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Technical Note

Radioactive waste sampling for characterisation - A Bayesian upgrade

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ABSTRACT

Presented in this paper is a methodology for combining a Bayesian statistical approach with Data Quality Objectives (a structured decision-making method) to provide increased levels of confidence in analytical data when approaching a waste boundary. Development of sampling and analysis plans for the characterisation of radioactive waste often use a simple, one pass statistical approach as underpinning for the sampling schedule. Using a Bayesian statistical approach introduces the concept of Prior information giving an adaptive sample strategy based on previous knowledge. This aligns more closely with the iterative approach demanded of the most commonly used structured decision-making tool in this area (Data Quality Objectives) and the potential to provide a more fully underpinned justification than the more traditional statistical approach. The approach described has been developed in a UK regulatory context but is translated to a waste stream from the Fukushima Daiichi Nuclear Power Station to demonstrate how the methodology can be applied in this context to support decision making regarding the ultimate disposal option for radioactive waste in a more global context.

1. Introduction

Radioactive waste management should be conducted using a decision-making approach based on criteria. This should be determined in advance and based on sufficient analytical data. The disposal of radioactive waste requires the waste to be characterised sufficiently to allow the most appropriate route for disposal to be determined. This usually requires the characterisation of the waste to demonstrate that the waste fits within predefined boundaries. When the characterisation indicates that the waste is close to a boundary, demonstrating the characterisation has been completed appropriately to make that determination becomes more important.

Radioactive waste management policy and approach in the UK has developed rapidly over the last twenty years and this has driven the development of a more comprehensive and detailed approach to waste characterisation. Since 1985, the UK has produced volumetric estimates, updated every three years, of radioactive wastes currently stored in facilities (Stock) or to be generated into the future (Arisings) [1]. Prior to 2004 all low-level radioactive wastes (LLW), were consigned to the UK's only low level waste repository (LLWR) located in Cumbria. LLW is defined as radioactive wastes below an upper activity level of 4 GBq/t Alpha and 12 GBq/t Beta gamma.

Based on the volumetric estimates and the known capacity of the LLWR it was identified that given the rate of waste production from ongoing operations and future planned decommissioning, the existing disposal approach was unsustainable. The predicted future waste arisings far exceeded the available capacity within the LLWR facility, and a new approach was needed.

It was recognised by the UK Government that not all wastes consigned to the LLWR were of sufficient risk to require such post disposal controls. Between 2005 and 2007 the formation of the Nuclear Decommissioning Agency (NDA) and subsequently LLWR Ltd allowed for a specific focus on the disposal of LLW under a National LLW Programme. A policy document was published [2] which introduced the principles of the waste management hierarchy [3] along with two definitions of very low-level waste (VLLW) types; low volume and high volume VLLW, into the management of radioactive waste.

The introduction of this policy facilitated the development of waste diversion which in turn has led to the development of a range of different options for the treatment and disposal of LLW, particularly VLLW. Options now exist for disposal by incineration, shallow

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landfill in permitted hazardous waste facilities or in the case of metals, decontamination/smelting for recovery, volume reduction or recycling.

A common definition used to define VLLW is that the total activity of the waste must fall below an upper limit of 200 Bq/g [4] but each disposal or recovery route has different acceptance criteria determining the types of waste and the activities/radionuclide concentrations they are able to accommodate. It is this move away from disposal of all LLW to a single facility, to the segregation of waste into LLW and VLLW categories and the resulting development of numerous options for disposal or recovery that in turn, has driven the requirement for more comprehensive and fully underpinned waste characterisation.

In Japan, Fukushima Daiichi Nuclear Power Station (NPS), owned by Tokyo Electric Power Company Holdings Inc. (TEPCO), is progressing with decommissioning and works completed to date have resulted in a large amount of radioactive waste [5]. Concerning waste management, TEPCO is intently working on safe storage including dehydration of secondary waste from water decontamination as well as on volume reduction of combustible waste by incineration. In parallel, research and development (R&D) for future processing and disposal of the waste is in progress. To provide essential information with respect to waste properties for R&D, radiochemical analysis has been carried out [6].

Waste continues to be generated from the site and as such the population size is not yet fixed. This makes it difficult to establish a statistical definition of the nature of the total waste. The long-term management approach to safely manage the waste originating from the severe nuclear reactor accident has not yet been established. Developing this approach brings difficulties with setting criteria for technical decision-making. As such a flexible methodology for characterising the wastes, which can be adapted for various waste types will be required. Furthermore, given the uncertainty regarding both the criteria and population, a methodology that can adapt to changes in understanding of the waste will be particularly valuable.

The Data Quality Objectives (DQO) Process uses systematic planning and data review (historic and future data) for creating sampling strategies for a wide range of purposes. The DQO methodology was originally developed by the United States of America Environmental Protection Agency's (USEPA) as their recommended planning process when using data to select between two opposing conditions (such as in decision making) [7]. The outputs of the DQO Process then define the performance criteria to be implemented.

Waste characterisation is typically based on the analysis of a number of samples from the waste and comparing this against the specified boundaries. This can be done in a structured and auditable way within the DQO framework [7]. Disposal facilities require clear evidence that the mean activity is less than the specified upper boundary limit (e.g. 200 Bq/g for VLLW in the UK [4]) and this requires statistical hypothesis testing. The number of samples is then driven by the level of confidence required to demonstrate this difference. Where the average activity is close to the upper boundary it becomes more difficult to demonstrate this and so more samples are required. Sample sizes are traditionally calculated using Frequentist statistics, though this approach has some notable limitations particularly when dealing with uncertainty around the magnitude of the expected mean and the size and shape of the associated variability around this mean. It also relies on the assumption that the data is normally distributed which is not always the case.

The iterative approach of the DQO methodology is more aligned to the fundamental principles of Bayesian statistics, where understanding of the distribution about the mean can be updated based on an adaptive approach where new sampling is undertaken, or stop decisions made, on the basis of information from the previous campaign. It also allows for the appropriate level of confidence within a characterisation plan to be agreed without the sampling demanded becoming excessive or onerous.

This paper presents an approach which combines the DQO structured decision-making approach with Bayesian statistics to assist decision makers when waste is expected to be close to disposal boundaries. This approach is demonstrated using an example waste arising from the Fukushima Daiichi nuclear facility with disposal criteria taken from the UK regulatory framework.

2. Materials and methods

2.1. Iterative approach with DQO process

Full details of the DQO approach are found in the USEPA's referenced document [7] and it is not intended to discuss or critique the DQO methodology within this paper. However, it is important to consider the main steps in the process if the methodology is to be followed.

The DQO methodology comprises of seven main stages or steps and at the highest level, establishes a hypothesis for testing and ultimately determines how many samples are needed to test the hypothesis. The first steps allow the hypothesis to be clearly articulated (Step 1: State the problem, Step 2: Identify the Goal of the Study) along with the information requirements necessary to test the hypothesis (Step 3). Step 4 defines the boundaries of the study to be undertaken whilst Steps 5 and 6 describe the analytical approach to be taken and specify the performance/acceptance criteria necessary to determine if the hypothesis has been proven. Ultimately this process identifies in step 7, what additional data, if any, needs to be collected (the number of samples) to fully test the hypothesis to the required level of certainty if this has not already been achieved.

The approach to the DQO process in this paper has been simplified to demonstrate the methodology used (Fig. 1). Step 1 of the original process remains as it is considered crucial. Accurate framing of the question to be answered is critical if the process is to be a success. Steps 2 through to 5 are, in this example, combined and simplified. This part of the approach is establishing what data in an ideal scenario would be required to answer the requirements posed by Step 1 and what data is actually available (prior information). The focus is then placed on the third area which combines Steps 6 and 7 of the DQO process. Here we consider in more detail how the iterative approach of Bayesian statistics aligns with the iterative approach of the DQO methodology to deliver the most appropriate sampling and analysis program for the waste.

The strength of this approach is it is not necessary to complete the testing of the hypothesis in a single pass through the process. If it becomes apparent that the information available in the early stages of the process will not support the development of a sampling plan which will deliver the required levels of confidence, then some data collection can be undertaken and the process revisited until it can be demonstrated that the final data collection program can deliver the levels of confidence needed in the characterisation. A key strength is that if there is insufficient confidence to consign waste to disposal/further treatment then one or more further iterations may be undertaken. The Bayesian approach specifically builds on prior knowledge and is ideally suited to this iterative approach.

2.2. T-test with traditional (frequentist) approach

In order to demonstrate that the specified waste is below the VLLW/LLW boundary, it is ideal to take random samples from the

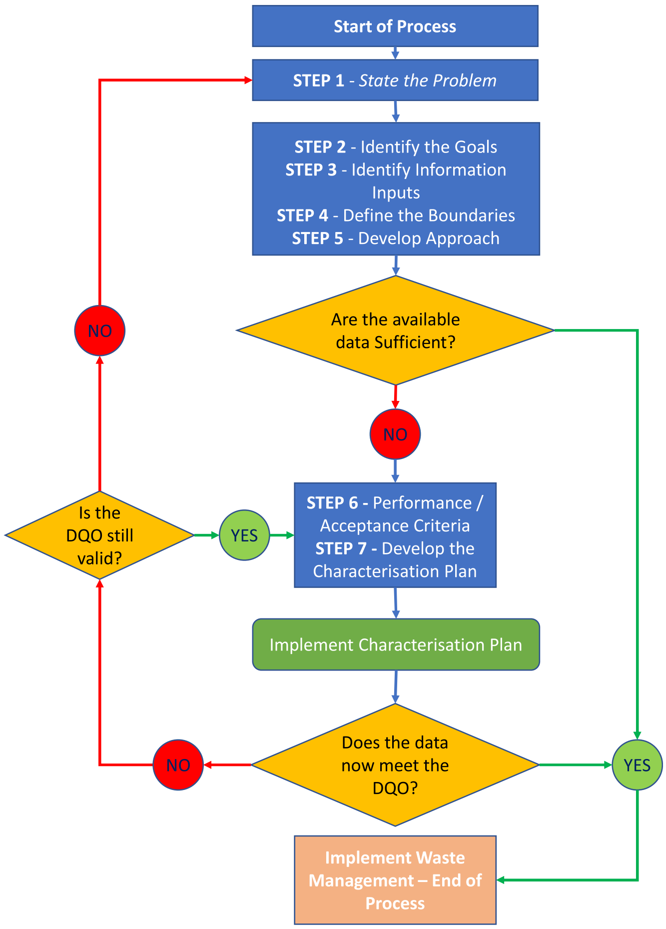

Fig. 1. Simplified DQO process model.

waste and compare this against those boundaries. These boundaries are referred to as the decision-making critical criteria values (C_c). The standard approach to these comparisons is to use a one sample *t*-test to compare sampled data against the C_c value, a methodology recommended in the guidance outlined in the Environmental Protection Act (The Act: EPA Part IIA) [8] and by the independent CL:AIRE organisation [9], for re-use or disposal from nuclear licensed sites [10]. Using the typical *t*-test formulae:

$$t_0 = \frac{(\bar{x} - C_c)}{s/\sqrt{n}} \quad (1)$$

Where t_0 is the *t*-test value, $\bar{x}$ is the mean contaminant concentration, C_c is the critical concentration of contaminants (VLLW/LLW boundary) and s is the sample standard deviation.

In Frequentist hypothesis testing we compare our test statistic (the theoretical mean μ) against our null and alternative hypotheses. This is described below for our case.

The null hypothesis is: $H_0: \ \mu > C_c$

The alternative hypothesis $H_1: \ \mu \le C_c$

Where, $\alpha = P(Reject\ H_0 \mid H_0\ TRUE)$ and $\beta = P(Accept\ H_0 \mid H_1\ TRUE)$ are referred to as the Type I and Type II errors respectively. Frequentist Power $(1 - \beta)$ conversely refers to the probability that the null hypothesis is correctly rejected $P(Reject\ H_0 \mid H_1\ TRUE)$.

For waste categorisation 95 % confidence is typically required to reject the null hypothesis, this corresponds to a significance level (Type I error rate) of $\alpha = 2.5\%$. The confidence interval for the mean is calculated as being between $\mu \pm t_{(\alpha,df)} \frac{s}{\sqrt{n}}$, where $df = n - 1$. This Frequentist confidence interval is an estimate of the amount of uncertainty associated with the sampling set but is often misinterpreted. If we were to find μ from a single sample set and we were to repeat this process 100 times (gaining 100 sets of independent samples) we would expect the true value of μ to lie within the 95 % confidence intervals 95 times out of 100.

Fig. 2 illustrates three scenarios where the mean and 95 % confidence interval of a sampling set is given relative to the critical criteria Cc value. Clear conclusions can be easily drawn from (i) and (iii). In scenario (i) the mean from the tested samples is clearly below the Cc value and when considering the range of results from those samples compared to the value of the mean, there is strong evidence to suggest that the theoretical mean of the sampled waste is below the Cc value. In this case the waste would be acceptable to the disposal facility or for further treatment. In scenario (iii) the mean appears to be above the Cc and as such the waste would not be accepted by the disposal facility and an alternative disposal route would need to be considered. Scenario (ii) is not definitive either way. The mean is less than the Cc value but the variability about that mean overlaps the Cc value. The disposal facility would not accept this as sufficient evidence to accept the waste. However, clearly there is a palpable reward for establishing a better understanding of this variability around the true mean. This is not however easily achieved with Frequentist hypothesis testing.

This approach to inference has two main limitations in our context.

1) The alternative hypothesis is never considered to be accepted; rather that the null hypothesis is rejected.

Whilst some understanding of risk with regards to false negatives (Type II errors) can be established through discussion of the Frequentist Power $(1 - \beta)$, this is often overlooked and does not directly attribute a spread of likelihoods to the alternative hypothesis. The lack of information about probabilities associated with the alternative hypothesis gives a somewhat incomplete picture for decision makers who may be considering the relative risks of continuing with a sampling campaign against alternative waste categorisation costs.

2) It gives limited opportunities for repeat sampling and incorporating previous knowledge/experience.

If decision makers choose to continue with sampling categorisation this is made more complex when using Frequentist hypothesis testing, which does not lend itself simply to iterative testing. If the outcome of sample set 1 is that more samples need to

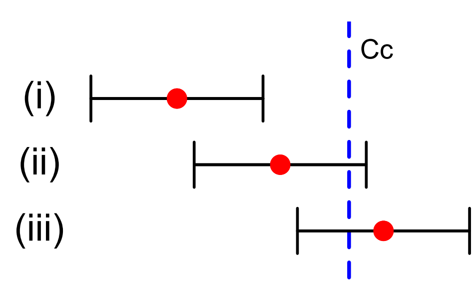

Fig. 2. Illustration of Frequentist means and confidence intervals against critical criteria Cc.

be taken, technically some adjustment should be applied to correct for the inflation of type I errors. This is known as multiplicity [11] and it has the effect of increasing the number of samples required because it increases the probability of observing chance findings.

2.3. Introducing Bayesian statistics

Bayesian statistics is not limited to the data collected but can use information from a wide number of sources, incorporating prior knowledge or beliefs as well as information from sampled data. In our case we are still interested in understanding the mean and uncertainty around that mean, however in Bayesian statistics that uncertainty is described as a credible interval. There are subtle but distinct differences between the Frequentist confidence interval and the Bayesian credible interval. Fig. 3 shows an example of a probability density function (pdf) curve; here we show a gaussian curve for simplicity. This shows the most credible value for the mean has a probability density of 0.4, in this case the probability that the mean is any other value decreases with distance from the peak. The shape of this distribution can vary depending on assumptions inputted by the user, which is a great advantage of using a Bayesian method compared to Frequentist methods which in the majority of cases, assume that the data is sampled from a "fixed" Normal distribution.

This image includes the critical criteria value Cc which shows here a small probability the that true mean is greater than the Cc value. This is more intuitive than the description of a Frequentist confidence interval.

In Frequentist statistics the data x may be described as normally distributed with $x = N(\mu, \sigma)$. For Bayesian statistics rather than μ and σ being fixed values (i.e. $\mu = mean(x)$ and $\sigma = sd(x)$), these are instead probability distributions described by their own hyperparameters. For instance, μ might be $\mu \sim N(M_\mu, S_\mu)$ where M_μ and S_μ are hyperparameters describing a normal distribution and σ might be $\sigma \sim Uniform(S1_\sigma, S2_\sigma)$ where $S1_\sigma$ and $S2_\sigma$ are hyperparameters describing a uniform distribution. Here μ and σ can take any distribution, not just normal or uniform.

The Bayesian method is based on an iterative process as shown in Fig. 4. It begins by defining a "Prior" distribution for each parameter. This is a baseline understanding of the distribution of the parameters used to characterise our waste. The scale and shape of these distributions can be determined from a variety of sources. This information may come from some initial sampling, expert opinion or inferences from the known source of the waste. Data is then collected and used to give a likelihood. The likelihood gives a distribution independently of the prior and tells us the likelihood of our mean taking each value given the observed data. This is analogous to the Frequentist likelihood function. The posterior distribution describes the uncertainty around the mean given the observed data. The posterior distribution is proportional to the prior multiplied by the likelihood. This concept is demonstrated graphically in the example probability density function curves also given in Fig. 4.

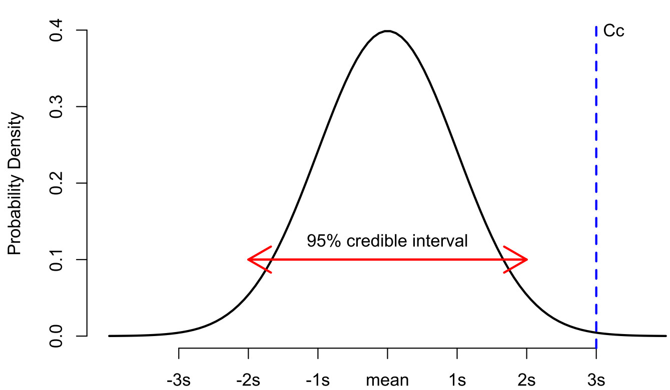

Fig. 3. Illustration of a Bayesian credible interval.

Following the logic of Bayes' rule the posterior pdf of θ (where θ is a parameter such as the mean) is given by

$$f(\theta|y) = \frac{f(\theta)f(y|\theta)}{f(y)} \tag{2}$$

Where $f(\theta)$ represents the prior and $f(y|\theta)$ represents the likelihood. For continuous $\theta, f(y) = \int f(\theta)f(y|\theta)d\theta$

Whilst it is possible to evaluate this integral by hand when the prior is conjugate (having the same distribution as the posterior), the process is simplified by using computer based iterative algorithms to estimate these probability density functions. This also allows for the extension of the methodology to use non-conjugate priors. The software used in this paper is from the statistical software R [12], using JAGS [13].

2.4. Bayesian sample size estimation for the one-sample T-test

John Kruschke [14] developed a Bayesian estimation alternative to the *t*-test called Bayesian Estimation Supersedes the T-test (BEST). The methodology has been included as a package [15] within the statistical programming software R [12]. In Frequentist *t*-testing it is assumed that the test statistic "*t*" as given from Equation (1) follows the t-distribution. However, BEST assumes that it is the data, x, that follows the *t*-distribution. The diagram in Fig. 5 shows the parameters that feed into the determination of x, these are μ, σ and ν. In our scenario x should be thought of as the standardised equivalent of the raw data, such that $x_i = \frac{r_i - \bar{r}}{s}$, where $r_i$is the raw observation, $\bar{r}$ and s are the mean and standard deviation of the r_i respectively.

The prior distributions of the parameters are $\mu \sim N(M_\mu, S_\mu)$, $\sigma \sim \Gamma(M_\sigma, S_\sigma)$ and $\upsilon \sim \Gamma(M_\upsilon, S_\upsilon)$. The parameter υ represents the degrees of freedom with smaller values of υ being better able to accommodate outliers, reducing the impact of these outliers on the estimates of μ and σ.

It is assumed that the raw data x, follows a t-distribution giving a likelihood function as $x \sim t(\mu, \sigma, \upsilon)$.

The purpose of the analysis is to find the posterior probability that μ is less than the critical value C_c given the observed data x, written as $P(\mu \le C_c|x)$.

Bayesian Power can be interpreted as "the probability of meeting the goals of the study given initial information or assumptions about the population parameters" [15]. In this case it is the chance that the posterior probabilities will be less than Cc, if a certain number of data points are collected from the specified population. Power estimates can be given retrospectively to determine whether sufficient samples have been collected to confidently interpret the results. They can also be used prospectively to inform sample size estimation and decision making for sampling campaigns. This is done through repeat simulation for a specified number of samples and comparing the likely distribution of the underlying parameters given the sampled data.

BEST is an example of an "off the shelf" package that could be used for robust estimation of the mean and standard deviation of a sample to accommodate outliers [14] as a Bayesian alternative to the Frequentist *t*-test. There are alternative approaches and indeed this approach could be modified to allow for a greater variety of underlying distributions of the parameters to be considered.

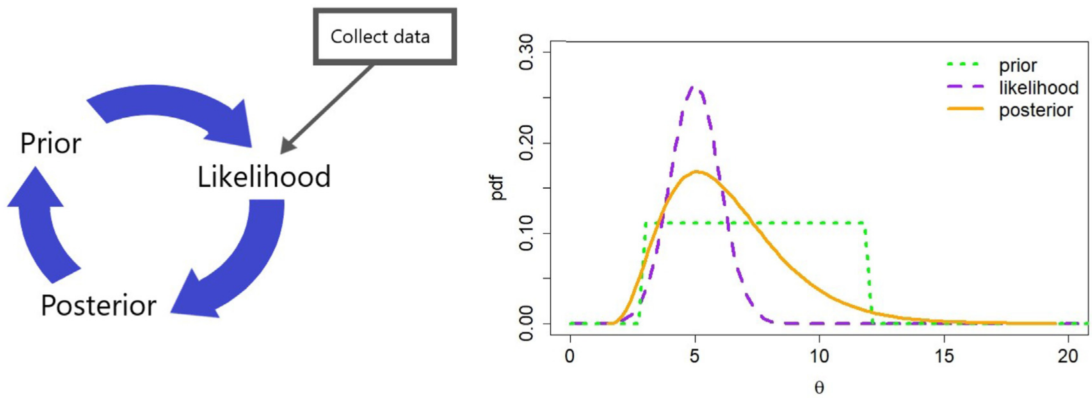

Fig. 4. The Bayesian Cycle.

2.5. *Demonstration based on accident waste*

Based on the UK regulatory framework for waste disposal and to demonstrate the iterative approach using Bayesian t-tests, trials of the DQO process were undertaken for the selected waste of Fukushima Daiichi NPS.

The UK has developed an approach to characterisation over the last 20 years, driven in part by implementation of the long-term waste strategy [2]. In Japan, there is a less well-established strategy for the disposal of radioactive wastes. Applying the UK regulatory framework to data around VLLW/LLW boundary taken from a selected Fukushima Daiichi NPS waste streams is presented to demonstrate this approach.

At Fukushima Daiichi NPS, diverse waste, which includes rubble, secondary waste from contaminated water treatment, used protective clothes, vegetation and soil, has been generated and stored inside the site. Within this paper the fallen trees have been used as an example to demonstrate the methodology developed.

To install facilities and decontaminated water tanks, trees inside the site were mostly cut down, and comprises 134,000 m^3 [16] of waste. The trees were separated into two sub-categories of waste for temporary storage. The leaves, small twigs and branches comprising the most contaminated materials were chipped and placed in covered rows and the less contaminated trunks stored separately; whole and open to the atmosphere. It was planned to incinerate them until 2025 at a new incineration facility, which is being built [17]. Leaf and branch of living trees inside the site were sampled and subject to radiochemical analysis. The data obtained was used to calculate probability density distributions for use with Bayesian statistics [18].

3. Results

3.1. *Problem statement*

The problem statement was defined as *"To provide relevant information to characterise the properties (radiological, chemical & physical) of fallen trees for incineration and disposal via available UK*

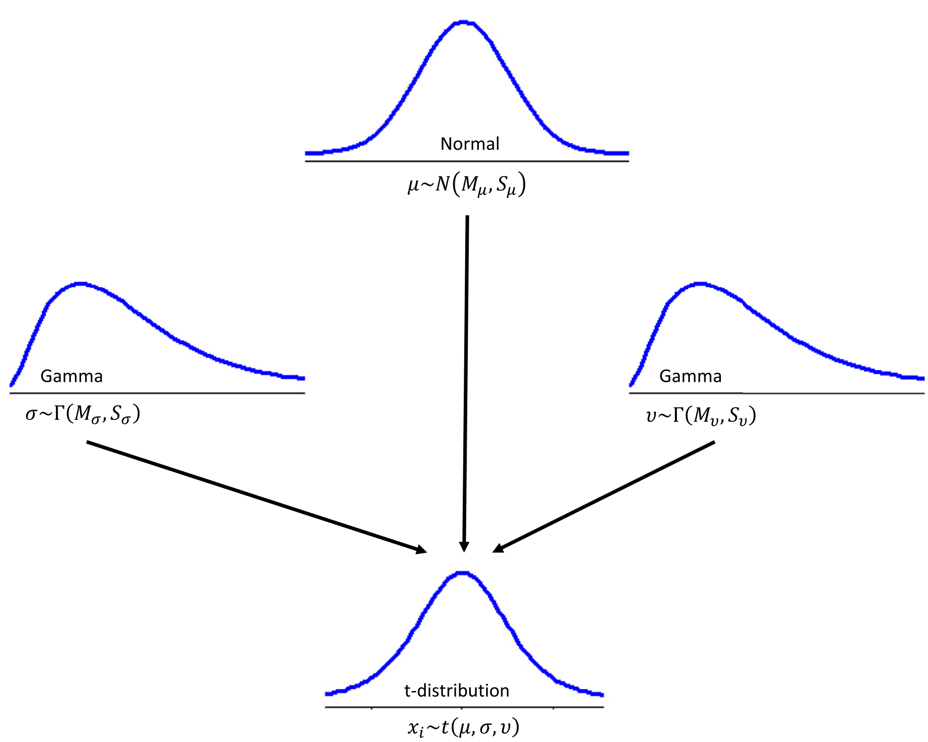

Fig. 5. Image of distributions used in the BEST package [15] based on examples given in vignette.

routes."

The goals were defined as;

(i) Identify percentage volumes of fallen trees aligned against categories within the waste hierarchy (re-use, recycle, disposal) with the aim being to move as much waste up the hierarchy, away from the disposal option as possible.
(ii) Identify the requirements for incineration.
(iii) Identify the requirements for disposal.

The available data for use in the assessment was limited to;

- The origin of the waste stream.
- The known volume of the waste stream.
- The methods of segregation and storage of the two subsets within the waste stream.
- The method of contamination of the bulk material prior to becoming waste.
- Seven samples had been taken from one subset of the waste (leaves, twigs and branches) and analysed for a range of radionuclides.
- If incinerated, the volume of the waste for disposal would be reduced to 0.5 % of the original waste volume [19].

3.2. Baseline information

Fig. 6 shows a histogram of the available data collected from the leaves and branches of the trees. This data was taken prior to incineration and the levels of activity place this waste well below the upper threshold for LLW and in one sample also below the VLLW threshold.

The initial assessment conservatively assumed that all activity within the waste remains within the ash (there would be no loss of activity up the stack during incineration although in reality this is known to occur) and the target is to achieve an incinerated waste form suitable for long term disposal at an VLLW site (Data Assumption 1).

No analytical data was available for the trunks therefore current (prior) information is based on activity in the trunks being $0.001\times$ *Activity in leaves/branches* [20] (Data Assumption 2). Based on this information the ash was estimated to be classified as approximately VLLW as shown in Fig. 7 with a mean activity of 159 Bq/g and a standard deviation of 103 Bq/g.

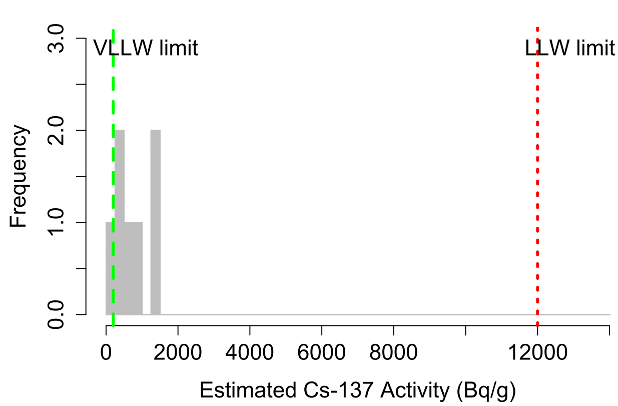

Fig. 6. Available sample data for Cs-137 (Bq/g).

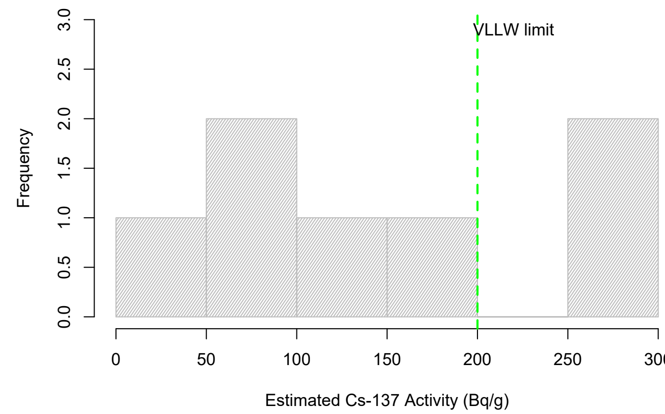

Fig. 7. Estimated activity in the ash for Cs-137 (Bq/g).

3.3. Frequentist Power and sample sizes

The Frequentist approach gives a 95 % confidence interval around the mean of (83, 235) and *t*-value of -1.054 ($p = 0.1661$ where p (known as the p-value) is the probability that the t-value will be at least as extreme as -1.054 if the null hypothesis $H_0: \mu > C_c$ is true) when compared with the Critical value C_c of 200. Clearly more data is required to confidently determine if the true mean is less than the Critical value.

Fig. 8 shows a Frequentist Power curve by various sample sizes. This is based on a Type I error rate of $\alpha = 0.05/2$, $C_c = 200$, $\bar{x} = 159$and $s = 103$ (where the effect size $d = (159 - 200)/103$) and has been calculated for a range of samples sizes using the pwr.t.test() in the package pwr [21] within the statistical programming software R [12]. Fig. 8 indicates that 90 % power is achieved at approximately n = 60.

However, these calculated powers rely on the assumptions around the prior information to be true, or at least sufficiently conservative to achieve this power. This approach gives very limited information with which to make decisions and brings with it quite high risks.

3.4. Bayesian Power and sample sizes

The relevant sample sizes were assessed using the Bayesian approach taken from the BEST package described in section 2.4. The data presented in Fig. 7 was used in the BESTmcmc() function to establish an initial posterior distribution. This was based on broad priors specified by Kruschke [15] $\mu \sim N(159, 1000 \times 103)$, $\sigma \sim \Gamma(103, 103 \times 5)$, and $\upsilon \sim \Gamma(30, 30)$. The probability densities of the mean are given in Fig. 9 (note to assist in interpretation, the unstandardised data is shown). Here it is clear that the 95 % HDI (46.5, 267) overlaps the Cc value of 200 Bq/g (the upper boundary of VLLW).

This initial posterior was then used in the BEST function BESTpower() to estimate prospective power for various sample sizes of interest. The results of these calculations are presented in Fig. 10. From this Figure it can be read that even with a sample size in the order of two hundred samples under these assumptions, only a Power of approximately 75 % is likely to be achieved. The difference in Power estimates (when compared with the Frequentist approach) arises from the Bayesian Power calculations incorporating information about reliability of using a small sample size (n = 7) to establish μ and σ by simulation from the raw data. These are assumed to be fixed values in the Frequentist approach. The above example uses wide priors based on limited data. Percentage

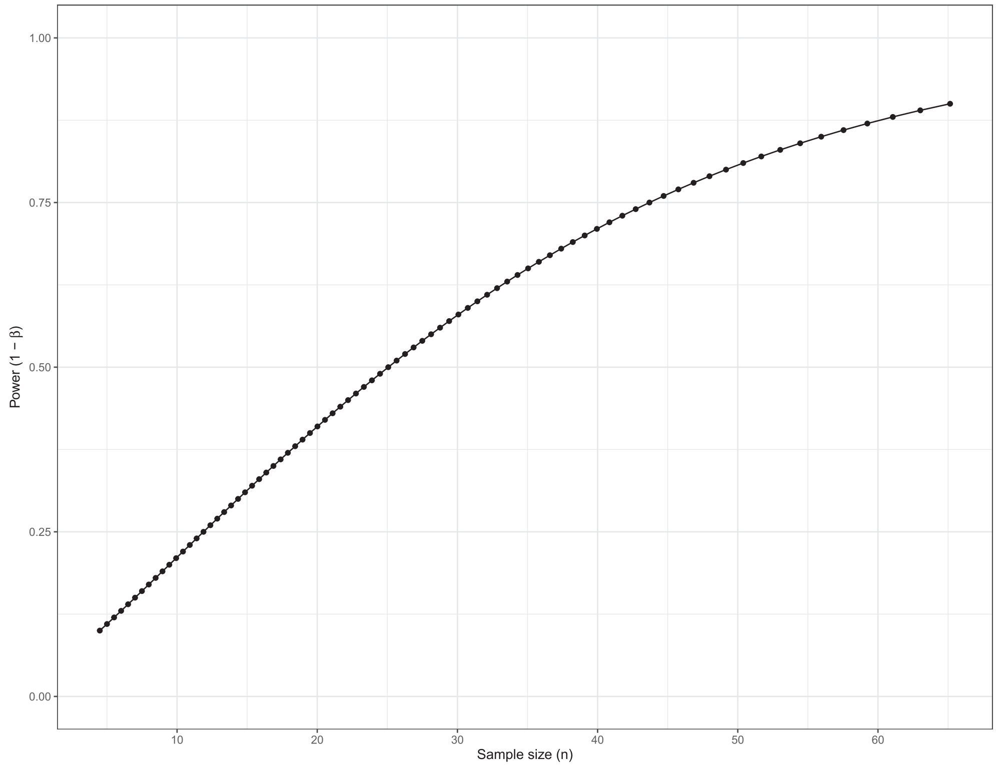

Fig. 8. Frequentist Power against sample size.

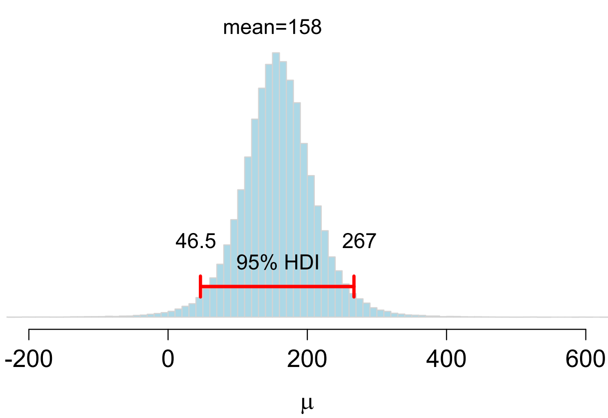

Fig. 9. Posterior Distribution for the mean (from 7 related samples).

powers more analogous to the Frequentist approach can be found by artificially narrowing the distribution around each of the parameters μ, σ and υ. For instance, an alternative posterior could be found using the values taken from the above posterior as a prior, reducing the variability around μ, σ and υ to 1 % and incorporating the same likelihood. The power estimate from this alternative posterior is approximately 88 % for a sample size of n = 60.

The overall message however is that it is much more difficult to be confident about the expected results from a future sampling campaign than is indicated by the simple Frequentist sample size power curve. Without this information, decision makers may be overly confident in their expectations of achieving a successful study outcome.

Clearly taking such a large number of samples is unlikely to be practicable. It is also unwise, because our prior understanding is based on extrapolation from a related population (leaves and twigs) as well as two key assumptions on how the activity is concentrated within the ash (Data Assumptions 1 & 2 given in section 3.2). This is true for both Bayesian and Frequentist sample size estimates. However, a key strength of the Bayesian approach is that any new data from a further sampling campaign will improve our understanding of the mean and associated uncertainty, without risking the outcome of the project on a specific number of samples. This information allows the decision makers to be fully aware of the risk and benefits of further sampling.

The number of samples required for the first round of sampling typically depends on practical considerations and should be guided by an understanding of the Power. Fig. 10 indicates that this is likely to be at approximately twenty samples since additional samples have a relatively small impact on this probability. The Bayesian analysis then incorporates this new data to determine whether additional sampling and analysis is required to further facilitate decision making.

4. Discussion

The Bayesian approach better complements the DQO methodology where iterative sampling is anticipated. Here practical constraints and an understanding of expected statistical gains from increased sample sizes led decision makers to use a first round of sampling that mitigated the risks of failure (through a better understand of Power arising from uncertain prior information). This led to an initial sample size of twenty compared to the alternative frequentist sample size of fifty-six. This reduction in sample numbers represents both a significant cost saving as well as demonstrating ALARP. The time taken for operatives to undertake sampling and analysis work is significantly reduced along with their associated dose uptake.

From the scenario presented there are three possible outcomes post sampling. These are presented in Fig. 11.

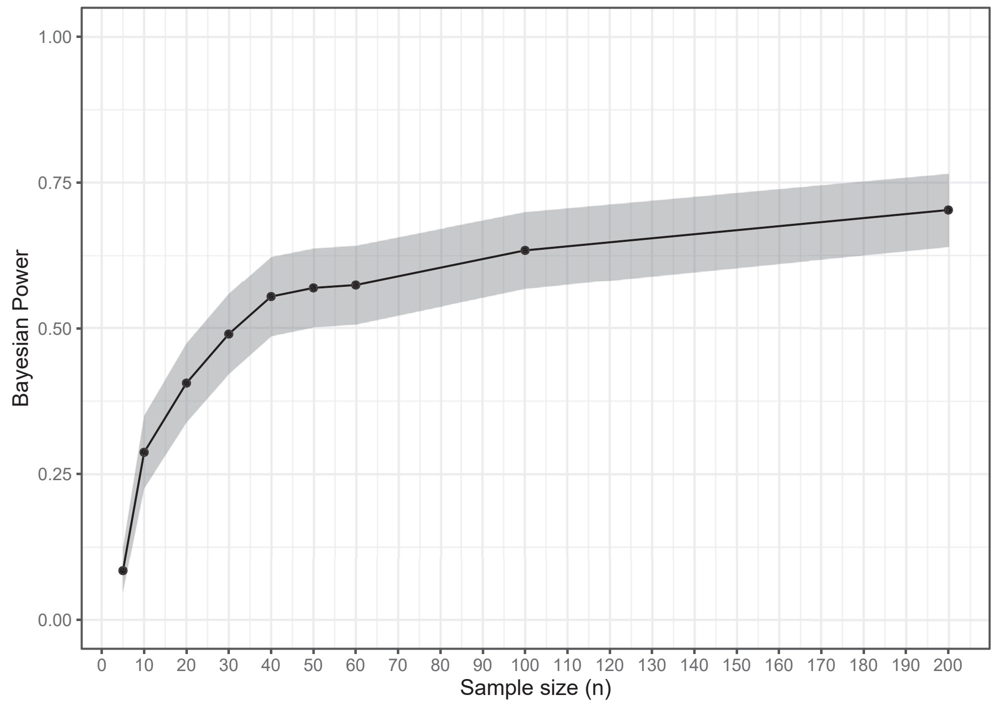

Fig. 10. Bayesian Power against sample size.

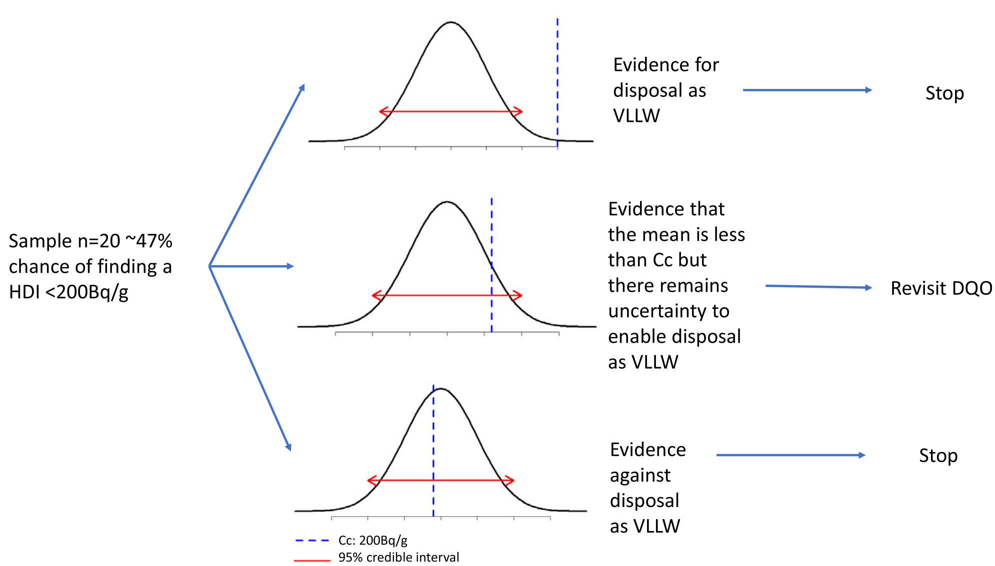

Fig. 11. Potential outcomes of sampling.

1) Firstly, the new data may be sufficiently different to our assumed prior such that the posterior distribution moves clearly into the VLLW categorisation. No further work is required, and the waste would be accepted by the disposal facility.
2) Secondly, the new data results in a posterior mean greater than the Cc value and so moves clearly into LLW categorisation. Therefore, it is highly unlikely that further sampling would result in successful evidence to dispose of the waste as VLLW.
3) Thirdly, as shown in the middle scenario in Fig. 11 the most likely mean is less than the *Cc* value, but the credible interval still overlaps the Cc value. In this circumstance the team would revisit the DQO process and determine how many more samples would be required to confidently narrow the credible interval. Allowing stakeholders to make data driven decisions with a greater understanding of perceived risk of success/failure based on more appropriate data.

This methodology clearly demonstrates a benefit of using a Bayesian approach over a Frequentist approach when developing a characterisation plan, particularly when the waste is near a classification boundary.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.net.2021.07.042.

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