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RESEARCH ARTICLE

Effects of a Flipped Classroom using Khan Academy and Mathematical Modeling on Overcoming Difficulties in Learning Mathematics

Jiyoon Lee[1], Dongjo Shin[2]

[1] Visiting Scholar, Boise State University

[2] Professor, Mathematics Education, Pusan National University

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Abstract

This study examined difficulties middle school students have in learning mathematics and proposed a flipped classroom consisting of Khan Academy activities, small-group problem solving, and mathematical modeling to help improve their learning. A mixed-method approach was used to identify difficulties students have in learning mathematics, explore how the flipped classroom helped them reduce the learning difficulties identified, and examine if there were differences in students' mathematics achievement and their affective characteristics after participating in the flipped classroom. Qualitative analyses showed that students had difficulties in understanding mathematical concepts and finding effective ways to learn as well as negative views towards learning mathematics. This study also found that each activity of the flipped classroom had a different impact on student learning. Before class, the Khan Academy activities were most likely to help students understand mathematical concepts. In class, small-group problem solving activities were most helpful for students who had trouble finding effective learning methods and environments. Mathematical modeling activities were most likely effective in changing students' negative views towards mathematics. A quantitative analysis showed that the flipped classroom not only significantly improved the students' mathematics achievement, but also positively affected their confidence and motivation and how much they valued learning mathematics.

Keywords Mathematics Education, Flipped Classroom, Khan Academy, Mathematical Modeling, Learning Difficulty in Mathematics.

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- Corresponding Author: DONGJO SHIN, email: djshin@pusan.ac.kr

I. INTRODUCTION

Mathematics is a fundamental subject necessary for understanding other domains such as statistics, science, engineering, information technology, and management. Many students, however, struggle with learning mathematics. The most recent UNESCO report shows that more than half of students around the world are not achieving the minimum proficiency levels in mathematics (UNESCO Institute for Statistics, 2017). Although students in East Asian countries have consistently taken the top places in mathematics achievement in large-scale international comparisons of mathematics achievement, they were less likely to enjoy and be motivated in learning mathematics (Zhu & Leung, 2011). Early research on students' struggles with mathematics attributed this to a lack of understanding of the basics, like number facts, visual-spatial aspects of mathematics, and symbol systems (e.g., Shalev et al., 2001). Students' difficulties in learning mathematics can also be attributed to teachers' teaching methods, especially in East Asian countries. In their cross-cultural study, Cai and Wang (2010) found that teachers in China tended to focus on abstract reasoning about mathematical concepts, solid mathematical knowledge, and memorization before understanding. Previous studies showed that memorization- and procedure-based thinking was a significant factor leading to students' difficulties in learning mathematics (Lithner, 2000) and that performance-based instruction, unlike instruction that focused on understanding, was likely to undermine students' interest in mathematics (Carmichael, Callingham, & Watt, 2017).

With the rapid advance in instructional technology, the flipped classroom model has been drawing the attention of researchers in education over the last decade. Although the ways in which teachers manage a flipped classroom vary (Chiang & Chen, 2017), its core idea is to reverse "the traditional arrangement of delivering basic knowledge in class to the time outside the classroom" (Hwang & Lai, 2017, p. 185). Recent reviews on flipped classrooms, however, have reported challenges teachers encounter when implementing them; the challenges are mainly related to the out-of-class stage. The most frequently reported challenge by teachers was the extra time required for pre-recording video lectures (Akçayır & Akçayır, 2018; Lo, Hew, & Chen, 2018). Other pedagogical issues with implementing the flipped classroom were related to difficulties in managing students' learning status and in providing them with feedback and help on their out-of-class work (Lai & Hwang, 2016). To overcome the barriers reported in the previous studies on the flipped classroom, the current study used the Khan Academy platform, which provides numerous video clips and practice exercises about mathematics so that teachers do not have to spend a large amount of time creating instructional videos before class (Zengin, 2017). Khan Academy also allows teachers to monitor their students' learning status in real time and make a discussion board if necessary (Cargile & Harkness, 2015; Ruipérez-Valiente, Muñoz-Merino, Leony, & Kloos, 2015).

As for the in-class stage, researchers have revealed that students are more likely to be engaged in mathematics if real-world problems are used in class activities (Lo & Hew, 2017; Tawfik & Lilly, 2015). A review on flipped math classrooms, however, reported that only a few studies used real-world problems when engaging students in the in-class activities (Lo et al., 2018). In this respect, mathematical modeling can be considered a suitable in-class activity because it generally "begins with a situation in the real world" (de Almeida, 2018, p. 19) and provides the opportunity for students to resolve a real-world problem using their mathematical knowledge (Bliss & Libertini, 2016; Blum & Ferri, 2009; Çekmez, 2019). In addition, developing students' modeling competency has been emphasized in mathematics education reforms (e.g., the Common Core State Standards Initiative [CCSSI], 2010). Therefore, the current study integrated modeling activities into the flipped classroom.

Many studies have examined the effectiveness of the flipped classroom approach in mathematics education, and some have further explored how the flipped classroom affects students of different backgrounds and cognitive and affective levels (e.g., Bhagat, Chang, & Chang, 2016; Gilboy, Heinerichs, & Pazzaglia, 2015; Hung, Sun, & Liu, 2019). Few studies, however, have examined the effects of specific components of the flipped classroom on the learning outcomes of students with different types of difficulties in learning mathematics. Moreover, more than 80% of studies on flipped mathematics classrooms were conducted in the context of higher education (Yang, Lin, & Hwang, 2019). To deal with these research gaps, this study aims to investigate the effects of a flipped classroom using Khan Academy and mathematical modeling (FCKAMM) on the learning outcomes of middle school students with distinct difficulties in learning mathematics. The research questions that guided this study were:

1. What are the difficulties in learning mathematics that middle school students have?
2. How does FCKAMM help middle school students reduce their difficulties in learning mathematics?
3. Are there significant differences in middle school students' mathematics achievement and their affective characteristics after participating in FCKAMM?

II. RELATED LITERATURE

Flipped Classroom

Over the last decade, growing attention has been paid to the flipped classroom as it provides students with the opportunity to participate in meaningful learning activities, instructor-guided problem solving, and discussions (Chen, Wang, Kinshuk, & Chen, 2014). In contrast, students in traditional lecture-based learning environments have to learn numerous mathematical concepts during class and thus have no time to engage in and discuss mathematics with others. Therefore, the flipped classroom model results in a paradigm shift from the teacher-centered approach to a student-centered approach (Kong, 2014). Many research studies have demonstrated positive effects of the flipped classroom on students' mathematical achievement. Bhagat et al. (2016) revealed that the flipped classroom approach positively affected high school students' mathematical achievement and motivation. Further analysis indicated that low achievers were more likely to improve their mathematical performance when engaging in a flipped classroom. In addition, research studies demonstrated positive effects on students' motivation, perceptions, and attitudes towards learning mathematics when they engaged in flipped classroom environments (e.g., Gilboy et al., 2015). Hwang and Lai (2017), for example, showed that an interactive e-book-based flipped classroom approach improved students' self-efficacy for learning mathematics, especially for students who had lower self-efficacy. Despite the benefits of implementing the flipped classroom, researchers have still raised doubts about its feasibility due to the extra burden on teachers to create educational videos and the difficulty of managing student learning outside of class (Akçayır & Akçayır, 2018; Lo et al., 2018; Yang et al., 2019).

Researchers have designed flipped classrooms in different ways. Lo and Hew (2017), for example, integrated an instructional design framework, the First Principles of Instruction, into their flipped mathematics classroom to examine its impact on high school students' cognitive and affective characteristics. Zengin (2017) used the Khan Academy platform and mathematics software (Geogebra) to examine the effect of a flipped classroom on undergraduate students' mathematics achievement. However, because a lack of knowledge about and experience in designing a flipped classroom can affect the efficacy of this approach (Overmyer, 2015), some researchers have voiced a need for further research drawing on an appropriate design principle or a conceptual framework (Lo et al., 2018; Song, Jong, Chang, & Chen, 2017).

Khan Academy

Founded in 2008 by Salman Khan, Khan Academy aims at creating an online learning environment where countless instructional video clips and exercises from the elementary to college level are provided for free. Khan Academy provides personalized self-paced tutoring and immediate feedback to students based on their learning progress data (Cargile & Harkness, 2015; Phillips & Cohen, 2015; Zengin, 2017) and points and badges that may motivate students to learn mathematics outside of their classrooms (Abramovich, Schunn, & Higashi, 2013). In addition, Khan Academy provides learning analytics which allows teachers to see if their students watched a video clip, started an exercise, or struggled with an exercise (Ruipérez-Valiente et al., 2015) and modify their instruction accordingly (Cargile & Harkness, 2015). Although only a few empirical studies have been conducted to examine its effectiveness in teaching and learning mathematics, the results indicated that Khan Academy positively decreased students' mathematics anxiety and improved their confidence in mathematics as well as their achievement (e.g., Murphy, Gallagher, Krumm, Mislevy, & Hafter, 2014). Furthermore, a flipped classroom integrated with the Khan Academy demonstrated potential to improve college students' understanding and retention of an advanced mathematical concept with visualization (Zengin, 2017). Therefore, the current study integrated Khan Academy into the flipped classroom to provide students with individualized learning and to help teachers understand students' learning progress outside of the classroom.

Mathematical Modeling

The current reforms in mathematics education emphasize the importance of integrating mathematical modeling into the teaching and learning of mathematics (e.g., CCSSI, 2010). Although the definition of mathematical modeling varies among researchers (Kaiser & Sriraman, 2006), it mainly concerns "a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena" (Bliss & Libertini, 2016, p. 8). In a modeling activity, students are engaged in simplifying a complex real-world phenomenon by identifying essential variables (Kaiser & Schwarz, 2006), translating the simplified reality into a mathematical expression, finding the mathematical solution, and interpreting the solution in the context of the real world phenomenon (Blum & Ferri, 2009). Studies have indicated that mathematical modeling activities promote interdisciplinary learning (Delice & Kertil, 2015), motivate students to engage in mathematics, and deepen their understanding of mathematics (Zbiek & Conner, 2006). As an important cognitive process required by mathematical thinking, mathematical modeling activities also encourage students to use various representation systems to better understand mathematical concepts and real-world situations (Ferrucci & Carter, 2003). Although prior studies have reported the importance of learning mathematics through modeling (Ç ekmez, 2019; Zbiek & Conner, 2006), researchers have not yet examined the effect of mathematical modeling within the flipped classroom model on students' mathematical achievement and attitude towards mathematics. The current study integrated modeling activities into the flipped classroom to explore their impact on students' learning.

III. METHODS

Participants and Procedure

The flipped classroom in this study was based on a project designed to provide learning opportunities for Korean middle school students who had trouble learning mathematics. To be clear, these were not students with learning disabilities in mathematics (e.g., a neurodevelopmental deficit or dyscalculia) but rather those who wanted to engage in the flipped classroom due to difficulties they had in learning mathematics regardless of their abilities. We asked the students to participate in a pre-lesson using Khan Academy and describe the reasons for applying for this project. After the pre-lesson, we analyzed the students' activities in Khan Academy and finally invited 18 students who actively participated in the pre-lesson.

Design and Implementation

The flipped classroom in this study was designed based on Bloom's revised taxonomy of the cognitive processes of learning: remembering, understanding, applying, analyzing, evaluating, and creating (Krathwohl, 2002). Traditional instruction was likely to focus on lower levels of the taxonomy such as remembering and understanding. However, Gilboy et al. (2015) demonstrated that, in such an innovative learning environment as a flipped classroom, teachers have the potential to create a learning environment in which students engage the higher-level cognitive processes of learning by analyzing relationships between mathematical concepts, evaluating different solution strategies, and creating new problems and solutions. Chiu and Mok (2017) stated that using the higher levels of cognition in Blooms taxonomy allows students to deepen their conceptual understanding of mathematics. Wenglinsky (2002) demonstrated that both low- and high-performing students can benefit from lessons designed to promote both lower- and higher-order thinking.

Therefore, we designed our flipped classroom by creating three activities based on different levels of cognition according to Bloom’s taxonomy, which consisted of 20 one-hour sessions and extra out-of-class learning with Khan Academy. During their out-of-class time, the students were asked to watch video clips about a concept of function based on their own level of cognition and solve relevant exercises given by Khan Academy. In class, the students were engaged in small-group problem solving activities based on their learning with Khan Academy[1]. We encouraged the students to participate actively in small-group discussions to solve advanced mathematics problems together. Each group of students solved problems collaboratively by sharing their own knowledge and by breaking complex problems down into more manageable parts. Furthermore, the students were engaged in modeling activities with their peers. Using a data set, for example, the students were asked to create a mathematical model for estimating how old their own companion animal is based on an existing set of data. They were also asked to analyze real-world data and create a mathematical model for predicting how many people will use social networking services such as Facebook and Instagram. Since there was no a single correct answer in the modeling activities, students were asked to justify their solutions.

Data Collection

To address the first and second research questions, we collected qualitative data using two sets of open-ended questionnaires. Before the class, the students were asked to describe in detail their difficulties in learning mathematics. After the class, we asked the students to write how each activity of the flipped classroom affected their learning. To investigate the impacts of the flipped classroom on learning outcomes (i.e., research question 3), pre- and post-tests of students’ mathematics achievement and affective factors were administered. The pre-test and post-test of mathematics achievement were comprised of 20 short-answer questions and were designed to be roughly equivalent in level of difficulty. Three mathematics teachers participated in ensuring the validity of the tests, and the preliminary versions of the assessments were pilot tested with an initial sample of 5 students. The 20 questions were given five points for each correct response, resulting in a total of 100 points. The pre- and post-questionnaire on affective factors were identical and

[1] The 18 participants were assigned to heterogeneous groups based on their prior achievement level.

modified from those developed by the National Assessment of Educational Achievement in South Korea (Lee, Rim, Park, Seo, & Kim, 2016) to measure students' affective domains related to mathematics (e.g., "I am good at mathematics" and "I enjoy learning mathematics"). The questionnaire was comprised of 20 to assess the four affective domains of confidence, interest, value, and motivation and used a 4-point Likert-type scale (1="Totally disagree" to 4="Totally agree"). In terms of reliability, Cronbach's alphas for each affective domain were between .82 and .92.

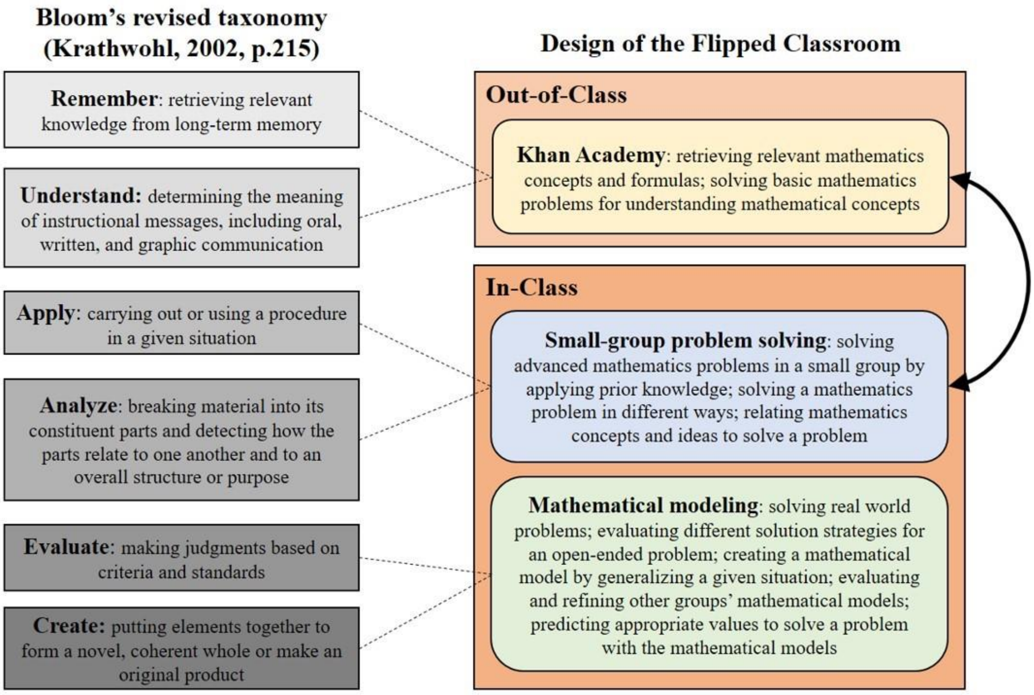

Figure 1. Design of the flipped classroom based on Bloom's revised taxonomy

Data Analysis

The data collected from the pre- and post-test were analyzed using the Wilcoxon signed-rank test to examine the extent to which the students' cognitive and affective domains changed. The data collected from the students' written responses were analyzed using an inductive content analysis (Elo & Kyngäs, 2008) to identify themes of students' difficulties in learning mathematics and to examine the ways in which the flipped classroom activities helped them reduce the difficulties identified. We first read through the students' written responses and selected units of analysis. A unit of analysis was a text chunk from the students' responses that represented a single theme relevant to our research questions (Elo & Kyngäs, 2008). We then codified key topics regarding students' difficulties in learning mathematics in order to identify emerging categories. Using the categories of the students' learning difficulties (i.e., difficulty in understanding concepts, negative views towards learning mathematics, difficulty in finding effective ways of learning), two authors individually coded the students' written responses to the question regarding the impacts of the flipped classroom on learning mathematics. There was 83.2% agreement between the two authors. Discrepancies were resolved through discussion with a third researcher.

IV. RESULTS

Difficulty in Learning Mathematics

Content analysis revealed that middle school students in South Korea typically had three types of difficulties in learning mathematics: difficulty in understanding mathematical concepts, a negative view towards learning mathematics, and difficulty in finding an effective way of learning. Table 1 details the types of difficulties in learning mathematics and the sub-codes for each category. Perhaps unsurprisingly, the first learning difficulty that middle school students mentioned was related to cognitive factors. Some students said that they had difficulty in understanding basic mathematical concepts (e.g., "Because I gave up math when I was in the second year of middle school, I was not able to even begin to solve problems regarding a quadratic equation and a quadratic function" [Student #7]). However, more students reported that they have trouble understanding mathematical concepts and applying them to different situations (e.g., "Because I tend to memorize mathematical concepts and formulas, I'm often confused about which formula I have to use in a different problem" [Student #17]). In other words, these students would solve a mathematics problem by memorizing a concept or a formula, but they had difficulty in solving a novel problem in a different context due mainly to their lack of conceptual understanding.

The second category that we identified was students' negative views towards mathematics and learning mathematics, which are related to their affective characteristics. Specifically, the students had no confidence or motivation in mathematics and even had a negative attitude toward mathematics. Student #3, for example, said, "I feel like mathematics is necessary, but I cannot get interested in mathematics and my grades in mathematics did not get any better." Even a student who was good at mathematics expressed a negative attitude toward mathematics. Student #17 mentioned, "I was interested in mathematics in the elementary levels, but when I learned advanced mathematics concepts in middle school, I hit a wall. So, my thoughts on mathematics became more and more negative."

Interestingly, one of the most prevalent difficulties that students had was a lack of effective learning strategies and environments. This finding indicated that although students wanted to study mathematics, they had no idea how to do so effectively and efficiently and had difficulty with self-directed learning. The students' comments below are indicative of this type of response:

Student #8: I'm frustrated that I can't make it work compared to the amount of time I spend.
Student #2: I tried to study alone because I had a passion for math, but I realized that there was a limit to studying alone.
Student #11: I studied by myself watching online class videos or asking a friend, but mostly I slept while watching class videos. And it was hard to ask a friend because he was reluctant, and I usually didn't understand his explanation.

It was obvious that students' difficulties in learning mathematics were related to both the cognitive (e.g., "mathematics is difficult") and affective (e.g., "mathematics is hard, boring, and useless") dimensions of learning mathematics. Our findings, however, suggested that it was also important to consider environmental factors (i.e., providing a learning environment for self-directed learning after school) to help students reduce their difficulties in learning mathematics.

Table 1. Difficulties in learning mathematics

Category	Code	Percentage\
Difficulty in understanding concepts	Difficulty in conceptual understanding and application (15.3%) Limited understanding of basic concepts (7.7%)	23%
Negative views towards learning mathematics	Lack of confidence (15.3%) Lack of interest (15.3%) Negative attitude (7.7%)	38.5%
Difficulty in finding effective ways of learning	Inefficiency in learning (19.2%) Difficulty with self-directed learning (15.3%) No place to ask (3.8%)	38.3%

Note. The percentage indicates the proportion of analysis units corresponding to each category (and code) in the total analysis unit.

Effect of the Flipped Classroom on Reducing Difficulties in Learning Math

In this section, we present how engagement in the flipped classroom affected the difficulties students had in learning mathematics by looking more closely at each component of the flipped classroom and by associating it with the types of difficulty in learning mathematics that students reported (see Figure 2). The thickness of each link in Figure 2 illustrates the extent to which each component of the flipped classroom helped students reduce types of difficulties in learning mathematics. In the subsequent sections, we provide more qualitative descriptions of these relationships with representative examples from the students' written comments.

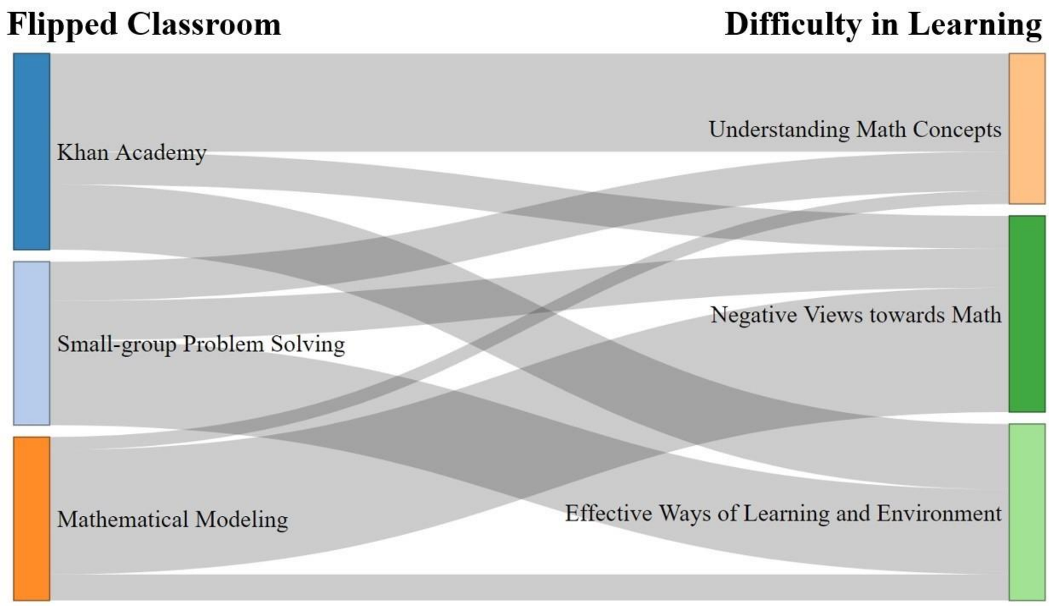

Figure 2. The relationship between flipped classroom activities and types of difficulty in learning mathematics

Learning in Khan Academy. The greatest contribution of Khan Academy activities was to improve students' mathematical knowledge acquisition (15 out of 30 units of analysis) by enhancing their understanding of basic mathematical concepts (e.g., "It certainly seems to have helped with the conceptual understanding" [student #11]), connecting different concepts (e.g., "It was really nice because there were other concepts that were explained to me in order to get to that concept" [student #7]), providing the opportunity to learn more (e.g., "I think it helped me understand things other than what I had learned in the classroom" [student #1]), and reviewing what they have learned (e.g., "If you don't know or get confused, it was good because I can check it again in Khan Academy" [student #18]). Khan Academy also provided a learning environment (10 out of 30 units of analysis) in which the students were given customized learning (e.g., "It was good that I could solve various kinds of problems that fit into my level by using KA" [student #6]) and a good educational resource for future learning (e.g., "I plan to study math using KA even after this class. I also recommended it to my younger brother, so he will study math hard" [student #2]). In some cases (5 out of 30 units of analysis), the students commented that it increased their learning motivation (e.g., "Through a system where I can collect points and badges, I was able to participate with more interest and enthusiasm" [student #10]) because of game-like elements given by Khan Academy.

Small-Group Problem Solving. The students in the current study commented that the in-class small-group problem solving activities helped them by providing an effective learning environment (13 out of 25 units of analysis). Specifically, the students mentioned that the collaborative problem-solving activities were particularly useful for them. Some students were able to understand mathematical concepts thanks to their friends' explanations (e.g., "I think I could solve a problem that I couldn't solve without my friends through a discussion class" [student #13]), while others understood a mathematical concept more deeply by explaining it to their peers (e.g., "I think my math ability improved as I explained the problem through discussion among the members" [student #14]). There were also a few students who noted both types of benefits. Student #19, for example, said,

> Because there were students with different levels, I could understand a problem more deeply in the process of explaining the problem to a friend who doesn't know the problem. Furthermore, when I don't know, the friend I know explained it to me. Because we discussed what we did not understand, it was a class that wasn't boring.

Mathematical Modeling. The mathematical modeling activities had the greatest impact on the students' affective aspects (19 out of 25 units of analysis). In other words, the experience of modeling a real-world problem using mathematics was very effective to improve the students' perceptions of mathematics. These activities led many students to recognize the value and necessity of mathematics as well as their interest in it. Student #9, for instance, said,

> I thought that even if I studied a function in math, I would never use them in real life. But as we drew the graph of the function and predicted the number of users in the future, it was so interesting, and I came to know why I had to learn the concept of a function.

Student #10 said, "It was really amazing to be able to express our life as a function like this. It was my first chance to learn that our lives are closely related to mathematics." However, a few students who had difficulty in understanding basic mathematical concepts expressed that these activities still felt difficult and boring.

Effect of the Flipped Classroom on Students' Mathematics Achievement and Affective Domains

To examine whether there were statistically significant differences in students' mathematical performance and affective factors after participating in the flipped classroom, the Wilcoxon signed-rank test was used. Table 2 shows that students' mathematics achievement significantly increased after the flipped classroom activities. As for the affective domains, there were significant changes in students' confidence and motivation and how much they valued learning mathematics between the pre- and post-test, but no difference was found in terms of interest in mathematics ($p = .361$). Notable change in the students' affective domains was found in terms of how much they valued mathematics (see

Figure 3). Before the flipped classroom, the students were not likely to think that mathematics was valuable (mean score of 1.97). However, after the flipped classroom, this domain showed the greatest increase among the four affective domains (mean score of 3.47).

Table 2. Descriptive statistics and results of Wilcoxon signed-rank test

		Pre-test mean (*SD*)	Post-test mean (*SD*)	*Z*	*p*
Mathematics performance		53.89 (22.20)	69.17 (26.53)	2.60	< 0.01
Affective domains	Confidence	2.22 (.46)	2.65 (.75)	2.25	< 0.05
	Interest	2.76 (.27)	2.93 (.56)	.91	.361
	Value	1.97 (.29)	3.47 (.43)	3.70	< 0.001
	Motivation	2.35 (.39)	2.96 (.64)	2.39	< 0.05

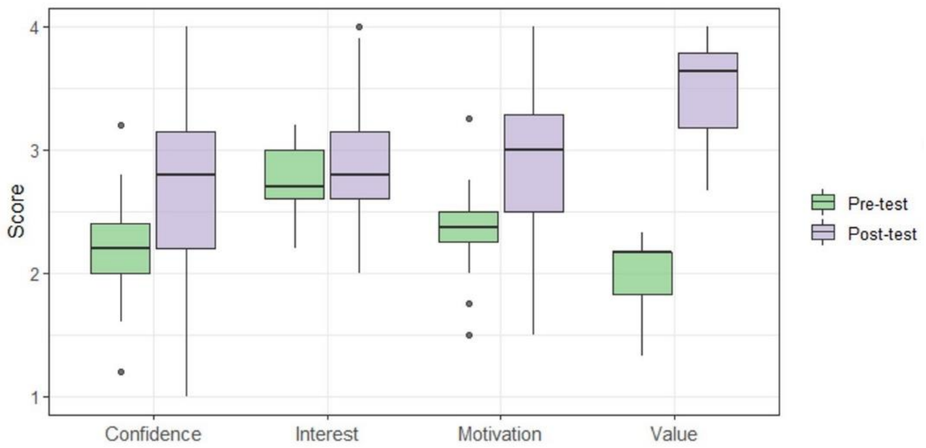

Figure 3. Boxplots illustrating changes in affective domains between pre- and post-test

V. CONCLUSION

The purpose of this study was to identify difficulties Korean middle school students have in learning mathematics and examine the effects of the FCKAMM on reducing the students' learning difficulties. The main findings of this study are discussed in the subsequent sections.

First, middle school students tended to have difficulty in understanding mathematical concepts, a negative view towards learning mathematics, and difficulty in finding an effective way of learning. It is unsurprising that students in South Korea have difficulty in understanding mathematics conceptually and have a negative attitude toward mathematics. However, it is noted that students are having a hard time finding the ways in which they can learn mathematics effectively. This indicates that teachers should provide their students learning environments or programs that allow them to self-directed mathematics learning.

Second, each activity of the FCKAMM had a different impact on student learning. Before class, the Khan Academy activities were most likely to help students understand mathematical concepts. In class, small-group problem solving activities were most helpful for students who had trouble finding effective learning methods and environments. Mathematical modeling activities were most likely effective in changing students’ negative views towards mathematics. In specific, research on the flipped classroom approach has demonstrated that students’ lack of self-regulated learning during out-of-class activities decreases the effectiveness of flipped classrooms, especially if the students are lower level (Lai & Hwang, 2016). It may be that Khan Academy was effective in the flipped classroom we implemented because it allowed students to watch and review videos at their own pace (Bhagat et al., 2016; Low & Hew, 2017), provided them with opportunities to explore the relationship between different mathematics concepts (Murphy et al., 2014), and encouraged them to engage in the activities using quizzes and badges (Abramovich et al., 2013). In other words, Khan Academy facilitated self-regulated personalized learning. This indicates that Khan Academy can be a useful tool to increase the effectiveness of flipped mathematics classroom.

In addition, the students in our study benefited from learning collaboratively when they engaged in small-group problem solving activities, which mirrors previous studies reporting positive effects of peer interactions on student learning in a flipped classroom (e.g., Lo & Hew, 2017). Specifically, collaborative learning in the flipped classroom not only helped students solidify their own mathematical understanding but also supported their peers’ learning in mathematics, which resonates with the findings of Sergis, Sampson, and Pelliccione (2018) that collaborative activities were beneficial for all students regardless of their achievement levels. Indeed, many students in the current study believed that discussion-based problem solving activities are effective ways to promote their mathematics learning. This finding indicates that teachers who rely heavily on traditional mathematics instruction, as described in Cai and Wang (2010), should provide their students with opportunities to engage in interactive learning with peers.

Third, the FCKAMM not only significantly improved the students’ mathematics achievement, but also positively affected their confidence and motivation and how much they valued learning mathematics, which is in line with the findings of Bhagat et al. (2016). When we considered this finding in comparison with the qualitative results, it was likely that mathematical modeling activities contributed to improving the students’ negative attitudes toward learning mathematics, especially significant improvement in their perception of the value of mathematics. However, we found no significant increase in the students’ interest in mathematics. This may result from the fact that low-achieving students had difficulty modeling real-world problems with mathematics, which mirrors previous research (e.g., Delice & Kertil, 2015). This indicates that mathematical modeling activities must be designed in consideration of the student's level of mathematical understanding.

Although the present study did not provide evidence of students improving their higher-order thinking through modeling activities, the findings are still significant because students in East Asian countries tend to have lower levels of motivation, interest, and confidence in mathematics as well as a lower perception of its value than those in Western countries (Zhu & Leung, 2011). This result indicates that the conventional design of a flipped mathematics classroom (i.e., watching instructional videos before class and engaging in small-group discussion in class) may not sufficiently promote students' learning and positive affect. Rather, integrating modeling activities into the flipped classroom model is critical because, based on expectancy-value theory, students' motivation, confidence, and perception of the value of mathematics have a long-term effect on their choice of a math-intensive career (Musu-Gillette, Wigfield, Harring, & Eccles, 2015).

We recognize that the generalizability of the findings from this study is limited. In addition, research has shown that students' online learning behaviors affect their learning outcomes in a flipped classroom (e.g., Hsiao et al., 2018). Although we did not include students' behaviors in the Khan Academy activities in our analysis, the platform provides "coach reports" that allow researchers and teachers to observe their students' learning progress in real time (Phillips & Cohen, 2015). Future studies may examine how students' engagement in Khan Academy lessons affects their learning outcomes and how teachers can use the data to promote teaching and learning of mathematics in their flipped classrooms.

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