=2}^{\infty} \frac{m^\nu (cm)^{n-1}}{\Gamma(n+\nu) \mathbb{E}_{\nu,c}(m)} a_n z^n,
$$

to prove that our claim is true $\mathbb{K}^m_{\nu,c} f \in \mathcal{K}(\lambda, \delta)$, in view of Lemma 1, we need to show that

$$
\sum_{n=2}^{\infty} n \left[n + \lambda n(n-1) - \delta \right] \frac{m^\nu (cm)^{n-1}}{\Gamma(n+\nu)} \frac{|a_n|}{\mathbb{E}_{\nu,c}(m)} \leq 1 - \delta.
$$

Now,

$$\sum_{n=2}^{\infty} n\big[n+\lambda n(n-1)-\delta\big]\frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)}\frac{|a_n|}{\mathbb{E}_{\nu,c}(m)}$$
$$\leq (A-B)|\tau|\sum_{n=2}^{\infty}\big[n+\lambda n(n-1)-\delta\big]\frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)}\frac{1}{\mathbb{E}_{\nu,c}(m)}$$
$$=\frac{(A-B)|\tau|}{\mathbb{E}_{\nu,c}(m)}\sum_{n=1}^{\infty}\big[(\lambda n^2+(1+\lambda)n+1-\delta\big]\frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu+1)}$$
$$=\frac{(A-B)|\tau|}{\mathbb{E}_{\nu,c}(m)}\Bigg\{\sum_{n=1}^{\infty}\lambda\big[(\nu+n)(\nu+n-1)+(\nu+n)(1-2\nu)+\nu^2\big]\frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu+1)}$$
$$+\sum_{n=1}^{\infty}\big[(\nu+n)(1+\lambda)-\nu(1+\lambda)+1-\delta\big]\frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu+1)}\Bigg\}$$
$$=\frac{(A-B)|\tau|}{\mathbb{E}_{\nu,c}(m)}\Bigg\{\lambda\sum_{n=1}^{\infty}\frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu-1)}+\big[2\lambda(1-2\nu)+1\big]\sum_{n=1}^{\infty}\frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu)}$$
$$+\big[(1-\lambda\nu)(1-\nu)-\delta\big]\sum_{n=1}^{\infty}\frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu+1)}\Bigg\}$$
$$=\frac{(A-B)|\tau|}{\mathbb{E}_{\nu,c}(m)}\Bigg\{\lambda\left(\frac{1}{m^2}\mathbb{E}_{\nu-2,c}(m)-\frac{m^{\nu}}{\Gamma(\nu-1)}\right)$$
$$+\big(2\lambda-2\lambda\nu+1\big)\left(\frac{1}{m}\mathbb{E}_{\nu-1,c}(m)-\frac{m^{\nu}}{\Gamma(\nu)}\right)$$
$$+\big[(1-\lambda\nu)(1-\nu)-\delta\big]\left(\mathbb{E}_{\nu,c}(m)-\frac{m^{\nu}}{\Gamma(\nu+1)}\right)\Bigg\}$$
$$\leq 1-\delta$$

by the given hypothesis. This completes the proof of Theorem 2.3. □

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Bilal ŞEKER
Department of Mathematics, Faculty of Science, Dicle University
Diyarbakır, Türkiye.
E-mail: bilal.seker@dicle.edu.tr

Sevtap SÜMER EKER
Department of Mathematics, Faculty of Science, Dicle University
Diyarbakır, Türkiye.
E-mail: sevtaps@dicle.edu.tr

Bilal ÇEKİÇ
Department of Mathematics, Faculty of Science, Dicle University
Diyarbakır, Türkiye.
E-mail: bilalc@dicle.edu.tr