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RESEARCH ARTICLE

Geometry: Do High School Mathematics Teachers really Need it?

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Abstract

A debate about the importance of geometry courses has existed for years. The questions have revolved around its significance to students and teachers alike. This study looks to determine whether a teacher taking a college-level geometry course has a positive relationship with their students' algebraic reasoning skills. Using data from the High School Longitudinal Study 2009 (HSLS09: Ingels et al., 2011, 2014), it was determined that 9th-grade teachers who took a college-level geometry course had a significant positive association with their students' 11th-grade algebraic reasoning scores. This study suggests that teachers who take geometry during college have a lasting effect on their students. The implications of these findings and how they may affect higher education are discussed.

Keywords: geometry, teacher training, mathematical knowledge for teaching, HSLS:09

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I. INTRODUCTION

The Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA) determined that geometry is one of the weakest mathematics subjects for American students (Ginsburg et al., 2005). These studies determined that, relative to the average score across all content areas, performance in geometry was significantly lower in grade 8 and at age 15 (Ginsburg et al., 2005). In a study looking at the geometry content-area knowledge of preservice and in-service teachers, Aslan-Tutak and Adams (2015) noted that teachers, especially new teachers, struggled with the topic of geometry. During their investigation, Aslan-Tutak and Adams noted that many of their first-year teachers found themselves thoroughly reviewing the topics before feeling comfortable instructing their students. Many beginning teachers are expected to teach geometry when they have done very little geometry themselves since they were in high school (Jones, 2000). Aslan-Tutak and Adams noted that the preservice teachers involved in their study mentioned that they enjoyed their experiences in their high school geometry classes where they had several hands-on experiences but felt they did not learn much geometry. However, the amount of geometry decreases as one moves through the high school mathematics curriculum until it all but disappears at the college level (Jones, 2000).

Teachers in the United States are likely to devote more instructional time to making mathematics relevant to students by providing real-world context and meaning to the topics and concepts presented during instructional time (Aslan-Tutak & Adams, 2015; Ginsburg et al., 2005). Carpenter et al. (1988) emphasized that subject matter knowledge held by the teacher strongly influences their use of pedagogical tools. However, some teachers struggle to understand the usefulness of geometry and experienced difficulty implementing a geometric approach to solving problems (Mousoulides & Gagatsis, 2004). Mousoulides and Gagatsis (2004) argue that a geometric approach to problem-solving is closely related to developing a better understanding of algebra concepts such as equations, graphs, and functions. Some teachers admitted to perceiving geometry as different than mathematics due to the lack of algebraic topics in geometry (Aslan-Tutak & Adams, 2015). The limited amount of research focused on the knowledge of geometry for teaching concludes that beginning teachers are not equipped with the necessary content and pedagogical content knowledge of geometry (Aslan-Tutak & Adams, 2015; Browning et al., 2014; Jones, 2000; Swafford et al., 1997). The questions that arise are: 1) Is it necessary for all teachers to take geometry during college, and 2) What are the benefits of teachers taking a geometry class during their collegiate careers?

II. LITERATURE REVIEW

Background

The argument about whether to teach geometry in high school and what material that course should contain is not new (Battista, 2007; Gonzalez & Herbst, 2006; Usiskin, 1980). The focus of this argument has often centered around the use of proofs and logic within the geometry course. Little attention has been paid to the coursework taken by teachers during their collegiate careers and how that coursework may affect their future students' mathematical abilities (e.g., algebraic reasoning, spatial reasoning, or generalizing mathematical concepts) within the classroom. The questions then become; do preservice teachers *need* to take geometry during their college careers, and what effect does having experience in geometry have on their students' learning.

The Conference Board of the Mathematical Sciences (CBMS) (2012) recommended that the coursework taken by prospective mathematics teachers be tailored to the work of teaching. Their recommendation suggests that the coursework taken by preservice mathematics teachers include courses designed specifically for prospective teachers to "provide opportunities for future teachers to learn the mathematics they need to know to be well-prepared beginning teachers who will continue to learn new mathematical content and deepen their understanding of familiar topics" (Conference Board of the Mathematical Sciences [CBMS], 2012, p. 5). This recommendation also included creating methods courses that incorporate content knowledge related to the mathematics that prospective teachers will teach, along with mathematical knowledge for teaching (Ball et al., 2008; Conference Board of the Mathematical Sciences, 2012; Murray & Star, 2013). However, as Murray and Star (2013) noted, schools have struggled to make a "substantial change in the types of mathematics content courses that prospective teachers take" (p. 1297).

A recent study by Choi and Cox (2020) suggested that coursework in Calculus, Foundations of Mathematics, and an unnamed third course had statistically significant, positive effects on 9th-grade students' algebraic reasoning skills. The study used data collected by the High School Longitudinal Study - 2009 (HSLS:09) that consisted of the students' quintile rankings on a test of algebraic reasoning and the college courses taken by the students' 9th-grade mathematics teachers (Ingels et al., 2011). The HSLS:09 grouped similar college-level courses when asking teachers which coursework they had taken (Ingels et al., 2011). For example, the coursework that Choi and Cox refer to as "Calculus" consisted of calculus (i.e., Calculus I, II, and III), analysis, and differential equations (i.e., differential and partial differential equations).

By testing for an association between whether the teacher took a specific set of courses and the students' quintile scores on the test of algebraic reasoning, Choi and Cox (2020) found that while several of the course offerings were significantly associated, only two of the groupings had effect sizes worth mentioning, the courses mentioned above of Calculus and Foundations. After conducting a stepwise regression to determine the number of courses needed to affect students' quintile scores positively, Choi and Cox suggested that the positive impact on student achievement ended after the third mathematics course. However, the identity of this third course was not determined (Choi & Cox, 2020). Based on Choi and Cox's data analysis, a possible candidate for this third course could be coursework in geometry. This notion is due to the significant association geometry had with student quintile scores ($p < .001$) and effect size that lies just outside the realm of feasibly small (*Cramer's V* = .06) (Choi & Cox, 2020).

Connections between Algebra and Geometry

Important connections exist between algebra and geometry. Both use symbols as variables, constants, labels, and parameters (Dindyal, 2004). The meaning and usage of variables as placeholders or unknown elements is a notion that transcends secondary and postsecondary mathematics. Variables and unknowns are related to the broader idea of algebraic thinking and reasoning (Dindyal, 2004). These variables often lead to representations and generalizations to solve complex, non-routine problems (Blanton & Kaput, 2005; J. J. Kaput, 2008; Mullis et al., 2005). A prime example of these generalizations appears when using area formulas. These formulas are often generated and explained in a geometry course and later applied in algebra courses. The ability to prove and create generalizations and formulas (e.g., area, perimeter, and interior angles given polygons) are vital components of the idea known as algebraic reasoning (Blanton & Kaput, 2005). As Blanton and Kaput (2005) described, algebraic reasoning is "a process in which students generalize mathematical ideas from a set of particular instances, establish those generalizations through the discourse of argumentation, and express them in increasingly and age-appropriate ways" (p. 413).

Another meaningful connection between geometry and algebra is the use of problem-solving and modeling skills. Successful problem solving involves interpreting word problems, processing geometric and algebraic concepts and facts, and applying the facts to solve problems (Novak & Tassell, 2017). One skill needed when creating models to solve problems is using various modes of representation. These representations include both geometric representations (e.g., figures and shapes to represent binomial multiplication) and algebraic representations (e.g., equations, functions, and formulas) (Novak & Tassell, 2017). These problem-solving skills are essential aspects of algebraic reasoning (Blanton & Kaput, 2005).

Algebraic and Geometric Approach to Understanding Functions

Mousoulides and Gagatsis (2004) suggested that "functions are among the most important unifying ideas in mathematics" (p. 385) and form one of the most important ideas in all mathematics in terms of understanding and exploring other topics within the subject. Functions often appear in geometry in the form of formulas, such as area formulas and the Pythagorean Theorem. Although the process may be complicated, building a deep understanding of the concept of functions is essential for success in mathematics (Mousoulides & Gagatsis, 2004).

Mousoulides and Gagatsis (2004) suggested two perspectives to approach understanding functions: process and object. The process (or algebraic) perspective perceives a function as the linking of x and y values in the manner found in most math textbooks: for each *x-value*, there is a corresponding *y-value* (Mousoulides & Gagatsis, 2004). This straightforward concept of a function leads students to see the function as an isolated entity not necessarily used in other settings. Students who view functions this way can substitute a value for x into an equation and find a corresponding y (Mousoulides & Gagatsis, 2004).

Using the object (or geometric) perspective of a function or relation allows the students to view the representations as complete entities or objects (Mousoulides & Gagatsis, 2004). Viewing the representations in this manner enables students to see graphs of functions as a single object that can be picked up as a whole and moved (e.g., rotations and translations) (Mousoulides & Gagatsis, 2004). Mousoulides and Gagatsis (2004) found that students who used this approach to functions often performed better when solving more complex problems. Their study also showed that a geometric approach allowed students to manipulate functions as an entity, allowing them to find connections and relations between different representations involved in the problems, which leads the students to create generalizations to use when solving more complex problems (Blanton & Kaput, 2005; Mousoulides & Gagatsis, 2004). Ultimately, Mousoulides and Gagatsis (2004) suggested a close relationship between using a geometric approach to functions and a better understanding of equations, graphs, and functions in general. Understanding these topics is necessary for success in algebra (Blanton & Kaput, 2005; Dindyal, 2004; J. J. Kaput, 2008; Mousoulides & Gagatsis, 2004).

III. PURPOSE

We know from the previous work of Choi and Cox (2020) that during the students' 9th-grade year coursework in Calculus/Analysis/Diff. Equations and Foundations/Logic/History showed a significant association in positively affecting students' scores on the algebraic reasoning test. Choi and Cox also stated that coursework in geometry had a significant association with the 9th-grade students' scores. However, given the effect size was so small (*Cramer's V* = .06), they chose not to include it in their results.

In this study, we look to determine how students' algebraic reasoning is affected by the teachers' geometry coursework and the possibility of a long-term effect on the students' understanding. We will use the HSLS:09 data set gathered for the first follow-up when the students were in their 11th-grade year of high school (Ingels et al., 2014). The data collected from the teachers is still from the students' previous 9th-grade teachers (Ingels et al., 2011, 2014). Using this data will allow us to determine if the effects of the 9th-grade teacher continue beyond that single year.

IV. METHODS

The High School Longitudinal Study of 2009 (HSLS:09) was designed to explore secondary to postsecondary transition plans, paths in and out of STEM fields, and the educational and social experiences that affect a student's journey along those paths (Ingels et al., 2011). We gleaned data from this study, including (a) students' quintile level membership on a test of algebraic reasoning taken during their third year of high school and (b) the college mathematics coursework of their 9th-grade mathematics teachers.

Among 24,658 students initially asked to participate in the study, 23,503 responded to the survey. After treating the missing data, we finalized 15,054 students' data paired with their mathematics teachers' data. In 2011, the students took the Algebraic Reasoning assessment for a second time to provide information about their mathematical understanding (Ingels et al., 2014). The students' scores on this assessment were reported as quintile levels. The study also provided data from the students' 9th-grade mathematics teachers (n = 5710). Among the various types of data provided by the teachers, we focused on their college-level mathematics coursework. The teachers responded either 'yes (=1)' or 'no (=0)' for each content course they took (see Table 1). We must note that the courses listed are as the HSLS:09 survey asked the teachers.

Table 1. Percent of students' teachers who took each specific course during college

Course Taken	%
College Algebra	89.80
Applied Mathematics	38.84
Calculus/Analysis/Differential Equations	92.20
Discrete Mathematics	56.38
Foundations/Logic/History	60.48
College Geometry	81.04
Number Theory	50.13
Probability/Statistics	85.53

V. ANALYSIS

To identify the association that student quintile levels membership has with teachers' mathematics coursework, we ran a χ^2 test of independence followed by a post hoc analysis. Each statistical analysis performed employed the statistical software SPSS (IBM, 2021). During the post hoc analysis, we examined the adjusted residuals from the χ^2 test to determine each association's effect on each variable. To identify which quintiles were affected most by the association and limit the likelihood of a Type 1 error, we used a Bonferroni adjusted z-score.

VI. RESULTS

We conducted a χ^2 test of independence using the students' quintile levels and teachers' mathematics coursework during college and observed significant associations between the quintile levels and all courses except for Number Theory (see Table 2). Based on Sun, Pan, and Wang's (2010) suggestion, we used 0.07 as the cutoff for a small effect size. For this reason, even though all courses were statistically significant, we are only considering those whose Cramer's V value for effect size is greater than the cutoff of 0.07. This cutoff decreases the number of significant courses to only three: Calculus, Foundations, and Geometry.

Table 2. χ^2 Tests and significance – 2009 HSLS Data

	Pearson Chi-square	Significance	Cramer's V
College Algebra	20.47*	<.001	.04
Applied Mathematics	13.99*	.007	.03
Calculus/Analysis/Diff EQ	335.66*	<.001	.15
Discrete Mathematics	59.06*	<.001	.06
Foundations/Logic/History	92.46*	<.001	.08
Geometry	69.77*	<.001	.07
Number Theory	10.11	.039	.03
Statistics	44.04*	<.001	.05
No Mathematics Courses	48.68*	<.001	.06

*association is significant at a level p<.01

Using only those courses with an effect size larger than 0.07, we conducted a post hoc analysis to determine where the association occurs (see Table 3). Comparing the adjusted residuals, we found that the calculus group showed significant associations with former students' scores who tested in the first, second, fourth, and fifth quintiles. This finding suggests that a 9th-grade mathematics teacher who took courses within the Calculus group, as described by HSLS:09, significantly decreased the number of 11th-grade students who fall into the lower quintiles and increased the number who test into the higher quintiles. Looking at Table 3, one can see that Geometry and Foundations coursework also significantly affects students who test into the first and fifth quintiles.

Table 3. Adjusted residuals for χ^2 post hoc analysis

Quintile Level	Calculus	Foundations	Geometry
1	-16.2*	-7.9*	-7.8*
2	-3.8*	-2.1	-0.5
3	1.6	-0.6	1.6
4	5.5*	2.7*	1.2
5	10.1*	6.4*	4.4*

Note. *Adjusted residuals are significantly different at an $\alpha = .1$ according to a Bonferroni adjustment

VII. DISCUSSION

In examining the effects that a 9th-grade mathematics teacher's college-level coursework has on their former students two years after completing their class, we see that coursework in calculus, foundations, and geometry has lasting effects on students. These courses significantly decreased the number of students who test in the lower quintile levels while increasing the number of students in the higher quintiles. As coursework in calculus and foundations is viewed as fundamental and foundational courses for most mathematics majors, one must note the presence of geometry coursework. Many 9th-grade mathematics teachers do not teach geometry. However, we see evidence of a significant effect on the 11th-grade students' scores whose 9th-grade teachers took a college-level geometry course. We assume that these 11th-grade students are not taking geometry and are most likely in a higher-level algebra or pre-calculus course. However, teachers who took geometry during college have a positive impact on the learning experiences of those students.

As Mousoulides and Gagatsis (2004) indicated, a close relationship exists between using geometric approaches to functions and a better understanding of equations, graphs, and functions in general. Perhaps, those students whose teachers had a better understanding of this geometric approach to problem solving and functions developed similar skills while in their 9th-grade mathematics courses. These skills could then be used to build a deeper understanding of equations, graphs, and functions as they become more prevalent in the higher-level mathematics courses. These geometric problem-solving skills could be used to better understand concepts such as binomial or function multiplication, families of graphs of functions, and even inverse functions. These concepts often view the ideas of graphs and functions not as a list of independent ordered pairs but as a single graph or entity. This singular view of the idea is the basis of a geometric approach to problem-solving (Mousoulides & Gagatsis, 2004). This study suggests that geometry coursework may influence student performance in algebraic reasoning.

The implications of this study suggest that schools of education reevaluate the course requirements for mathematics education majors. We see evidence of a significant effect on algebraic reason skills for high school students whose teachers took geometry during college. This finding leads to the conclusion that all mathematics teachers, both elementary and secondary teachers, can benefit from taking geometry during their collegiate careers. These courses allow future teachers to build confidence in teaching geometry and develop the skills to apply and illustrate a geometric approach to understanding concepts such as functions, as mentioned by Mousoulides and Gagatsis (2004). As CBMS (2012) suggested, these courses should approach high school mathematics from an advanced standpoint, considering particular high school topics and developing them in depth. Creating a specialized course for prospective teachers allows for the development of mathematics that is useful in the teachers' professional lives. Moreover, by designing a specialized geometry course to deepen the preservice teachers' understanding of geometry, teachers may experience less anxiety about teaching the subject.

The work involved in this study focused on the association between high school students' algebraic reasoning skills and the college-level mathematics coursework taken by their 9th-grade mathematics teachers. However, it is reasonable to believe that students learn algebraic reasoning skills from all their teachers, even those at the elementary level. The information ultimately handed down to the students begins with the teachers (Wayne & Youngs, 2003). By ensuring that all preservice teachers have experience with and knowledge of problem-solving skills such as the geometric perspective purposed by Mousoulides and Gagatsis (2004), schools of higher education will provide students of all ages the opportunity to learn and develop these skills beginning in elementary school and not having to wait until high school to experience it. More work must be done in this area to develop a course designed for elementary education majors to ensure they receive the skills necessary to build geometric problem-solving skills in younger students. There is a need to conduct more research in this area to determine what skills are necessary for students to learn at different levels.

Many beginning teachers emphasized a lack of confidence in teaching geometry due to a lack of knowledge of the subject (Aslan-Tutak & Adams, 2015). By providing teachers with the skills and confidence to teach geometry, they develop skills applicable to other mathematics subjects. This shift in confidence and growth in teaching ability may positively impact their students' problem-solving and algebraic reasoning skills. Students can then put these skills to use in situations beyond the mathematics classroom.

VIII. LIMITATIONS AND FUTURE STUDIES

We focused on 11th-grade students and their 9th-grade teachers in this study. Further studies that extend to all grade levels of students and mathematics teachers will provide a better picture of the connection between the teachers' college mathematics coursework and their students' outcomes.

We would also like to note that the data source did not delineate some courses. For example, the course referred to as "Calculus" included Calculus, Analysis, and Differential Equations. The analysis treated this range of classes as a single course offering even though it could consist of as many as seven courses. In future studies, we would like to separate these into individual course offerings to see which course affected the students' learning.

References

Aslan-Tutak, F., & Adams, T. L. (2015). A study of geometry content knowledge of elementary preservice teachers. *International Electronic Journal of Elementary Education*, *7*(3), 301–318.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389–407.

Battista, M. T. (2007). The development of geometric and spatial thinking. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 2, pp. 843–908). Information Age Pub.

Blanton, M. L., & Kaput, J. J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. *Journal for Research in Mathematics Education*, *36*(5), 412–446. https://doi.org/10.2307/30034944

Browning, C., Edson, A., Kimani, P., & Aslan-Tutak, F. (2014). Mathematical content knowledge for teaching elementary mathematics: A focus on geometry and measurement. *The Mathematics Enthusiast*, *11*(2), 333–383. https://doi.org/10.54870/1551-3440.1306

Carpenter, T. P., Fennema, E., Peterson, P. L., & Carey, D. A. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education*, *19*(5), 385–401. https://doi.org/10.5951/jresematheduc.19.5.0385

Choi, K. M., & Cox, W. (2020). College coursework on high school teacher performance. A paper presented at the 2020 *International Conference of KSME, Seoul, Korea*.

Conference Board of the Mathematical Sciences. (2012). *Issues in mathematics education: The mathematical education of teachers II* (Vol. 17). Mathematical Association of America.

Dindyal, J. (2004). Algebraic thinking in geometry at high school level: Students' use of variables and unknowns. In I. Putt, R. Faragher & M. McLean (Eds.), *Mathematics education for the third millennium: Towards 2010 (Proceedings of the 27th annual conference of the Mathematics Education Research Group of Australasia, Townsville*) (pp. 183-190). MERGA, Inc.

Ginsburg, A., Cooke, G., Leinwand, S., Noell, J., & Pollock, E. (2005). Reassessing U.S. international mathematics performance: New findings from the 2003 TIMSS and PISA. In *American Institutes for Research*. The American Institutes for Research. https://eric.ed.gov/?id=ED491624

Gonzalez, G., & Herbst, P. G. (2006). Competing arguments for the geometry course: Why were American high school students supposed to study geometry in the twentieth century. *The International Journal for the History of Mathematics Education, 1*(1), 7–33.

IBM. (2021). *SPSS Statistics* (Version 28) [Mac]. IBM Corporation.

Ingels, S., J., Pratt, D., J., Herget, D., R., Burns, L., J., Dever, J., A., Ottem, R., Rogers, J., E., Jin, Y., & Leinwand, S. (2011). *High school longitudinal study of 2009 (HSLS:09) base-year data file documentation*. U.S. Department of Education: National Center for Educational Statistics. http://nces.ed.gov/pubsearch

Jones, K. (2000). Teacher knowledge and professional development in geometry. *Proceedings of the British Society for Research into Learning Mathematics*, *20*(3), 109–114.

Kaput, J. J. (2008). What Is algebra? What is algebraic reasoning? In J. Kaput J., D. W. Carraher, & M. L. Blanton, *Algebra in the early grades* (1st ed., pp. 5–18). Routledge.

Mousoulides, N., & Gagatsis, A. (2004). Algebraic and geometric approach in function problem solving. *International Group for the Psychology of Mathematics Education*, *3*, 385–392. https://eric.ed.gov/?id=ED489596

Mullis, I. V. S., Martin, M. O., Ruddock, G. J., O'Sullivan, C. Y., Arora, A., & Erberber, E. (2005). TIMSS 2007 assessment frameworks. In *International Association for the Evaluation of Educational Achievement*. International Association for the Evaluation of Educational Achievement. https://eric.ed.gov/?id=ED494654

Murray, E., & Star, J. R. (2013). What do secondary preservice mathematics teachers need to know? Content courses connecting secondary and tertiary mathematics. *Notices of the American Mathematical Society*, *60*(10), 1297. https://doi.org/10.1090/noti1048

Novak, E., & Tassell, J. L. (2017). Studying preservice teacher math anxiety and mathematics performance in geometry, word, and non-word problem solving. *Learning and Individual Differences*, *54*, 20–29. https://doi.org/10.1016/j.lindif.2017.01.005

Sun, S., Pan, W., & Wang, L. L. (2010). A comprehensive review of effect size reporting and interpreting practices in academic journals in education and psychology. *Journal of Educational Psychology*, *102*(4), 989–1004. https://doi.org/10.1037/a0019507

Swafford, J. O., Jones, G. A., & Thornton, C. A. (1997). Increased knowledge in geometry and instructional practice. *Journal for Research in Mathematics Education*, *28*(4), 467–483. https://doi.org/10.5951/jresematheduc.28.4.0467

Usiskin, Z. (1980). What should not be in the algebra and geometry curricula of average college-bound students? *The Mathematics Teacher*, *73*(6), 413–424. https://doi.org/10.5951/MT.73.6.0413

Wayne, A., J., & Youngs, P. (2003). Teacher characteristics and student achievement gains: A review. *Review of Educational Research*, *73*(1), 89–122.