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COMMUTATIVITY CRITERIA OF PRIME RINGS INVOLVING TWO ENDOMORPHISMS

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Abstract. This paper treats the commutativity of prime rings with involution over which elements satisfy some specific identities involving endomorphisms. The obtained results cover some well-known results. We show, by given examples, that the imposed hypotheses are necessary.

1. Introduction

All along this paper, A denotes a non-trivial associative ring, $Z(A)$ denotes the center of A. For each $a, b \in A$, we set $[a, b] = ab - ba$ and $a \circ b = ab + ba$.

Let A be a ring. A is said to be semi-prime if $aAa = \{0\}$ implies $a = 0$ for each $a \in A$, A is said to be prime if $aAb = \{0\}$ implies $a = 0$ or $b = 0$ for each $a, b \in A$, and A is said to be two-torsion free if $2a = 0$ implies $a = 0$ for each $a \in A$.

An involution of A is an anti-automorphism of order 2, that is a map $* : A \to A$ such that $(a + b)^* = a^* + b^*$, $(a^*)^* = a$, and $(ab)^* = b^*a^*$. Hermitian (resp. skew-hermitian) elements are $a \in A$ such that $a^* = a$ (resp. $a^* = -a$). Set $H(A)$ (resp. $S(A)$) the set of hermitian (resp. skew-hermitian) elements of A. If $Z(A) \subseteq H(A)$, then $*$ is said to be of the first kind. Otherwise, there exists a non hermitian element $z \in Z(A)$. Then, $z - z^* \in Z(A) \cap S(A)$ and $z - z^* \neq 0$. Thus, $Z(A) \cap S(A) \neq \{0\}$ and $*$ is said to be of the second kind.

The study of additive mappings on rings equipped with involution was introduced by Brešar et al. [2]. Recently, there is considerable interest in the commutativity criteria of rings with involution given by the existence of some additive mappings of A such as derivations, generalized derivations, automorphisms acting on appropriate subsets of rings (see [1, 3–5, 7, 8]).

Recently, the authors in [6] explore the commutativity of a prime ring with involution of the second kind $(A, *)$ admitting a homomorphism $g : A \to A$ satisfying any one of the following conditions:

- $\phi(xx^*) + xx^* \in Z(A)$ for all $x \in A$.
- $\phi(xx^*) - xx^* \in Z(A)$ for all $x \in A$.

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- $\phi(xx^*) + x^*x \in Z(A)$ for all $x \in A$.
- $\phi(xx^*) - x^*x \in Z(A)$ for all $x \in A$.

In this paper, we consider two homomorphisms ϕ and ψ of a two-torsion free prime ring A which are not both trivial. Then, we prove that A is commutative once one the following conditions is satisfied:

- $\phi(x)\psi(x^*) + xx^* \in Z(A)$ for all $x \in A$.
- $\phi(x)\psi(x^*) - xx^* \in Z(A)$ for all $x \in A$.
- $\phi(x)\psi(x^*) + x^*x \in Z(A)$ for all $x \in A$.
- $\phi(x)\psi(x^*) - x^*x \in Z(A)$ for all $x \in A$.

2. Lemmas

All along this section, $(A, *)$ will denote a two-torsion free prime ring with involution of the second kind.

Lemma 2.1. *Let ϕ be a non-trivial endomorphism of A. If $\phi(x) - x \in Z(A)$ for each $x \in A$ (resp. $\phi(x) - x^* \in Z(A)$ for each $x \in A$), then A is an integral domain.*

Proof. (1) Suppose that $\phi(x) - x \in Z(A)$ for each $x \in A$. Then we have $\phi(x)x = x\phi(x)$ and

$$(\phi(x) - x)(\phi(x) + x) = \phi(x)^2 - x^2 = \phi(x^2) - x^2 \in Z(A).$$

Thus, $\phi(x) - x = 0$ or $\phi(x) + x \in Z(A)$. Set $A_0 = \{x \in A \,|\, \phi(x) = x\}$ and $A_1 = \{x \in A \,|\, \phi(x) + x \in Z(A)\}$. Since $\phi \neq \mathrm{id}_A$, $A \neq A_0$. However, A_0 and A_1 are additive sub-groups of A and $A = A_0 \cup A_1$. Thus, $A = A_1$, which means that $\phi(x) + x \in Z(A)$ for all $x \in A$. So, $2x \in Z(A)$ for each $x \in A$ and A is commutative.

(2) Suppose that $\phi(x) - x^* \in Z(A)$ for each $x \in A$. Then,

$$(\phi(x)-x^*)(\phi(x)+x^*) = \phi(x)(\phi(x)+x^*)-(\phi(x)+x^*)x^* = \phi(x^2)-(x^2)^* \in Z(A).$$

Then, either $\phi(x) = x^*$ or $\phi(x) + x^* \in Z(A)$. The sets $A_0 = \{x \in A_0 \,|\, \phi(x) = x^*\}$ and $A_1 = \{x \in A_1 \,|\, \phi(x) + x^* \in Z(A)\}$ are additive sub-groups of A with $A = A_0 \cup A_1$. Then, either $A = A_0$ or $A = A_1$. If $A = A_0$, then $xy = \phi((xy)^*) = \phi(y^*x^*) = \phi(y^*)\phi(x^*) = yx$ for each $x, y \in A$. Thus, A is commutative. Now, if $A = A_1$, using our assumption, we obtain $2x^* \in Z(A)$, and so A is commutative. □

Lemma 2.2. *Let ϕ be a non-trivial endomorphism of A. The following are equivalent*:

1. *$\phi(x)x^* - xx^* \in Z(A)$ for all $x \in A$.*
2. *$x\phi(x^*) - xx^* \in Z(A)$ for all $x \in A$.*
3. *A is an integral domain.*

Proof. (3) $\Rightarrow$ (1) and (3) $\Rightarrow$ (2) are trivial.

(1) $\Rightarrow$ (3) We suppose that A is non commutative. We have

$$\phi(x)x^* - xx^* \in Z(A) \quad \text{for all } x \in A. \tag{2.1}$$

The linearization of (2.1) gives

$$(\phi(x) - x)y^* + (\phi(y) - y)x^* \in Z(A) \quad \text{for all } x, y \in A. \tag{2.2}$$

Replacing y by yh in (2.2), with $h \in H(A) \cap Z(A)$, we obtain

$$(\phi(x) - x)y^*h + (\phi(y)\phi(h) - yh)x^* \in Z(A) \quad \text{for all } x, y \in A. \tag{2.3}$$

Multiply (2.2) by h and use (2.3), we acquire

$$\phi(y)(\phi(h) - h)x \in Z(A) \quad \text{for all } x, y \in A. \tag{2.4}$$

Thus,

$$\phi(y)(\phi(h) - h)xt \in Z(A) \quad \text{for all } t, x, y \in A. \tag{2.5}$$

Since A is non commutative, by (2.4) and (2.5), we get $\phi(y)(\phi(h) - h) = 0$. Since $\phi(h) - h \in Z(A)$, we have either $\phi(h) = h$ or $\phi = 0$. However, in the last case, the hypothesis becomes $xx^* \in Z(A)$ for all $x \in A$, which means that A is commutative, a contradiction. Thus $\phi(h) = h$ for all $h \in H(A) \cap Z(A)$. Following (2.2), we have $(\phi(x) - x)h \in Z(A)$, and this implies that $\phi(x) - x \in Z(A)$. Using Lemma 2.1, we obtain that A is commutative, which is a contradiction. Consequently, A is commutative.

(2) $\Rightarrow$ (3) Similar to the proof of (1) $\Rightarrow$ (3) with slight modifications. □

3. Some criteria of commutativity of prime rings with involution

All along this section, A will denote a two-torsion free prime ring with involution of the second kind $*$.

Theorem 3.1. *Let ϕ and ψ be two endomorphisms of A not both trivial. The following are equivalent*:

(1) $\phi(x)\psi(x^*) - xx^* \in Z(A)$ *for all* $x \in A$.
(2) $\phi(x)\psi(x^*) + xx^* \in Z(A)$ *for all* $x \in A$.
(3) A *is an integral domain.*

Proof. (3) $\Rightarrow$ (1) and (3) $\Rightarrow$ (2) are trivial.

(1) $\Rightarrow$ (3) Let's suppose that A is not commutative. Then we have

$$\phi(x)\psi(x^*) - xx^* \in Z(A) \quad \text{for all } x \in A. \tag{3.1}$$

Seeing Lemma 2.2, we can suppose that ϕ and ψ are both non-trivial.

Let $z \in Z(A) \setminus \{0\}$. Easily, we have $\phi(z)\psi(z^*) \in Z(A)$.

Let xz instead of x in (3.1), we get

$$\phi(x)\phi(z)\psi(z^*)\psi(x^*) - xx^*zz^* \in Z(A) \quad \text{for all } x \in A. \tag{3.2}$$

That is

$$\phi(x)\psi(x^*)\phi(z)\psi(z^*) - xx^*zz^* \in Z(A) \quad \text{for all } x \in A. \tag{3.3}$$

Hence,

$$[\phi(x)\psi(x^*), r]\phi(z)\psi(z^*) - [xx^*, r]zz^* = 0 \quad \text{for all } x \in A. \tag{3.4}$$

So,

$$[xx^*, r](\phi(z)\psi(z^*) - zz^*) = 0 \quad \text{for all } x \in A. \tag{3.5}$$

The non-commutativity of A yields that

$$\phi(z)\psi(z^*) = zz^* \quad \text{for all } z \in Z(A). \tag{3.6}$$

Linearizing (3.1), we obtain

$$\phi(x)\psi(y^*) + \phi(y)\psi(x^*) - xy^* - yx^* \in Z(A) \quad \text{for all } x, y \in A. \tag{3.7}$$

Taking $y = z$ and replacing x by xz, we get

$$(\phi(x) + \psi(x^*) - x - x^*)zz^* \in Z(A) \quad \text{for all } x \in A. \tag{3.8}$$

Thus,

$$\phi(x) + \psi(x^*) - x - x^* \in Z(A) \quad \text{for all } x \in A. \tag{3.9}$$

For this reason, $\phi(z) + \psi(z^*) \in Z(A)$. Therefore, for each $x \in A$, $\psi(x)(\phi(z) + \psi(z^*)) = (\phi(z) + \psi(z^*))\psi(x)$. Since $\psi(x)\psi(z^*) = \psi(z^*)\psi(x)$, we conclude that $\psi(x)\phi(z) = \phi(z)\psi(x)$. Then, $\phi(z) \in Z(\psi(A))$. On the other hand, $[\phi(x)\psi(x^*) - xx^*, \phi(z)] = 0$. Thus, $[xx^*, \phi(z)] = 0$, and so $\phi(z) \in Z(A)$. Similarly, $\psi(z) \in Z(A)$. In particular, for each $s \in S(A) \cap Z(A)$, we have $\phi(s) \in Z(A)$ and $\psi(s) \in Z(A)$.

Replacing x by xs in (3.9), we get

$$\phi(x)\phi(s) - \psi(s)\psi(x^*) - xs + x^*s \in Z(A) \quad \text{for all } x, y \in A. \tag{3.10}$$

Taking $y = s$ in (3.7), we get

$$-\phi(x)\psi(s) + \phi(s)\psi(x^*) + xs - x^*s \in Z(A) \quad \text{for all } x, y \in A. \tag{3.11}$$

Adding (3.10) and (3.11), we obtain

$$(\phi(x) + \psi(x^*))(\phi(s) - \psi(s)) \in Z(A) \quad \text{for all } x \in A. \tag{3.12}$$

Consequently, $\phi(s) = \psi(s)$ or $\phi(x) + \psi(x^*) \in Z(A)$ for each $x \in A$. However, the second case together with our assumption means that $x + x^* \in Z(A)$ for each $x \in A$. So, A is commutative, which is a contradiction. Thus, $\phi(s) = \psi(s)$.

From (3.6), we have that $\phi(s)\psi(s) = s^2$. So, $\phi(s)^2 = s^2$. Thus, since $\phi(s) \in Z(A)$, we deduce that either $\phi(s) = s$ or $\phi(s) = -s$.

Case 1 "$\boldsymbol{\phi(s) = \psi(s) = s}$". Replacing x by xs in (3.9), we obtain that

$$\phi(x) - \psi(x^*) - x + x^* \in Z(A) \quad \text{for all } x, y \in A. \tag{3.13}$$

Adding (3.9) and (3.13), we obtain

$$\phi(x) - x \in Z(A) \quad \text{for all } x, y \in A. \tag{3.14}$$

Hence, by Lemma 2.1, A is commutative, which is a contradiction.

Case 2 "$\phi(s) = \psi(s) = -s$". Replacing x by xs in (3.9), we obtain that

$$-\phi(x) + \psi(x^*) - x + x^* \in Z(A) \quad \text{for all } x, y \in A. \tag{3.15}$$

Subtracting (3.9) form (3.15), we obtain

$$\phi(x) - x^* \in Z(A) \quad \text{for all } x, y \in A. \tag{3.16}$$

Then, by Lemma 2.1, A is commutative, which is a contradiction.

Consequently, A is commutative.

(2) $\Rightarrow$ (3) Similar to the proof of (1) $\Rightarrow$ (3) with slight modifications. □

Taking $\phi = \psi$ in the previous result, we obtain the following consequence.

Corollary 3.2 ([6], Theorem 3.1). *Let ϕ be a non-trivial endomorphism of A. The following are equivalent*:

(1) $\phi(xx^*) - xx^* \in Z(A)$ *for all* $x \in A$.
(2) $\phi(xx^*) + xx^* \in Z(A)$ *for all* $x \in A$.
(3) *A is an integral domain.*

Theorem 3.3. *Let ϕ and ψ be two endomorphisms of A. The following are equivalent*:

(1) $\phi(x)\psi(x^*) - x^*x \in Z(A)$ *for all* $x \in A$.
(2) $\phi(x)\psi(x^*) + x^*x \in Z(A)$ *for all* $x, y \in A$.
(3) *A is an integral domain.*

Proof. (3) $\Rightarrow$ (1) and (3) $\Rightarrow$ (2) are trivial.

(1) $\Rightarrow$ (3) If $\phi = \psi = \mathrm{id}_A$, then A is clearly commutative. If one of ϕ and ψ is the identity and the other one is non-trivial, the result follows immediately from Lemma 2.2. Therefore, we can suppose that both of ϕ and ψ are non-trivial.

Assume that A is non-commutative.

As in the proof of Theorem 3.1, we prove that

$$\phi(x) + \psi(x^*) - x - x^* \in Z(A) \quad \text{for all } x \in A, \tag{3.17}$$

and that $\phi(s) = \psi(s) = s$ or $\phi(s) = \psi(s) = -s$ for each $s \in Z(A) \cap S(A)$. The treatment of the both cases is similar to the one doing in Theorem 3.1, and in the both cases we obtain that A is commutative, which is a contradiction. Consequently, A is commutative.

(2) $\Rightarrow$ (3) Similar to the proof of (1) $\Rightarrow$ (3). □

Taking $\phi = \psi$ in the previous results, we obtain the following consequence.

Corollary 3.4 ([6], Theorem 3.3). *Let ϕ be an endomorphism of A. The following are equivalent*:

(1) $\phi(xx^*) - x^*x \in Z(A)$ *for all* $x \in A$.
(2) $\phi(xx^*) + x^*x \in Z(A)$ *for all* $x \in A$.
(3) *A is an integral domain.*

Our first example shows that the condition "* is of the second kind" required in Theorems 3.1 and 3.3 is necessary.

Example 3.5. Let $M_2(\mathbb{Z})$ be the ring of 2×2 matrices with entries in $\mathbb{Z}$. For each $n, m, p, q \in \mathbb{Z}$, we set

$$\begin{pmatrix} n & m \\ p & q \end{pmatrix}^* = \begin{pmatrix} q & -m \\ -p & n \end{pmatrix}.$$

Clearly $(M_2(\mathbb{Z}), *)$ is prime with involution of the first kind. Since $XX^* \in Z(M_2(\mathbb{Z}))$ for all $X \in M_2(\mathbb{Z})$, then, for $\phi = 0$, the conditions of Theorems 3.1 and 3.3 are satisfied, but $M_2(\mathbb{Z})$ is not commutative.

The next example shows that in the context of semi-prime rings, Theorems 3.1 and 3.3 turn to be false.

Example 3.6. Let us consider $A = M_2(\mathbb{Z}) \times \mathbb{C}$ provided with the involution of the second kind τ defined by $\tau(X, z) = (X^*, \overline{z})$ with $*$ is as in the later example. For $\phi = 0$, the conditions of Theorems 3.1 and 3.3 are satisfied. However, A is not commutative.

4. Commutativity criteria for prime rings involving two endomorphisms

In this section, we give analogue results to Theorems 3.1 and 3.3 in rings without necessarily involution.

All along this section, A will denote a two-torsion free prime ring.

Theorem 4.1. *Let ϕ and ψ be two endomorphisms of A that are not both trivial. The following are equivalent*:

(1) $\phi(x)\psi(y) - xy \in Z(A)$ *for all* $x, y \in A$.
(2) $\phi(x)\psi(y) + xy \in Z(A)$ *for all* $x, y \in A$.
(3) *A is an integral domain.*

Proof. The implications (3) $\Rightarrow$ (1) and (3) $\Rightarrow$ (2) are trivial.

(1) $\Rightarrow$ (3) We have

$$\phi(x)\psi(y) - xy \in Z(A) \quad \text{for all } x, y \in A. \tag{4.1}$$

If $\phi = \mathrm{id}_A$ or $\psi = \mathrm{id}_A$, then the result follows easily. Therefore, we may assume that $\phi \neq \mathrm{id}_A$ and $\psi \neq \mathrm{id}_A$.

Suppose that A is non-commutative.

Let's prove that $Z(A) \neq \{0\}$. Assume that $Z(A) = \{0\}$. Then, (4.1) becomes

$$\phi(x)\psi(y) - xy = 0 \quad \text{for all } x, y \in A. \tag{4.2}$$

Replacing x by tx in (4.2), with $t \in A$, we get

$$(\phi(t) - t)xy = 0 \quad \text{for all } t, x, y \in A. \tag{4.3}$$

Since $\phi \neq \mathrm{id}_A$, we deduce that $A = \{0\}$, which is a contradiction. Accordingly, $Z(A) \neq \{0\}$.

Let $z \in Z(A) \setminus \{0\}$ and take $x = y = z$ in (4.1). We get $\phi(z)\psi(z) \in Z(A)$.

Let xz instead of x and zy instead y in (4.1), we obtain

$$\phi(x)\phi(z)\psi(z)\psi(y) - z^2xy \in Z(A) \quad \text{for all } x, y \in A. \tag{4.4}$$

Hence, since $\phi(z)\psi(z) \in Z(A)$, we obtain that

$$\phi(z)\psi(z)\phi(x)\psi(y) - z^2xy \in Z(A) \quad \text{for all } x, y \in A. \tag{4.5}$$

That is

$$\phi(z)\psi(z)[\phi(x)\psi(y), r] - z^2[xy, r] = 0 \quad \text{for all } x, y, r \in A. \tag{4.6}$$

Using (4.1), we conclude that

$$(\phi(z)\psi(z) - z^2)[xy, r] = 0 \quad \text{for all } x, y \in A. \tag{4.7}$$

Hence, since A is non-commutative, we deduce that $\phi(z)\psi(z) = z^2$.

Now, take $y = z$ and let xz instead of x in (4.1), we get

$$\phi(x)\phi(z)\psi(z) - xz^2 \in Z(A) \quad \text{for all } x \in A. \tag{4.8}$$

That is

$$(\phi(x) - x)z^2 \in Z(A) \quad \text{for all } x \in A. \tag{4.9}$$

Then, since $z \neq 0$, we obtain that

$$\phi(x) - x \in Z(A) \quad \text{for all } x \in A. \tag{4.10}$$

Using Lemma 2.1, we obtain that A is commutative, which is a contradiction. So A is a commutative ring.

(2) $\Rightarrow$ (3) Similar to the proof of (1) $\Rightarrow$ (3) with slight modifications. □

Theorem 4.2. *Let ϕ and ψ be two endomorphisms of A. The following are equivalent*:

(1) $\phi(x)\psi(y) - yx \in Z(A)$ *for all* $x, y \in A$.
(2) $\phi(x)\psi(y) + yx \in Z(A)$ *for all* $x, y \in A$.
(3) *A is an integral domain.*

Proof. (3) $\Rightarrow$ (1) and (3) $\Rightarrow$ (2) are trivial.

(1) $\Rightarrow$ (3) Assume that A is non-commutative. We have

$$\phi(x)\psi(y) - yx \in Z(A) \quad \text{for all } x, y \in A. \tag{4.11}$$

Let's prove that $Z(A) \neq \{0\}$. Suppose that $Z(A) = \{0\}$. Then (4.11) becomes

$$\phi(x)\psi(y) - yx = 0 \quad \text{for all } x, y \in A. \tag{4.12}$$

Replacing y by yA in (4.12), we get

$$\phi(x)\psi(y)\psi(A) - yrx = 0 \quad \text{for all } r, x, y \in A. \tag{4.13}$$

That is

$$yx\psi(A) - yrx = 0 \quad \text{for all } r, x, y \in A. \tag{4.14}$$

So

$$x\psi(A) - rx = 0 \quad \text{for all } r, x \in A. \tag{4.15}$$

Replacing x by tx, we obtain

$$tx\psi(A) - rtx = 0 \quad \text{for all } r, t, x \in A. \tag{4.16}$$

Left multiplying (4.15) by t, we get

$$tx\psi(A) - trx = 0 \quad \text{for all } r, t, x \in A. \tag{4.17}$$

From (4.16) and (4.17), we conclude that

$$[r, t]x = 0 \quad \text{for all } r, t, x \in A. \tag{4.18}$$

So, A is commutative, which is a contradiction. Thus $Z(A) \neq \{0\}$.

Let z be a nonzero element of $Z(A)$. Taking $x = y = z$ in (4.11), we get $\phi(z)\psi(z) \in Z(A)$. Replacing x by xz and y by zy in (4.11), we get

$$\phi(x)\phi(z)\psi(z)\psi(y) - z^2yx \in Z(A) \quad \text{for all } x, y \in A. \tag{4.19}$$

Since $\phi(z)\psi(z) \in Z(A)$, we get

$$\phi(z)\psi(z)\phi(x)\psi(y) - z^2yx \in Z(A) \quad \text{for all } x, y \in A. \tag{4.20}$$

Commuting this with r, we arrive at

$$\phi(z)\psi(z)[\phi(x)\psi(y), r] - z^2[yx, r] = 0 \quad \text{for all } r, x, y \in A. \tag{4.21}$$

That is

$$(\phi(z)\psi(z) - z^2)[yx, r] = 0 \quad \text{for all } r, x, y \in A. \tag{4.22}$$

Since A is non-commutative, we get $\phi(z)\psi(z) = z^2$.

Take now $y = z$ and replacing x by xz in (4.11), we get

$$\phi(x)\phi(z)\psi(z) - xz^2 \in Z(A) \quad \text{for all } x \in A. \tag{4.23}$$

That is

$$(\phi(x) - x)z^2 \in Z(A) \quad \text{for all } x \in A. \tag{4.24}$$

Since z^2 is a nonzero element of $Z(A)$, we conclude that

$$\phi(x) - x \in Z(A) \quad \text{for all } x \in A. \tag{4.25}$$

Using Lemma 2.1, we obtain that A is commutative, which is a contradiction. Consequently, A is commutative.

(2) $\Rightarrow$ (3) Similar to the proof of (1) $\Rightarrow$ (3) with slight modifications. $\square$

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