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A Fractional Integration Analysis on Daily FX Implied Volatility: Long Memory Feature and Structural Changes

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Abstract

Purpose - The purpose of this paper is to analyze the dynamic factors of the daily FX implied volatility based on the fractional integration methods focusing on long memory feature and structural changes.
Design/methodology/approach - This paper uses the daily FX implied volatility data of the EUR-USD and the JPY-USD exchange rates. For the fractional integration analysis, this paper first applies the basic ARFIMA-FIGARCH model and the Local Whittle method to explore the long memory feature in the implied volatility series. Then, this paper employs the Adaptive-ARFIMA-Adaptive-FIGARCH model with a flexible Fourier form to allow for the structural changes with the long memory feature in the implied volatility series.
Findings - This paper finds statistical evidence of the long memory feature in the first two moments of the implied volatility series. And, this paper shows that the structural changes appear to be an important factor and that neglecting the structural changes may lead to an upward bias in the long memory feature of the implied volatility series.
Research implications or Originality - The implied volatility has widely been believed to be the market's best forecast regarding the future volatility in FX markets, and modeling the evolution of the implied volatility is quite important as it has clear implications for the behavior of the exchange rates in FX markets. The Adaptive-ARFIMA-Adaptive-FIGARCH model could be an excellent description for the FX implied volatility series

Keywords: Adaptive-ARFIMA-Adaptive-FIGARCH Model, ARFIMA-FIGARCH Model, FX Implied Volatility, Local Whittle Method, Long Memory Feature, Structural Changes.
JEL Classifications: C14, C22, F31

Ⅰ. Introduction

Analyzing the evolution of the implied volatility of the financial assets is quite important since it has clear implications for the behavior of the asset prices and for practitioners in terms of profitable trading strategies. Also, the implied volatility has widely been believed to be the market's best forecast regarding the future volatility in financial markets. A number of studies have focused on this predictive power of the implied volatility. Among them, several papers such as Fleming, Ostdiek, and Whaley (1995), Jorion (1995), Fleming (1998), Bates (2000)

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and Christen and Hansen (2002) have confirmed that implied volatility outperforms other volatility measures in forecasting future volatility in equity market and currency options markets.

And, the paper of Pong et al. (2004) has argued that currency traders in FX markets may possess additional information about future events on top of what is provided by historical data, making implied volatility potentially more accurate forecasts of future volatility than those based on history alone. The study of Martin and Zein (2004) also has presented that the implied volatility possesses the information that the others do not contain for the USD-JPY exchange rates. Thus, the accurate implied volatility modeling in financial markets can be valuable for numerous market participants and benefit investors facing risk management issues in international portfolios. And, the optimal modeling of the implied volatility in financial time series data is essential for a variety of risk assessment and trading purpose. In this context, the empirical analysis of the implied volatility dynamics for financial assets has been attracted much attention of financial economics (Kim and Kim, 2003; Ederington and Lee, 1996; Fung and Hsieh, 1991).

In particular, some extant empirical papers have applied several time series models with either long memory feature or non-linear structural changes for the analysis of the implied volatility (Pavlidis, Shackleton and Voukelatos, 2012; Kellard, Jiang and Wohar, 2015, 2010; Dunis, Kellard and Snaith, 2013; Konstantinidi, Skidopoulos and Tzagkaraki, 2008; Nielsen, 2007; Bandi and Perron, 2006)[1). They have considered the difficulty in choosing either the long memory feature or the structural changes for representing the most appropriate process to describe the implied volatility series. Generally, there exists some evidence that the features of long memory fractionally integrated behavior and/or generic non-linear structural changes occur within the implied volatility. Thus, it could be important to represent both the long memory feature and the structural changes in the implied volatility series. Following the previous papers which have presented several models for both the long memory feature and the structural changes in financial time series data (Diebold, 1988; Lamoreaux and Lastrapes, 1990; Lobato and Savin, 1998; Beine and Laurent, 2000; Morana and Beltratti, 2004; Martens, van Dijik, and de Pooter, 2004), this paper suggests an appropriate model for the FX implied volatility series to represent both the long memory feature and the structural changes.

This paper first analyzes the long memory feature in the FX implied volatility series of the daily EUR-USD and JPY-USD exchange rates by using the basic ARFIMA-FIGARCH model of Baillie, Han, and Kwon (2002) and the Local Whittle model of Taqqu and Teverovsky (1997) for the comparison. The long memory parameters estimated from the basic ARFIMA-FIGARCH model in both the conditional mean and the conditional variance process of the FX implied volatility series are found to be statistically significant and they are in consistent with the results from Local Whittle estimation. There exits strong evidence of the long memory features in both the conditional mean and the conditional variance process of the FX implied volatility series. Since neglecting structural changes may overestimate the long memory parameters as pointed by Granger and Hyung (2004) and Diebold and Inoue (2001), this paper applies a combined model involving both the stochastic long memory feature and the deterministic structural changes for the analysis of the FX implied volatility series.

For the purpose, this paper uses the adaptive modification of the basic ARFIMA-FIGARCH model, designated as the Adaptive-ARFIMA-Adaptive-FIGARCH model of Baillie and Morana

1) Becker, Clements, and McClelland (2009) and Busch, Christensen, and Nielsen (2011) have used jump models for the implied volatilities of foreign exchange rates and S&P500 stock prices.

(2012), which appears to be capable of replicating many of the observed characteristics in the FX implied volatility series. The Adaptive-ARFIMA- Adaptive-FIGARCH model is obtained by augmenting the basic ARFIMA-FIGARCH model with time dependent intercept terms which evolve according to a flexible Fourier form (FFF) (Gallant, 1984) in the conditional mean process and the conditional variance process in order to consider the structural changes and the long memory feature simultaneously. As presented by the simulation studies of Baillie and Morana (2012), the Adaptive-ARFIMA-Adaptive-FIGARCH model is found to be the better representation for the sharper structural changes which may occur as background noise with the long memory adjustment to shocks and the QMLE method performs well in terms of estimation of the model's parameters in moderately sized samples. And, the Adaptive-ARFIMA-Adaptive-FIGARCH model has an advantage of being computationally straightforward since the model does not require pre-testing for the numbers of structural change points nor does it require any smooth transition between regimes (Baillie and Morana, 2009).

The estimation results show that the estimates of the long memory parameters in the conditional mean process and the conditional variance process of the FX implied volatility series are in general smaller in magnitude compared to the results of the basic ARFIMA-FIGARCH model. The results imply that there exists the strong evidence of the deterministic structural changes in the FX implied volatility series and that the long memory parameters could be overestimated due to the neglected structural changes. And, this model selection strategy can provide a useful representation of time series such as FX implied volatility and deserves consideration for applying in a wider range of contexts.

The plan of the rest of the paper is as follows: Section II provides statistical evidence of the long memory feature in the FX implied volatility series of the EUR-USD and JPY-USD rates. For the purpose, the basic ARFIMA-FIGARCH model is adopted in order to investigate the long memory feature which may be related with the structural changes. Also, the semi-parametric Local Whittle estimation method is used for the comparison. Section III is concerned with the structural changes in the FX implied volatility series by using the Adaptive-ARFIMA-Adaptive-FIGARCH model with allowing for the long memory feature in both the conditional mean and the conditional variance process of the FX implied volatility series. Finally, section IV concludes briefly.

II. Long Memory Feature in FX Implied Volatility

This section is concerned with the long memory feature of the FX implies volatility series of the daily EUR-USD and JPY-USD rates which are the most heavily traded in FX markets around the world. The FX implied volatility series are sampled from January 4, 2000 through December 31, 2018 providing by the Olsen Financial Technologies (OFT) which is one of the largest financial data providers in the world. Each quotation consists of the mid rates of bid and ask rates and is recorded in time to the nearest second at the closing time so that the implied volatility rates in this paper are the mid rates of the FX implied volatility series derived from the daily EUR-USD and JPY-USD exchange rates. After excluding the weekends and the worldwide holidays like Christmas day (Dec. 25) and New Year Day (Jan. 1) with relatively low trading activities in FX markets as suggested by Bollerslev and Domowitz (1993), the dataset of the implied volatility series realizes a sample of total 4926 observations. The descriptive statistics for the FX implied volatility series of the EUR-USD and the JPY-USD rates are provided in 〈Table 1〉.

Table 1. Descriptive Statistics for the Implied Volatility Series

	EUR-USD	JPY-USD
Mean	9.9230	10.3457
Variance	11.3465	12.6103
Q(20)	67889.5625	58078.1438
$Q^2(20)$	64549.7779	49395.9649
Skewness	1.1966	1.2336
Kurtosis	1.6521	1.8976
ρ_1	0.9641	0.9477

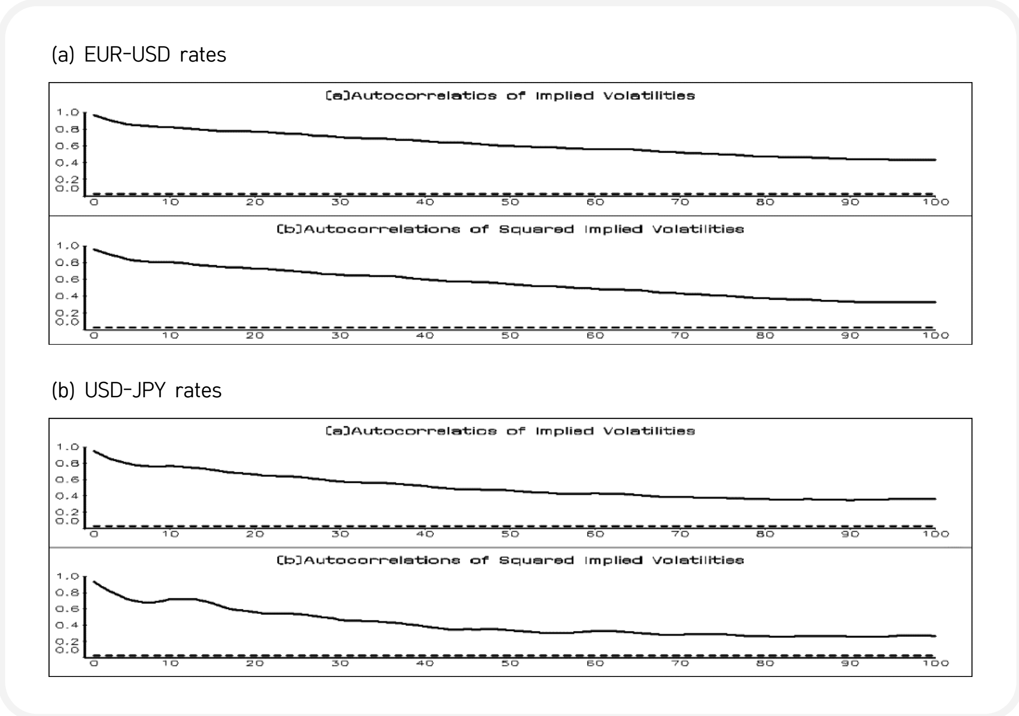
Note: 1. the Q(20) and $Q^2(20)$ are the modified Ljung-Box test statistics at 20 degrees of freedom based on the level values and the squared values.
2. ρ_1 is the first order of autocorrelation.

The sample means of the FX implied volatility of the EUR-USD rates and the USD-JPY rates are found to be 9.9 and 10.35 respectively, with the sample variances of 11.35 and 12.61. And, the Ljung-Box test statistics for the test of the serial correlations, Q(20) and Q2(20), calculated from the level of the FX implied volatility series and the squared values of the FX implied volatility series are 67,889 and 64,549 for the EUR-USD rates and 58,078 and 49,395 for the USD-JPY rates. The large test statistics indicate the existence of highly persistent autocorrelations in both the conditional mean process and the conditional variance process of the FX implied volatility series, which is the feature of the long memory process. In particular, the autocorrelations in the conditional mean process generally appear to be more persistent than in the conditional variance process for the FX implied volatility series. The problem of the serial correlation seems to be a little more significant in the FX implied volatility series of the EUR-USD rates than in the USD-JYP rates. This is also in consistent with the fact that the values of the first order autocorrelation (ρ1) in which the value of the FX implied volatility in the EUR-USD rate is slightly greater than in the USD-JPY rates. And, the FX implied volatility series of the EUR-USD and the USD-JPY rates appear not to be normally distributed since the values of the skewness are 1.19 and 1.23, and the values of the kurtosis are 1.65 and 1.89, which are different from the levels of the normal distribution, and they are all statistically significant[2)].

These finding can be confirmed by 〈Fig. 1〉 which plots the correlograms for the level values and the squared values of the FX implied volatility of the EUR-USD and the USD-JPY rates. The autocorrelations of the level values and the squared values of the FX implied volatility series all exist above the dotted lines representing the band in which there is no serial correlation at the 95% confidence level, and they all decay very slowly at the hyperbolic rate, which is the typical feature of the long memory feature. These correlograms also indicate the existence of the long memory feature in both the conditional mean process and the conditional variance process of the FX implied volatility series. And, the degree of the long memory feature seems to be a little more significant in the FX implied volatility series of the EUR-USD rates than in the USD-JPY rates.

2) According to Jarque and Bera (1987), the standard errors of the sample skewness and the sample kurtosis in their corresponding normal distributions are $(6/T)^{1/2}$ and $(24/T)^{1/2}$.

Fig. 1. Correlograms for the FX Implied Volatility Series



Thus, modeling the FX implied volatility series appears to be difficult since very complicated dynamics including the pronounced long memory feature is quite apparent in both its first two conditional moments. As presented by Baillie, Han, and Kwon (2002), the ARFIMA-FIGARCH model could be a good representation for the financial time series data which possess the long memory feature in both its conditional mean and conditional variance process. Thus, this section applies the basic ARFIMA-FIGARCH model to estimate the long memory parameters in the conditional mean and the conditional variance process of the FX implied volatility series[3]. The parametric modeling of the FX implied volatility is nonstandard and after considerable implementations, the following ARFIMA-FIGARCH model is specified. In particular, the specific model that is consistent with these stylized facts is the basic ARFIMA (p,d,q)-FIGARCH (m,b,n) process of Baillie, Han and Kwon (2002),

$$\varphi(L)\ (1 - L)^{d}(y_t) = \mu + \theta(L)\varepsilon_t, \qquad (1)$$

$$\varepsilon_t^2 = z_t\sigma_t^2, \qquad (2)$$

$$[1 - \beta(L)]\sigma_t^2 = \omega + [1 - \beta(L) - (1 - \psi(L))(1 - L)^{b}\ ]\varepsilon_t^2, \qquad (3)$$

where $\varphi(L)$, $\theta(L)$, $\beta(L)$ and $\psi(L)$ are polynomials in the lag operator of order p,q,m and n respectively with all their roots lying outside the unit circle, (d) an (b) are the fractional integration parameters in the conditional mean and the conditional variance process, z_t is i.i.d.(0,1) process, and t = 1,.., n.

3) Dunis, Kellard, and Snaith (2013) has used the similar ARFIMA model to analyze the implied volatility of the intra-day EUR-USD exchange rates.

The equations (1) through (3) are estimated by using non-linear optimization procedures to maximize the Gaussian log likelihood function,

$$\ln(L) = -(T/2)\ln(2\pi) - (1/2)\Sigma_{t=1,..n} [\ln(\sigma_t^2 + \varepsilon_t^2\sigma_t^{-2})] \quad (4)$$

Since most series are not well described by the Gaussian normal density in equation (4), subsequent inference is consequently based on the Quasi Maximum Likelihood Estimation (QMLE) technique of Bollerslev and Wooldridge (1992). Simulation evidence presented by Baillie, Han and Kwon (2002) shows that QMLE works quite well for joint estimation of the model in equations (1) through (3), including the case of when $0.5 < d < 1$. For the conditional variance process, asymptotic normality and consistency has only been derived for the FIGARCH model by Baillie (1996). And, the orders of the ARMA and FIGARCH polynomials in the lag operator are selected to be parsimonious and provide a proper model for the autocorrelation structure of the FX implied volatility series.

The exact parametric specifications of the models that best represent the degree of autocorrelation in the conditional mean process and the conditional variance process of the FX implied volatility series appear to be the ARFIMA (1,d,0)-FIGARCH (1,b,1) model for the EUR-USD rates and the ARFIMA (1,d,0)-FIGARCH (1,b,0) model for the USD-JPY rates. The estimation results are reported in 〈Table 2〉 applying the above models for the FX implied volatility series of the EUR-USD and the USD-JPY rates. While the long memory volatility parameters are found to be 0.44 and 0.41, the estimated long memory parameters in the conditional mean process are 0.63 and 0.58 for the EUR-USD and the USD-JPY rates. Since the estimated long memory parameter values are all statistically significant at 1% significance level thereby indicating that the long memory model generally provides a good fit for the FX implied volatility series. Thus, these results strongly support that there exists significant long memory feature in both the conditional mean process and the conditional variance process of the FX implied volatility series for the EUR-USD and the USD-JPY rates. These are quite consistent with the findings of 〈Table 1〉 and 〈Fig. 1〉 which show the apparent autocorrelations decaying more slowly at the hyperbolic rate in the implied volatilities and the squared volatilities in the EUR-USD rates and the USD-JPY rates.

Table 2. Estimation Results of Basic ARFIMA- FIGARCH Models for the FX Implied Volatility series

	EUR-USD	USD-JPY
μ	12.1805*** (0.9711)	9.7282*** (2.4273)
θ	0.3052*** (0.0319)	0.2957*** (0.0552)
d	**0.6313***** **(0.0268)**	**0.5803***** **(0.0538)**
b	**0.4499***** **(0.0894)**	**0.4173***** **(0.0850)**
ω	0.0266*** (0.0093)	0.0747*** (0.0215)
β	0.6086*** (0.0715)	0.2998*** (0.1057)
ψ	0.2638*** (0.0670)	- -
ln(L)	-5686.189	-6607.469

m_3	0.146	0.257
m_4	20.413	11.526
Q(20)	25.527	15.238
$Q^2(20)$	7.055	13.332

Notes: 1. robust standard errors are in parentheses below the corresponding parameter estimates.
2. ln(L) refers to the value of the maximized log likelihood function, the values of m_3 and m_4 are the skewness and kurtosis of the standardized residuals, and the Q(20) and $Q^2(20)$ are the modified Ljung-Box test statistics with 20 degrees of freedom based on the standardized residuals and squared standardized residuals.
3. (*,**,***) denote the statistical significance at 10%, 5% and 1% level respectively.

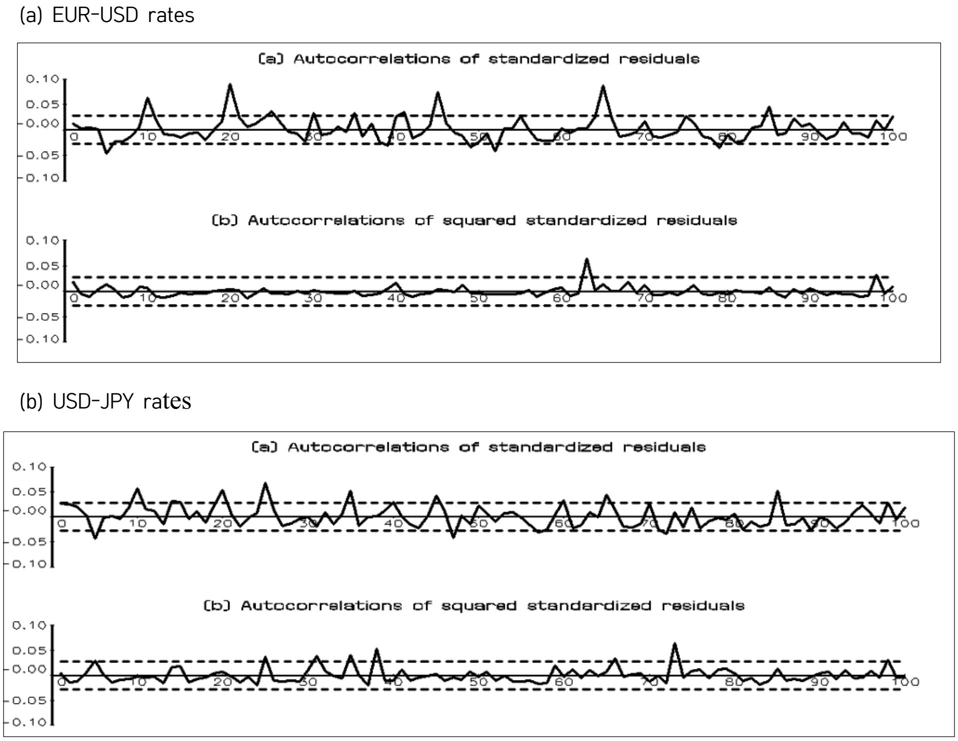
The strong long memory may possibly be related with neglecting some forms of the structural changes in the FX implied volatility series. Also, it is well known that the forms of structural changes can be mistakenly considered as the long memory feature (Diebold and Inoue, 2001). A relevant possibility is that the structural changes may provide an explanation of the empirical finding of the long memory in the FX implied volatility series. It is could be interesting to note that the estimated long memory parameters in the conditional mean process of the FX implied volatility series all exceed the region of stationary long memory ($d > 0.5$) implying that the conditional mean processes all appear to be non-stationary but mean-reverting while the conditional variance process seem to follow the stationary process with the estimated long memory parameters (b) whose values are all smaller than 0.5. And, the long memory feature in the EUR-USD rates appears to be slightly greater than that in the USD-JPY rates, and that the long memory feature in the conditional mean process is a little higher than in the conditional variance process for the FX implied volatilities as presented in Section I.

And, the estimated values of the Q(20) and the $Q^2(20)$ which are the modified Ljung-Box test statistics calculated from the standardized residuals show that there is no strong evidence of additional autocorrelation in the standardized residuals or the squared standardized residuals. Thus, this basic ARFIMA-FIGARCH model seems to match the dynamics of the FX implied volatility series for the EUR-USD rates and the USD-JPY rates. Furthermore, the results can be confirmed by the 〈Fig. 2〉 which presents the correlograms for the autocorrelations in the standardized residuals and the squared standardized residuals estimated from the basic ARFIMA-FIGARCH model with the dotted lines representing the band in which there is no serial correlation at the 95% confidence level. The figure shows that in each case the autocorrelations mostly fluctuate inside the band implying that there do not exist any significant serial correlations and the long memory features in the standardized residuals and the squared standardized residuals.

For the comparison, this paper also estimates the long memory parameters (d and b) in the FX implied volatility series by using Local Whittle semiparametric estimation method (Taqqu and Teverovsky, 1997)[4]. The motivation for using the local Whittle estimator is largely due to its performance in the presence of non i.i.d. densities, as documented by Taqqu and Teverovsky (1997). The Local Whittle estimator appears particularly desirable in situations where the long memory dependence of a time series is compounded by very non Gaussian, fat tailed densities. The papers of Taqqu and Teverovsky (1997, 1998) have reported the detailed simulation studies of various semi parametric estimators for long range dependence and find the Local Whittle estimator to perform well in extreme non Gaussian cases.

4) Nielsen (2007) has used the similar Local Whittle method to analyze the implied volatility dynamics.

Fig. 2. Correlograms for the Standardized Residuals of the FX Implied Volatility Series from ARFIMA-FIGARCH Model



If $f(v_j)$ is the spectral density of the series, then the local Whittle estimator only requires specifying the form of the spectral density close to the zero frequency. For a long memory process, $f(v_j) \approx g(d) \mid v_j \mid^{-2d}$, as $v_j \rightarrow 0$, and for g(d) which is some function of d which is the fractional integration parameter. The Local Whittle estimator of the fractional integration parameter (d) is obtained by minimizing the objective function,

$$R(d) = \ln[(1/m) \Sigma_{j=1,m} [I(v_j) v_j^{2d}] - (2d/m) \Sigma_{j=1,m} [\ln(v_j)], \quad (5)$$

where $I(v_j) = (2\pi T)^{-1} \mid \Sigma_{t=1,T} (y_t) \exp(itv_j) \mid^2$, and is the periodogram of the FX implied volatility series (y_t).

⟨Table 3⟩ presents the estimates of the Local Whittle method for the long memory feature in the FX implied volatility. The long memory parameters in the mean process (d) and the variance process (b) are estimated from the implied volatility series and the squared implied volatility series. The estimated long memory parameters in the conditional mean process of the FX implied volatility are 0.72 and 0.68, and the estimated parameters in the conditional variance process are 0.64 and 0.53 for the EUR-USD rates and the USD-JPY rates. Even though they are found to be a little greater than the values from the ARFIMA-FIGARCH model, the estimated long memory parameters from the Local Whittle method are all statistically significant at 1% significance level and they are generally in consistent with the ARFIMA-FIGARCH model supporting the existence of the long memory feature in the FX implied volatility series.

Table 3. Long Memory Parameters Estimated from Local Whittle Method for the FX Implied Volatility Series

	EUR-USD	USD-JPY
d	0.7262*** (0.0129)	0.6821*** (0.0132)
b	0.6487*** (0.0144)	0.5377*** (0.0141)

Notes: 1. Standard errors are in parentheses below the corresponding parameter estimates.
2. (*,**,***) denote the statistical significance at 10%, 5% and 1% level respectively.

Ⅲ. Structural Changes in FX Implied Volatility

As presented in the introduction, many previous studies have provided abundant motivations to allow for the possibility of the structural changes in the financial time series data including the FX implied volatility. In particular, the structural changes in the FX implied volatility could be related to the changes caused by traders' trading pattern, private information or public information in the FX markets (Kim and Kim, 2003). And, it should be noted that neglecting structural changes may lead to an upward bias in estimates of long memory parameters (Granger and Hyung, 2004; Diebold and Inoue, 2001). The main purpose of this study is to propose a model that is consistent with evidence of the long memory and the structural changes in the FX implied volatility series. And, the results in the previous section are strongly indicative of the long memory feature as being a reasonable description of the data generating process for the FX implied volatility series. Thus, it is important to consider the structural changes with the long memory feature and investigate the presence of neglected structural changes because otherwise the value of the long memory parameters may be overestimated (Baillie and Kapetanios, 2007).

This section investigates the possibility of the structural changes with the long memory feature in the FX implied volatility series of the EUR-USD and JPY-USD rates. One of the quite powerful approaches to account for the structural changes and the long memory feature is the Adaptive-ARFIMA-Adaptive-FIGARCH model of Baillie and Morana (2012) which allows the intercepts to be time varying in the conditional mean process and the conditional variance process for the structural changes with the long memory feature. As presented by Baillie and Morana (2012), the Adaptive-ARFIMA-Adaptive-FIGARCH model can be derived from the basic ARFIMA-FIGARCH model by directly allowing the intercepts to be time varying according to the flexible Fourier form of Gallant (1984). The flexible Fourier form model can allow for a very efficient modeling of structural changes since it does not require any pretests to determine the actual location of change points and add any estimation complexity.

Hence, the Adaptive-ARFIMA-Adaptive-FIGARCH model is formed from two basic components of the long memory feature and the deterministic time-varying intercepts which allow for the changes. Although the deterministic process modeled by the flexible Fourier form is smooth, it has been shown to be able to accurately approximate quite abrupt structural changes

(Baillie and Morana, 2012). The Adaptive-ARFIMA-Adaptive-FIGARCH model then allows for the long memory and structural changes in the conditional mean process and the conditional variance process of the FX implied volatility series. The Adaptive ARFIMA (p,d,q,k)-Adaptive-FIGARCH (m,b,n,h) model can be specified the following:

$$\varphi(L)\ (1 - L)^{d}(y_t) = \mu_t + \theta(L)\varepsilon_t, \tag{6}$$
$$\mu_t = \mu_0 + \Sigma_{j=1..k}[\gamma_{j,m} \sin(2\pi jt/T) + \delta_{j,m} \cos(2\pi jt/T)] \tag{7}$$
$$\varepsilon_t^2 = z_t\sigma_t^2, \text{ where } z_t \sim \text{i.i.d.}(0,1), \tag{8}$$
$$\sigma_t^2 = \omega_t + [1 - \beta(L) - (1 - \psi(L))(1 - L)^{b}\ ]\varepsilon_t^2, \tag{9}$$
$$\omega_t = = \omega_0 + \Sigma_{j=1..h}[\gamma_{j,v} \sin(2\pi jt/T) + \delta_{j,v} \cos(2\pi jt/T)] \tag{10}$$

From the above model on applying the restrictions $\mu_t = \mu$, $\gamma_{j,m} = \delta_{j,m} = 0$ (j=1,⋯,k) and $\omega_t = \omega$, $\gamma_{j,v} = \delta_{j,v} = 0$ (j=1,⋯,h), the basic ARFIMA-FIGARCH model can be obtained. Furthermore, by imposing the additional restrictions that d=b=0, then the simple ARMA-GARCH model is obtained. See Baillie and Morana (2012) for additional details.

The Adaptive-ARFIMA-Adaptive-FIGARCH model is based on the basic ARFIMA-FIGARCH model with time dependent intercepts (μ_t and ω_t) which are represented by a linear combination of harmonic terms. Following the method of Baillie and Morana (2012), the choice of the functional form for the time varying intercepts is deliberately chosen to be a flexible Fourier form (FFF) of Gallant (1984) and is well known to be an excellent approximation to a wide range of non-linear functions for the suitable choice of the orders of harmonic approximation (k and h) due to the universal approximation feature of trigonometric expansions[5)]. The estimation and the inference for the parameters of the above model can be carried out by means of the method of QMLE where the loglikelihood function is numerically maximized with respect to the parameters as in Baillie and Morana (2012). The procedure can implement simultaneous estimation of all the model's parameters including those in the flexible function forms which specify the time varying intercepts in the conditional process. From the results of Hosoya (1997) and Yajima (1988), it appears that the model satisfies the required conditions for the consistency and asymptotic Normality of the QMLE estimators.

One important factor to be considered for the practical implementation of the model is to select the appropriate orders of the trigonometric terms (k and h) in the flexible functional forms since the unnecessary inclusion of the trigonometric components can lead to some bias in the estimates of the long memory parameters as pointed out by Baillie and Morana (2012). Also, there is one interesting issue related to the interpretation of the structural changes whether or not they correspond to some important changes in FX markets which drive the pattern of the implied volatilities. However, it is difficult to distinguish these changes without more detailed information. Furthermore, there are no formal test available for detecting multiple structural changes in the long memory I(d) process with unknown number of changes (Granger and Hyung, 2004).

In this paper, the trigonometric terms are selected 3 and 4 for the FX implied volatility series of the EUR-USD rates and the USD-JPY rates based on the Schwartz Information Criterion (SIC) following the method of Baillie and Morana (2012). Thus, the selected models for the FX im-

5) Enders and Lee (2012) has used the similar FFF form for modeling of structural breaks in the mean of a stochastic process.

plied volatility series are Adaptive-ARFIMA (1,d,0,3)-Adaptive-FIGARCH (1,b,1,3) model for the EUR-USD rates and Adaptive-ARFIMA (1,d,0,4)-Adaptive-FIGARCH (1,b,0,4) model for the USD-JPY rates. The estimation results of the Adaptive-ARFIMA-Adaptive-FIGARCH model for the FX implied volatility series are presented in 〈Table 4〉. As expected, the results show that the long memory parameters in the conditional mean process of the FX implied volatility series are found to be smaller than the values from the basic ARFIMA-FIGARCH models with the estimated parameters (d) taking the values of 0.55 and 0.49 for the EUR-USD rate and the USD-JPY rates. In the case of the USD-JPY rates, the estimated value of the long memory parameter in the conditional mean process is below 0.5 implying the mean process is stationary and the non-stationarity seems to be disappeared. Similarly, the estimated long memory parameters (b) in the conditional variance process are 0.39 and 0.35 for the EUR-USD rate and the USD-JPY rates, which are also reduced compared with the values of the previous basic models. Thus, the Adaptive-ARFIMA-Adaptive-FIGARCH models generally find the less persistent long memory parameters in the FX implied volatility series than the basic ARFIMA-FIGARCH models.

Table 4. Estimations of the Adaptive-ARFIMA-Adaptive-FIGARCH Models for the FX Implied Volatility Series

	EUR-USD Rates	USD-JPY Rates
μ_0	8.8423*** (1.3493)	6.5557*** (1.1379)
$\gamma_{1,m}$	1.0781 (1.5533)	0.5722 (1.1799)
$\delta_{1,m}$	-0.8495 (0.8491)	-1.2718* (0.7215)
$\gamma_{2,m}$	-1.2736** (0.6121)	-0.6034 (0.5334)
$\delta_{2,m}$	2.7136 (0.9694)	-0.3937 (0.4358)
$\gamma_{3,m}$	1.2708* (0.7344)	4.0688*** (1.0024)
$\delta_{3,m}$	-0.5203 (0.5729)	0.9280 (0.6042)
$\gamma_{4,m}$	- -	-0.0670 (0.5315)
$\delta_{4,m}$	- -	-0.8318* (0.4643)
θ	0.3435*** (0.0327)	0.3142*** (0.0335)
d	**0.5544***** **(0.0251)**	**0.4971***** **(0.0246)**
b	**0.3976***** **(0.0256)**	**0.3586***** **(0.0221)**
β	0.5570*** (0.0411)	0.2093*** (0.0259)
ψ	0.2884*** (0.0337)	- -
ω_0	0.0426*** (0.0058)	0.1062*** (0.0074)
$\gamma_{1,v}$	-0.0087** (0.0039)	-0.0572*** (0.0096)
$\delta_{1,v}$	-0.0031 (0.0037)	-0.0581*** (0.0089)
$\gamma_{2,v}$	0.0278*** (0.0049)	0.0293*** (0.0101)

$\delta_{2,v}$	0.0290*** (0.0047)	-0.0038 (0.0077)
$\gamma_{3,v}$	-0.0006 (0.0041)	0.0489*** (0.0088)
$\delta_{3,v}$	-0.0181*** (0.0049)	-0.0277*** (0.0088)
$\gamma_{4,v}$	- -	-0.0603*** (0.0091)
$\delta_{4,v}$	- -	-0.0346*** (0.0089)
ln(L)	-5644.276	-6566.903
m_3	0.299	0.125
m_4	16.115	10.498
Q(20)	26.243	14.727
$Q^2(20)$	7.452	13.634
SIC	0.012	0.022
LR	830826	81.132

Notes: 1. The table is the same as Table 2 except the trigonometric parameters (γ,δ), the SIC which is the value of the Schwartz information criterion and the LR which is the Loglikelihood Ratio test statistics.
2.(*,**,***) denote the statistical significance at 10%, 5% and 1% level respectively.

These results present that the basic ARFIMA-FIGARCH models which neglect the structural changes lead to an upward bias and tend to overestimate the long memory parameters in the FX implied volatility series. Thus, the adaptive deterministic components in form of the time dependent intercepts in the conditional mean and the conditional variance process appear to be important and successful in accounting for the structural changes in the FX implied volatility. And, it may need to note that there still exists the long memory feature in the FX implied volatility series even after the structural changes are accounted for by the Adaptive-ARFIMA-Adaptive-FIGARCH models, which could be considered as the intrinsic feature in the FX markets as explained in Baillie (1996).

In addition, no strong evidence of serial correlation or instability can be detected in the conditional mean and the conditional variance process of the FX implied volatility series from the Ljung-Box test statistics. In particular, the Loglikelihood Ratio (LR) test statistics, denoted by LR, for testing the null hypothesis of the basic ARFIMA-FIGARCH model versus the Adaptive-ARFIMA-Adaptive-FIGARCH model are found to be 83.82 and 81.13 for the EUR-USD rates and the USD-JPY rates. Under the null hypothesis, the LR statistics follows an asymptotic $\chi 2$ distribution and the basic ARFIMA-FIGARCH models are rejected at 1% significance level. Thus, there can find the better improvement in fit and the reduction in the long memory parameters once the structural changes and the long memory feature are jointly modeled. These test statistics support the facts that the inclusion of the trigonometric components makes an important improvement to the general goodness of fit of the model for the FX implied volatility series and that the Adaptive-ARFIMA-Adaptive-FIGARCH model is superior to the basic ARFIMA-FIGARCH model for representing the structural changes and the long memory feature jointly in the FX implied volatility series, which is consistent with the findings of Baillie and Morana (2012).

Ⅳ. Conclusion

This paper addresses the issues in understanding the long memory feature and the structural changes in the FX implied volatility series of the daily EUR-USD and JPY-USD exchange rates. The paper first applies the basic ARFIMA-FIGARCH model and the Local Whittle method to analyze the long memory feature in the FX implied volatility series and find strong evidence of the long memory feature in the conditional mean and the conditional variance process of the FX implied volatility series. Then, the basic ARFIMA-FIGARCH model is generalized by allowing the intercepts to be time varying in the conditional mean process and the conditional variance, which is the Adaptive-ARFIMA-Adaptive-FIGARCH model with the ability to flexibly represent forms of the long memory feature and the structural changes together. The adaptive model can be estimated by means of the QMLE, which is conjectured to feature the usual optimal asymptotic features as the basic model. The empirical results of the adaptive model indicate the apparent presence of the long memory feature and the structural changes in the conditional mean and the conditional variance process of the FX implied volatility series.

Thus once structural changes are accounted for appropriately, the long memory feature can be reduced significantly indicating that the time dependent intercepts for the structural changes seem to be an important component and neglecting structural changes could lead to an upward bias and overstate the long memory feature in the FX implied volatility series. And, the adaptive model can make an important improvement to the goodness of fit of the model and that the adaptive model is the better than the basic model for modeling jointly the structural changes and the long memory feature in the FX implied volatility series. Consequently, this paper shows the statistical evidence that the FX implied volatility series in FX markets contain the dynamic factors, the long memory feature and the structural changes, and that the Adaptive-ARFIMA-Adaptive-FIGARCH model could be an excellent description and competitor with more elaborate models for the FX implied volatility series.

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