An Optical Graphene-silicon Resonator Phase Shifter
Suitable for Universal Linear Circuits

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This paper describes the construction of a phase shifter with low loss and small volume. To construct it, we use the two graphene layers that are separated by a hexagonal boron nitride (hBN) and embedded in a silicon waveguide. The refractive index of the waveguide is adjusted by applying a bias voltage to the graphene sheet to create an optical phase shift. This waveguide is a compact device that only has a radius of 5 μm. It has a phase shift of 6π. In addition, the extinction ratio (ER) is 11.6 dB and the insertion loss (IL) is 0.031 dB. Due to its unique characteristics, this device has great potential in silicon on-chip optical interconnection and all-optical multiple-input multiple-output processing.

Keywords : Microring resonator, Optical phase shifter, Photonic integrated circuit
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I. INTRODUCTION

Universal linear circuits are reprogrammable optical circuits that theoretically can implement all possible linear optical protocols. They are limited by their size [1, 2]. Universal linear circuits are essentially reconfigurable integrated waveguide devices fabricated in a SiO₂ chip. Carolan et al. [1] showed a device that allowed for universal linear optic transformations on waveguides by integrated Mach-Zehnder interferometers (MZIs). A universal linear circuit provides a potential platform for quantum computing, communication, and all-optical multi-input multi-output devices [3]. Unfortunately, the MZI device is large, and the SiO₂ chip size is limited. It is difficult to expand the general linear circuit because of the large size. Therefore, accommodating more optical devices with a limited universal linear circuit size is a critical research topic to maximize the number of optical protocols. In 2019, Sato and Enokihara [4] designed a universal linear circuit using a microring resonator to replace the MZI and phase shifter. This improvement significantly reduced the footprint of a universal linear circuit. In this paper, the two-port cascaded microring that functions as a phase shifter is completed by heating the microring to change the refractive index.

An optical modulator is a critical component that is used to control the fundamental characteristics of a carrier light that is propagating in a waveguide [5]. In integrated silicon photonics, the primary methods for implementing optical phase modulators include the use of the free-carriers plasma dispersion effect, thermo-optical, electro-optical effects, and deformation characteristics of materials. In the modulation method based on plasma dispersion effect, a P-N junction is usually used to change the refractive index, thereby completing phase modulation [6–10]. However, the modulation device using this method usually covers a small area [8] and consumes significant energy [11]. Another challenge is that these modulators are always accompanied by light absorption [12]. If a higher free carrier density and more substantial plasma dispersion are desired to be obtained, either aspect must accept a higher loss or increase driving voltage. Although the thermo-optical effect method, which is also widely used, can achieve low energy...
II. METHODS

2.1. Graphene Optoelectronic Properties

Our simulations are performed by the finite element method. The surface conductivity of a single layer of graphene can be derived from the Kubo formula [30]:

\[
\sigma_{\nu}(\mu) = \sigma_{0} \frac{4\mu}{\pi \hbar (\Gamma_{1} - i\omega)} + \\
\frac{1 + \frac{1}{\pi} \arctan \frac{h\omega - 2\mu}{h\Gamma_{1}}}{1 + \frac{1}{\pi} \arctan \frac{h\omega + 2\mu}{h\Gamma_{1}}} \\
\frac{i}{2\pi} \ln \left( \frac{(h\omega + 2\mu)^{2} + (h\Gamma_{1})^{2}}{(h\omega - 2\mu)^{2} + (h\Gamma_{1})^{2}} \right)
\]

(1)

where \(\omega\) is the angular frequency, corresponding to the wavelength \(\lambda = 1.55\ \mu\text{m}\), \(\omega = 1.216 \times 10^{15}\ \text{s}^{-1}\), \(r = 0.2\ \text{ps}\), and \(T = 300\ \text{K}\). The optical conductivity of undoped graphene is \(\sigma_{0} = e^{2}/(4\hbar) = 60.85\ \mu\Omega\). \(\hbar\) is the reduced Planck constant. The chemical potential \(\mu\) can be controlled by an applied voltage \(V\) [31], which is described by Eq. (2):

\[
\mu = \hbar v_{F} \sqrt{\frac{\pi \varepsilon_{\parallel} \varepsilon_{r}}{d}} \left| V - V_{0} \right|.
\]

(2)

where \(v_{F} = 1.1 \times 10^{6}\ \text{m/s}\) is the electronic group velocity [32] and \(\varepsilon_{r}\) is the relative dielectric constant of the spacer. \(d\) is the thickness of the spacer. \(V\) is the bias voltage. \(e\) is the elementary electric charge. For this design, we set \(V_{0} = 0\) [33]. When the chemical potential \(\mu\) is tuned, the equivalent permittivity can also be modified as

\[
\varepsilon_{\parallel} = 1 + i\frac{\sigma_{0}}{\varepsilon_{r} \frac{d}{\varepsilon_{0}}},
\]

(3)

\(\varepsilon_{0}\) is the permittivity of the vacuum. In this simulation, we set the graphene’s thickness as \(d_{g} = 0.7\ \text{nm}\).

2.2. Graphene-integrated Photonic Waveguides

This proposed GSRPS is based on a ring resonator configuration. Its structure is shown in Fig. 1(a). A ring waveguide is coupled with a straight waveguide, and graphene is...
integrated into the ring waveguide. The graphene does not shift the light mode coming from the silicon waveguide and hardly absorbs any light [21]. To enhance the interaction between graphene and light, we embed two graphene sheets horizontally in the middle of the hBN spacer layer. A silicon layer is deposited on the top to form a graphene-silicon hybrid waveguide. This design is shown in Fig. 1(c). The size of the waveguide is based on commercial SOI wafers that are 0.34 μm thick. Its width is 0.4 μm. Its ridge height is 260 nm. The graphene is placed in the middle of the ridge. The thickness of the hBN layer between the graphene flakes is 5 nm. The other two hBN layers above and below these graphene flakes are 7 nm thick. The exact size is shown in Fig. 1(b) and 1(d). The refractive index of waveguides is $n_{Si} = 3.476$, while the cladding and hBN spacer layers are $n_{SiO_2} = 1.44$ and $n_{hBN} = 2$, respectively. The simulations are based on incident light $\lambda = 1.55 \mu m$, and $T = 300 K$. The primary role of hBN is to encapsulate the graphene sheets to prevent contamination and provide electrical insulation. The manufacturing method for this graphene-silicon hybrid part can be made by transferring a single-layer graphene sheet grown by a chemical vapor deposition to a 7-nm thick isolation layer.

EMI in the waveguide is essentially a quantifying number to characterize the phase change and loss of the fundamental modes. The real part of EMI affects the phase of light, known as electro-refraction, and the imaginary part of the EMI is defined as electro-absorption. $N_{eff}$ corresponds to the real part of EMI, where $\alpha$ is the imaginary part. EMI has apparent fluctuations when the graphene’s chemical potential is modified. We varied the EMI.

We define the refractive index change $\Delta N_{eff}$ as $\Delta N_{eff} = N_{eff} - N_{eff, \mu=0}$, where $N_{eff}$ is the refractive index of the graphene hybrid waveguide at different chemical potentials. $N_{eff, \mu=0}$ is the refractive index of the graphene hybrid waveguide with the chemical potential $\mu$ at zero.
III. RESULTS

3.1. The Refractive Index Changes with Graphene Layers

Figures 2(a) and 2(b) show the refractive index changes $\Delta N_{\text{eff}}$ of TE and TM modes, respectively, in a ring waveguide integrated with 1–4 layers of graphene when the chemical potential ranges from 0 to 2 eV. Figures 2(c) and 2(d) show the corresponding loss changes. $\Delta N_{\text{eff}}$-TM and $\Delta N_{\text{eff}}$-TE have a dip at $\mu = 0.4$ eV. Both $\alpha$-TM and $\alpha$-TE have the lowest value at $\mu = 0.5$ eV. $\alpha$-TM and $\alpha$-TE are suppressed to a low level. It can be seen that under similar conditions, embedded graphene has a relatively significant influence on the TE mode.

With the same chemical potential, the more graphene layers, the more pronounced the change in EMI. That is, the smaller the value of the refractive index of the waveguide, the greater the loss. Considering the difficulty and practical requirements of manufacturing, the effect of double-layer graphene in the TE mode is cost-effective. Then, we determined that the structure of the phase shifter is designed as a double-layer graphene hybrid waveguide. Figures 3(a) and 3(b) illustrate the changes of the real and imaginary parts of the EMI. For a waveguide, when the chemical potential $\mu$ of graphene changes from 0.4 eV to 2.0 eV, the real part of the EMI of the TE and TM modes shows a quasi-linear decline. At the same time, the imaginary part of EMI $\alpha$, which represents the loss, remains at low level. This excellent characteristic meets the requirements of a phase modulator. Since most laser diodes operate with TE polarization, a TE mode waveguide is more compatible with on-chip optical integrated circuits [34]. Thus, our subsequent research focuses on the TE mode.

In addition, it should be noted that the TM simulation

![Fig. 2](image-url). The effective mode index (EMI) change curves against the graphene layer under different chemical potentials. (a) and (b) The $N_{\text{eff}}$ variations of TM and TE modes under different chemical potentials for single-layer, double-layer, triple-layer, and four-layer graphene structures. (c) and (d) $\alpha$ of TM and TE modes as a function of chemical potential for a single-layer, double-layer, triple-layer, and four-layer graphene structures.
data in this paper are dissimilar to some previous articles [28, 33, 35]. Because graphene is regarded as an isotropic sheet in those papers, the epsilon-near-zero effect graphene exhibits extremely strong absorption in the TM fundamental mode. Therefore, when the chemical potential \( \mu \) is adjusted to about 0.5 eV, the TM mode electric field is strongly concentrated. However, considering the two-dimensional nature, the electrons are tightly confined within an atomic monolayer. Therefore, the graphene is anisotropic. Since significantly enhanced light absorption will not occur, this paper adopts the anisotropic graphene model, which is consistent with [27] and [36].

### 3.2. Optical Phase Shifter Based on Graphene-silicon Microring Resonator

The ring resonator designed in this paper feeds back an output of a directional coupler to its input. This structure is called an all-pass ring resonator. For an all-pass ring resonator, shown in Fig. 1, the linear transfer function can be expressed [37] by Eq. (4):

\[
T = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{\alpha^2 - 2r \alpha \cos \varphi + r^2}{1 - 2r \alpha \cos \varphi + \alpha^2 r^2}.
\]  

(4)

In this equation, \( P_{\text{in}} \) is the incident optical power, \( P_{\text{out}} \) is the output optical power, \( r \) is the transmission coefficient of the coupling region, and \( \alpha \) is obtained from \( \kappa^2 + r^2 = 1 \) in a lossless coupler. \( \kappa \) is the field cross-coupling coefficient, and \( \alpha \) can be derived based on the coupled-mode theory [38]. \( \varphi \) is the roundtrip phase shift and can be written as

\[
\varphi = \frac{2\pi n_{\text{GSI}}^\text{GSI} L + 2\pi n_{\text{Si}}^\text{Si}(2\pi R - L)}{\lambda}.
\]  

(5)

where \( R \) is the radius of the microring and \( L \) is the coating length of a graphene sheet that is placed in the ring waveguide. \( \alpha \) is the loss coefficient of the graphene hybrid microring. It can be expressed as

\[
a = \frac{\exp(-\alpha_{\text{Si}} \cdot L)}{\exp[-\alpha_{\text{Si}} \cdot (2\pi R - L)]}.
\]  

(6)

where \( \alpha_{\text{Si}} \) and \( \alpha_{\text{GSI}} \) are the propagation loss of pure Si waveguides and graphene hybrid Si waveguides, respectively.

The total transmission phase-shift of the output field is

\[
\Delta \varphi = \arctan \frac{r \alpha \sin \varphi}{1 - r \alpha \cos \varphi} - \arctan \frac{\alpha \sin \varphi}{r - r \alpha \cos \varphi}.
\]  

(7)

The normalized modulation depth can be defined as the extinction ratio (ER) given by

\[
\text{ER} = 10 \log_{10} \frac{T_{\text{max}}}{T_{\text{min}}} = 10 \log_{10} \frac{(r + \alpha)^2 (1 - r \alpha)^2}{(r - \alpha)^2 (1 + r \alpha)^2},
\]  

(8)

and insertion loss can be obtained by Eq. (9):

\[
\text{IL} = 10 \log_{10} \frac{T_{\text{min}}}{T_{\text{max}}} = 10 \log_{10} \frac{(r - \alpha)^2}{(1 - r \alpha)^2}.
\]  

(9)

### IV. DISCUSSION

As mentioned in Section 2.2, the effective refractive index of the waveguide constantly changes with the applied chemical potentials. It does not reach saturation until 2 eV, and the loss is at a low level. This characteristic makes
phase shift possible. Therefore, the phase shift can be performed in regions where the absorption no longer changes. In addition, graphene on Si electro-absorption modulators and graphene on Si phase modulators are similar. The main difference lies in the applied voltage bias. For a phase modulator, the applied voltage bias must far exceed the Pauli blocking threshold. Therefore, it is suitable to achieve the phase modulator after 0.4 eV. Figure 4(a) shows the phase shift for the GSRPS when the chemical potential is 0.9 eV. For this potential, a phase shift of 6π can be obtained, which means that the tunable range can cover three free spectral ranges.

Next, we study the extinction ratio and insertion loss related to graphene coverage and chemical potential. As shown in Fig. 5(b), when the fixed chemical potential is 0.9 eV, the graphene coverage angles are 40°, 80°, and 120°. For these angles, the graphene coverages are 11%, 22%, and 33%, and the transmission values are 0.278, 0.284, 0.305, respectively. There is almost no difference between the three transmission curves. For these angles, the chemi-

**FIG. 4.** Phase shift curves and transmission spectra simulation under 0.9 eV chemical potential. (a) Corresponding phase shift when the chemical potential is 0.9 eV, and the graphene coverage angle is 40°. A phase shift of 6π can be obtained. (b) The chemical potential is 0.9 eV, the transmission spectra simulation of graphenesilicon resonator phase shifter (GSRPS) with 40°, 80°, and 120° graphene coverage angles.

**FIG. 5.** The chemical potential and the transmission value changes caused by different graphene coverage angles: (a) with 40°, 80°, and 120° graphene coverage angles, the chemical potential required for the phase shift of the graphenesilicon resonator phase shifter (GSRPS) to reach π, and (b) the fixed coverage angle is 40°. There are different extinction ratios when chemical potential is adjusted to 0.4 eV, 0.6 eV, and 0.9 eV, while the loss is maintained at a relatively low level.
cal potential required for the phase shift of the modulator to reach $\pi$ is shown in Fig. 5(a). At a coverage angle of 120°, the chemical potential required to achieve the $\pi$ phase shift is minimal. With a coverage angle of 40°, the changing trend of the phase curve is the smallest.

Finally, we simulated the corresponding extinction ratio and insertion loss of different chemical potentials at fixed coverage. The results are shown in Fig. 5(b). When the fixed graphene coverage angle is 40° and the chemical potential is adjusted to 0.4 eV, 0.6 eV, and 0.9 eV, the loss is still at relatively low level, but has different extinction ratios.

V. CONCLUSION

In this paper, we first investigated the influence of 1–4 layers of a graphene sheet on the EMI of a waveguide under the condition of a fixed waveguide size. In our simulation, graphene is anisotropic. The results show that the more graphene layers, the larger the impact on the EMI of the waveguide. By modifying the chemical potential and changing the properties of graphene, the corresponding changes in the EMI of the waveguide are produced.

Then, we propose a low loss and compact size graphene-silicon resonators phase shifter based on a double-layer graphene sheet. This device has a radius of only 5 $\mu$m. Two layers of graphene are separated by hexagonal boron nitride and embedded in the silicon waveguide. The change in the refractive index of the waveguide is adjusted by applying a bias voltage to the graphene to achieve an optical phase shift. The simulation shows that with proper graphene coverage, multiple operating points can be achieved. In this exemplary design, when the graphene coverage angle is 40°, a chemical potential of 0.9 eV can achieve a phase change of 6$\pi$. The GSRPS has an extinction ratio of 11.6 dB and a 0.035 dB insertion loss. This GSRPS allows the creation of energy-efficient communication and on-chip silicon optical interconnection. It also has excellent potential for all-optical multi-input multi-output processing.

In future work, we will verify and then optimize this design. In addition, we will combine it with a universal linear circuit and then integrate it into a complete integrated circuit.

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DISCLOSURES

The authors declare no conflicts of interest.

DATA AVAILABILITY

Data underlying the results presented in this paper are not publicly available at the time of publication, which may be obtained from the authors upon reasonable request.

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