J. Korean Soc. Math. Ed. Ser. D. (2022) 25(3), 171–173 ISSN 1226-6191
https://doi.org/10.7468/jksmed.2022.25.3.171 Online ISSN 2287-9943

EDITORIAL

Understanding the Critical Areas in Teaching and Learning of Geometry and Measurement

Dae S. Hong[1], Kyong Mi Choi[2]

[1] Professor, Department of Teaching and Learning, University of Iowa, dae-hong@uiowa.edu
[2] Professor, Department. of Curriculum, Instruction and Special Education, University of Virginia, kc9dx@virgina.edu

Accepted: September 27, 2022 / Published online: September 30, 2022

Geometry and measurement are important to students' future performance in mathematics and other STEM fields as they provide foundational knowledge for understanding elementary topics including multiplication, fractions, and advanced topics of calculus. However, in the domain of geometry and measurement, US students have shown poor performance on national and international tests (Mullis, Martin, Foy, & Hooper, 2016). Many studies have pointed out that what and how students learn geometry and measurement is mostly procedural basis without conceptual understanding. What is more alarming about teaching and learning of geometry and measurement is teachers also have demonstrated similar challenges as their students (Hong & Runnalls, 2020, 2022; Runnalls & Hong, 2019). With many formulas in geometry and measurement, it is well – known that students and teachers know how to use those formulas but without knowing why they work. When teachers do not possess conceptual understanding of geometry and measurement topic, it is highly likely that their lessons might be procedurally focused and students might not learn geometry and measurement conceptually. There could be many contributing areas to these results such as limitations in curriculum materials, procedurally focused lessons, previously taken mathematics classes by teachers and teachers' content and pedagogical knowledge. When planning mathematics lessons, teachers use a variety of resources, including textbooks, outside tasks, and other curriculum materials. While planning their lessons, their personal knowledge plays a critical role in their decision-making process in how they select and implement tasks. What this means is curriculum materials need to include tasks that promote reasoning and problem solving and address students' learning challenges. At the same time, teachers' lessons also need to include such tasks and questions that specifically address students' learning challenges and key conceptual ideas. In terms of conducting research in geometry and measurement, exploring these different areas to improve teaching and learning of geometry is one of the essential areas in mathematics education.

Pak's article in this issue deals with a novice teacher's attempt to implement conceptual ideas in learning length measurement. The article describes the challenges that a teacher faces in implementing such lessons, which indicates what we can do to support teachers when they are in teacher education program. His results show that although teachers' intentions were to have interactive lessons, where students have opportunities to share and explain their thinking, it was the teacher who overtook lessons and dominated class discussions. Being able to share students' thinking and using their thinking is one of effective teaching practices recommended by National Council of Teachers of Mathematics (National Council of Teachers of Mathematics [NCTM], 2014). It is well – known that when teachers dominated class discussions, teachers and students are not able to maintain cognitive demand (Henningsen & Stein, 1997). His results provide insights for teacher educators about challenges in implementing mathematics lessons that promote effective teaching practices.

Another area that will impact how students learn geometry and measurement is teachers' content knowledge. One of the issues in geometry and measurement is many elementary students were being taught by teachers who have limited geometry and measurement content knowledge. Especially in elementary level, there have been several studies that showed teachers' limited knowledge in understanding geometry. Cox's article in this issue raised the question about whether high school mathematics teachers need to take geometry class or not. Cox argues that with improved geometric thinking, students will be able to improve algebraic thinking and functional thinking as well. Her study used data collected by the High School Longitudinal Study – 2009 and tested if taking geometry class can positively impact students' success in geometry.

In addition to teachers' knowledge (both content and pedagogical), another critical area is planning lessons that promote effective teaching practices. According to NCTM, effective mathematics lessons will include cognitively demanding tasks, purposeful questions, using student's thinking and allowing students to struggle productively. Two studies in this issue describe possible ways to include such mathematics lessons in geometry.

Flores' article describes a systematic way to implement such lessons. She describes the concrete-representational-abstract integrated (CRA-I) sequence. CRA-I is a systematic approach to using multiple representations. When using CRA-I the teacher organizes lesson materials in a particular order. First, the teacher provides explicit instruction using concrete objects, two-dimensional representations, and abstract symbols (e.g., equations). In each lesson, the students solve problems or complete mathematical tasks using all three types of representation (concrete, representational, and abstract). While implementing CRA-I lessons, it is suggested that teachers need to ask purposeful questions to guide their students. What is interesting about CRA-I is it includes all representations together and the systematic fading to the abstract. Moreover, its explicit instruction can support student who struggle with mathematics.

Seshaiyer and Suh's article also shows an interesting and rigorous way of presenting geometry topics. His suggested lessons include learning by doing, challenge-based learning, mathematical modeling and connecting their lessons to learning trajectories.

In turn, students were able to become "doers" of mathematics who are challenged by rigorous mathematics problems. As students use different presentations and become doers of mathematics, students who use both Flores' and Seshaiyer's approaches can struggle productively as suggested by (NCTM, 2014).

In this special issue, we have four articles that explore different critical areas in geometry and measurement. We encourage mathematics education researchers and mathematics teacher educator to think about what it means for teachers to possess strong content and pedagogical knowledge in geometry and measurement, how to promote interactive classes that allow students to share and express their thinking, and how to develop geometry and measurement lessons that allow students to work on cognitively challenging problem and actively involved in problem solving process.

References

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*(5), 524-549. https://doi.org/10.2307/749690

Hong, D. S., & Runnalls, C. (2020). Examining preservice teachers' responses to area conservation tasks. *School Science and Mathematics, 120*(5), 262-272. https://doi.org/10.1111/ssm.12409

Hong, D. S., & Runnalls, C. (2022). "I don't think they need any help with the formula": Examining elementary preservice teachers' responses to volume tasks. *School Science and Mathematics, 122*(3), 155-168. https://doi.org/10.1111/ssm.12521

Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2016). *TIMSS 2015 International Results in Mathematics*. Retrieved from http://timssandpirls.bc.edu/timss2015/international-results/

National Council of Teachers of Mathematics [NCTM]. (2014). *Principles to actions: ensuring mathematical success for all*. NCTM.

Runnalls, C., & Hong, D. S. (2019). "Well, they understand the concept of area": pre-service teachers' responses to student area misconceptions. *Mathematics Education Research Journal, 32,* 629-651. https://doi.org/10.1007/s13394-019-00274-1