

## **BTS Based Improved BER for Stronger Channel User in Non-Uniform Source SSC NOMA**

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### **Abstract**

*In this paper, to improve further the bit-error rate (BER) performance of the stronger channel user in non-uniform source non-orthogonal multiple access (NOMA) with symmetric superposition coding (SSC), we propose a smart bit-to-symbol (BTS) mapping of SSC. First, the analytical expression for the total allocated power of the proposed BTS mapping scheme is derived, and then we show that the BER of the proposed BTS mapping scheme improves further, compared to that of the existing BTS mapping scheme. Moreover, based on the simulations, the signal-to-noise (SNR) gain of the proposed BTS mapping scheme over the existing BTS mapping scheme is calculated. In result, the proposed BTS mapping could be a candidate scheme for non-uniform source SSC NOMA with the SNR gain.*

**Keywords:** *NOMA, SSC, Lempel-Ziv coding, Superposition coding, Power allocation.*

### **1. Introduction**

Recently, the optimal maximum a-posteriori (MAP) receiver [1] for non-uniform source non-orthogonal multiple access (NOMA) [2-4] with symmetric superposition coding (SSC) [5] has been proposed to improve the bit-error rate (BER) performance for the stronger channel gain user. In NOMA, unipodal binary pulse amplitude modulation (2PAM) was investigated [6]. Asymmetric 2PAM NOMA without successive interference cancellation (SIC) was studied in [7]. Quadrature correlated superposition was studied in NOMA [8]. Low-correlated superposition coding for NOMA was investigated in [9]. Meanwhile, Lempel-Ziv coding [10] produces usually a non-uniform source. In this paper, to improve further the BER performance for the stronger channel gain user, we propose a smart bit-to-symbol (BTS) mapping. First, the closed-form expression for the total allocated power of the proposed BTS mapping scheme is derived. Then we show that the BER of the proposed BTS mapping scheme is improved, compared with that of the existing BTS mapping scheme. Moreover, we calculate the signal-to-noise (SNR) gain of the proposed BTS mapping scheme over the existing BTS mapping scheme. Lastly, the motivations of this paper are stated as follows: Usually, in NOMA, the lower power is allocated to the strong channel gain user than the weak channel gain user. Thus, it is essential to improve the performance of the strong channel gain user.

The remainder of this paper is organized as follows. In Section 2, the system and channel model are

described. the closed-form expression for the total allocated power of the proposed BTS mapping scheme is derived in Section 3. The results of simulations are presented in Section 4. Finally, the conclusions are presented in Section 5.

The main contributions of this paper are summarized as follows:

- We propose the smart BTS mapping of SSC in non-uniform source NOMA.
- Then, we derive an analytical expression for the total allocated power of the proposed BTS mapping scheme for non-uniform source NOMA with SSC.
- We show that the BER of the proposed BTS mapping scheme improves further, compared to that of the existing BTS mapping scheme.
- Moreover, we calculate the SNR gain of the proposed BTS mapping scheme over the existing BTS mapping scheme.

## 2. System and Channel Model

We consider a base station and two users in a downlink NOMA network. The complex channel coefficient is  $h_m$ ,  $m=1,2$ , between the  $m$ th user and base station, with  $|h_1| \geq |h_2|$ . The superimposed signal  $x = \sqrt{P_A \alpha} s_1 + \sqrt{P_A (1-\alpha)} s_2$  is transmitted by base station. Given the average total transmitted power  $P$  of  $x$ ,  $P_A$  is the average total allocated power.  $s_m$  is the signal for the  $m$ th user with the average unit power, and  $\alpha$  is the power allocation coefficient. The received signal  $r_m$  at the  $m$ th user is expressed as follows:

$$r_m = |h_m| x + n_m, \tag{1}$$

where  $n_m \sim N(0, N_0/2)$  is additive white Gaussian noise (AWGN). Assume that the information bits for the user-1 and user-2 are  $b_1, b_2 \in \{0,1\}$ . A joint probability mass function (PMF)  $P(b_1, b_2)$  is given as [1]

		$P(b_2)$		
		$P(b_2 = 0) = \frac{1}{2}$	$P(b_2 = 1) = \frac{1}{2}$	
$P(b_1, b_2)$				
$P(b_1 = 0) = 2\delta_{0,0}$		$P(b_1 = 0, b_2 = 0) = \delta_{0,0}$		$P(b_1 = 0, b_2 = 1) = \delta_{0,1} = \delta_{0,0}$
$P(b_1 = 1) = 1 - 2\delta_{0,0}$		$P(b_1 = 1, b_2 = 0) = \delta_{1,0} = \frac{1}{2} - \delta_{0,0}$		$P(b_1 = 1, b_2 = 1) = \delta_{1,1} = \frac{1}{2} - \delta_{0,0}$

(2)

The binary phase shift keying (BPSK) modulation  $s_1, s_2 \in \{+1, -1\}$  is used. The existing BTS for SSC schemes is given by [1]

$$\begin{cases} s_1(b_1 = 0, b_2 = 0) = +1 & s_1(b_1 = 0, b_2 = 1) = -1 \\ s_1(b_1 = 1, b_2 = 0) = -1 & s_1(b_1 = 1, b_2 = 1) = +1 \end{cases} \quad (3)$$

$$\begin{cases} s_2(b_2 = 0) = +1 \\ s_2(b_2 = 1) = -1 \end{cases}$$

### 3. Derivation of Total Allocated Power and Analytical Performance Comparison

We propose a new BTS mapping for SSC schemes, which is given by

$$\begin{cases} s_1(b_1 = 0, b_2 = 0) = -1 & s_1(b_1 = 0, b_2 = 1) = +1 \\ s_1(b_1 = 1, b_2 = 0) = +1 & s_1(b_1 = 1, b_2 = 1) = -1 \end{cases} \quad (4)$$

$$\begin{cases} s_2(b_2 = 0) = +1 \\ s_2(b_2 = 1) = -1 \end{cases}$$

Now we derive the closed-form expression for the total allocated power, and present an analytical performance comparison, with the following proposition.

*Proposition 1:* Based on the reference [1], assume that  $P(b_1 = 0, b_2 = 0) = \delta_{0,0} > \frac{1}{4}$ , without loss of generality. Then  $P_A^{(\text{proposed})} > P_A^{(\text{existing})}$ .

*Proof:* First, the total power of the existing BTS scheme is given by [1]

$$P_A^{(\text{existing})} = \frac{P}{1 + 2\sqrt{\alpha}\sqrt{(1-\alpha)}4\left(\delta_{0,0} - \frac{1}{4}\right)}. \quad (5)$$

Then the total allocated power of the proposed BTS scheme is derived as follow.

$$\begin{aligned} P &= \mathbb{E}[|x|^2] = \mathbb{E}\left[\left|\sqrt{P_A\alpha}s_1 + \sqrt{P_A(1-\alpha)}s_2\right|^2\right] = P_A\left(1 + 2\sqrt{\alpha}\sqrt{(1-\alpha)}\mathbb{E}[s_1s_2^*]\right) \\ &= P_A\left(1 + 2\sqrt{\alpha}\sqrt{(1-\alpha)}\left[\left(\frac{1}{2} - \delta_{0,0}\right)(+1)(+1) + \delta_{0,0}(-1)(+1) + \left(\frac{1}{2} - \delta_{0,0}\right)(-1)(-1) + \delta_{0,0}(+1)(-1)\right]\right) \\ &= P_A\left(1 + 2\sqrt{\alpha}\sqrt{(1-\alpha)}\left(-2\delta_{0,0} + 2\left(\frac{1}{2} - \delta_{0,0}\right)\right)\right) = P_A\left(1 + 2\sqrt{\alpha}\sqrt{(1-\alpha)}2\left(-\delta_{0,0} + \left(\frac{1}{2} - \delta_{0,0}\right)\right)\right) \\ &= P_A\left(1 + 2\sqrt{\alpha}\sqrt{(1-\alpha)}4\left(-\delta_{0,0} + \frac{1}{4}\right)\right), \\ \therefore P_A^{(\text{proposed})} &= \frac{P}{1 + 2\sqrt{\alpha}\sqrt{(1-\alpha)}4\left(\frac{1}{4} - \delta_{0,0}\right)}. \end{aligned}$$

Thus, we have  $P_A^{(\text{proposed})} > P_A^{(\text{existing})}$ , because  $\delta_{0,0} > \frac{1}{4}$ . Q.E.D.

One limitation of this proposed scheme is that the proposed BTS with a non-uniform source is effective to SSC schemes, i.e., not applicable to normal superposition coding schemes.

#### 4. Numerical Results and Discussions

We assume that  $\Sigma_1 = \mathbb{E}[|h_1|^2] = 1.8$  and  $\Sigma_2 = \mathbb{E}[|h_2|^2] = 0.2$ . The total transmitted signal to noise power ratio (SNR) is  $P/N_0 = 40$  dB (the noise power is normalized as unit power) and  $\delta_{0,0} = \frac{23}{48}$ .

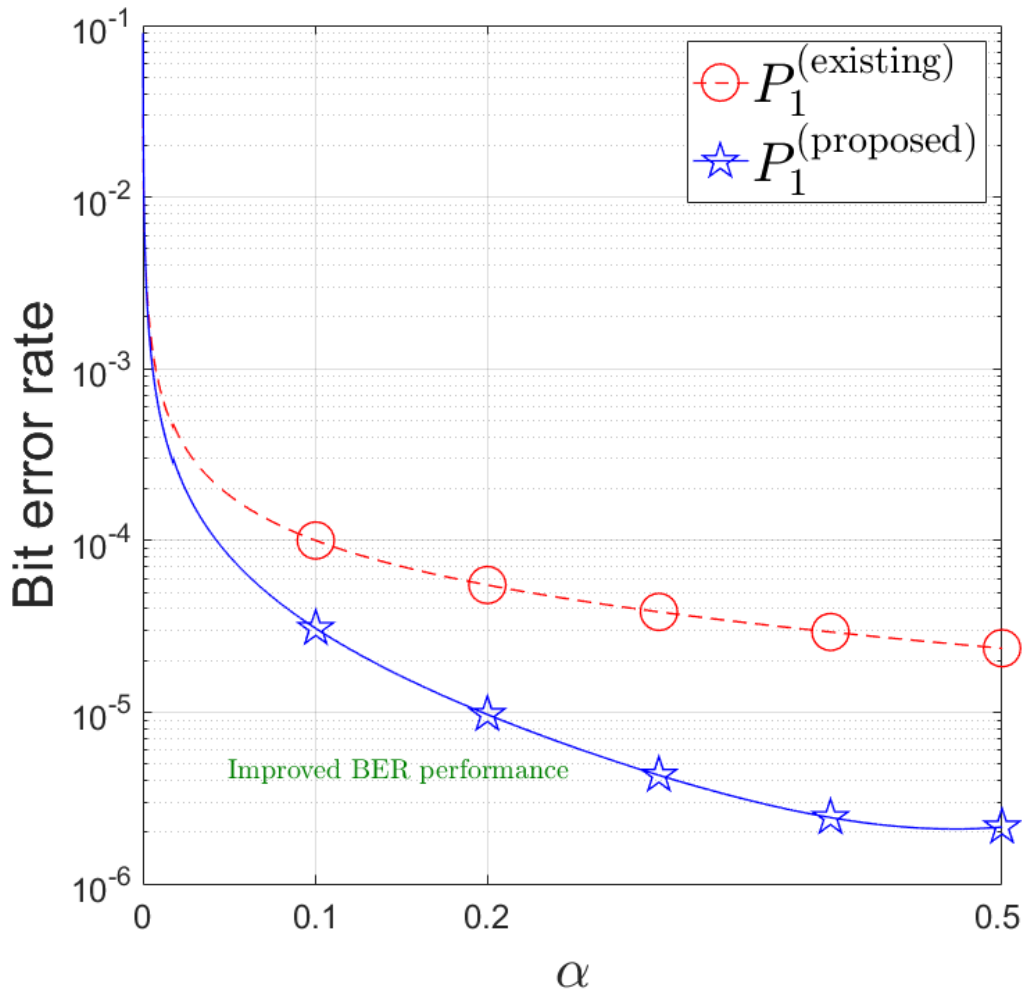
For a comparison, the approximate average BER can be expressed as [1]

$$P_1^{(\text{optimal MAP receiver: non-uniform})} \simeq +P(b_1 = 0)F \left[ \frac{\Sigma_1 P_A \left( \sqrt{\alpha} + \frac{N_0}{4\Sigma_1 P_A \sqrt{\alpha}} \log_e \frac{\delta_{0,0}}{\delta_{1,0}} \right)^2}{N_0} \right] - P(b_1 = 0)F \left[ \frac{\Sigma_1 P_A \left( 2\sqrt{1-\alpha} + \sqrt{\alpha} - \frac{N_0}{4\Sigma_1 P_A \sqrt{\alpha}} \log_e \frac{\delta_{0,0}}{\delta_{1,0}} \right)^2}{N_0} \right] + P(b_1 = 1)F \left[ \frac{\Sigma_1 P_A \left( \sqrt{\alpha} - \frac{N_0}{4\Sigma_1 P_A \sqrt{\alpha}} \log_e \frac{\delta_{0,0}}{\delta_{1,0}} \right)^2}{N_0} \right] + P(b_1 = 1)F \left[ \frac{\Sigma_1 P_A \left( 2\sqrt{1-\alpha} - \sqrt{\alpha} - \frac{N_0}{4\Sigma_1 P_A \sqrt{\alpha}} \log_e \frac{\delta_{0,0}}{\delta_{1,0}} \right)^2}{N_0} \right], \tag{7}$$

where

$$F(\gamma_b) = \frac{1}{2} \left( 1 - \sqrt{\frac{\gamma_b}{1+\gamma_b}} \right). \tag{8}$$

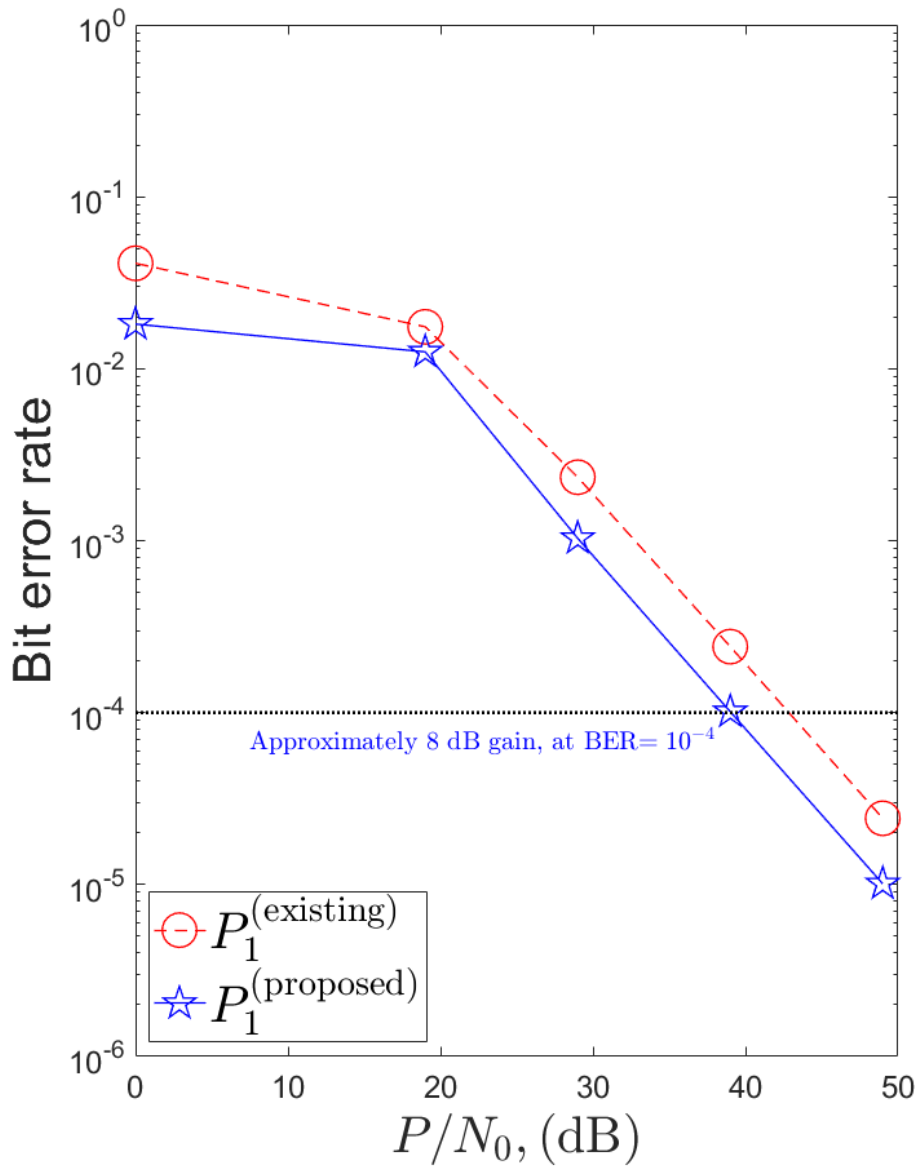
First, to compare the BER of the proposed BTS mapping scheme with that of the existing BTS mapping scheme, we depict the BERs, both for the proposed BTS mapping scheme and the existing BTS mapping scheme, in Fig. 1.



**Figure 1. Comparison of BERs of existing and proposed schemes for the non-uniform source SSC NOMA.**

As shown in Fig. 1, the BER performance of the proposed BTS mapping scheme improves, compared to that of the existing BTS mapping scheme, over the whole power allocation range, i.e.,  $0 \leq \alpha \leq 0.5$ . Note that such BER performance improvement of the proposed BTS mapping scheme can be achieved, only with a simple BTS design, without an increase of complexity and latency. In these simulations, we consider the various SNRs, i.e.,  $0 < P/N_0 < 5000 \simeq 33$  dB.

Second, to demonstrate the superiority of the proposed BTS mapping scheme over the existing BTS mapping scheme, the BERs are depicted versus the SNR,  $0 \leq P/N_0 \leq 50$  (dB), with the fixed power allocation,  $\alpha = 0.05$ , in Fig. 2.



**Figure 2. Comparison of BERs of existing and proposed schemes for the non-uniform source SSC NOMA, with varying SNR  $P/N_0$ .**

As shown in Fig. 2, the BER of the proposed BTS mapping scheme improves by about 8 dB, compared to that of the existing BTS mapping scheme, at the BER of  $10^{-4}$ . It should be noted that such SNR gain can be obtained without an increase of complexity.

## 5. Conclusion

In the paper, we proposed the smart bit-to-symbol BTS mapping of non-uniform source SSC NOMA. First, the analytical expression for the total allocated power of the proposed BTS mapping scheme was derived, and then we showed that the BER of the proposed BTS mapping scheme is improved, compared with that of the existing BTS mapping scheme. Moreover, based on the simulations, the SNR gain of the proposed BTS

mapping scheme over the existing BTS mapping scheme was calculated. As a future research, it would be interesting to provide more performance metrics, such as achievable data rates.

In result, the proposed BTS mapping could be considered for non-uniform source SSC NOMA, with the SNR gain, without an increase of complexity and decoding latency.

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