

Optimizing delivery routing problem for logistics companies based on Integer Linear Programming method

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Abstract

Currently, issues related to freight at Vietnamese logistics companies are becoming more and more urgent because of typical problems in Vietnam such as traffic, infrastructure, and application of information technology. This problem has been studied by applying many different approaches such as Integer Programming (LP), Mixed Integer Programming (MIP), hybrid, meta search, ... In this paper, we applied the ILP model in order to deal with the VRP problem in a small size logistics company which is very popular in Vietnam. The experiments showed promising results with some optimal solutions with some small extra costs.

Keywords: Vehicle Routing Problem (VRP); Linear programming (LP), Integer Linear Programming (ILP).

1. Introduction

Despite a rather long development process, the transport industry has recently seen many strong developments due to appropriate government policies, upgraded infrastructure, international integration as well as strong competition from foreign shipping companies. Currently, there are many companies that have been established or entered the freight transport market, but most of them are small companies with limited potential and have not been optimized in terms of operational processes and transportation capacity. Most of these companies are still operating according to the traditional model, and the application of IT to increase production efficiency has not been given much attention. However, with the trend of digital transformation, along with the participation of many international transport companies, there has been a great competitive pressure on domestic transport companies. The research and application of information technology is concentrated on large logistics companies, but also in small units, so it has revealed many limitations such as not being computerized, few people using information technology with high-level information technology. this object class.

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The difficulty of the research on optimization with LP is to understand and know how to model the real problems we are facing into linear programming problems as well as the compatible methods to solve them, thereby making best use of linear programming and interpreting the results obtained. New research works on LP are shown in [1][2][3][5].

The most challenging problem these logistics companies face is how to serve the maximum number of customers while minimizing the cost of resources such as vehicles, time, and humans. This problem is scientifically formulated as a VRP (Vehicle Routing Problem) problem. There are many methods available to solve the VRP which are basically classified into three groups: heuristics, and meta-heuristics and exact methods [1][6][7][8].

Recently, several sophisticated algorithms have emerged aiming at solving VRP with more and more instances. Yet, much effort has been put in, only problems with relatively 200 customers can be solved optimally and the required computing time is high [9]. That's why heuristics are necessary in practice, where a variety of objectives and side constraints are handled. Constructive Heuristics is one of the most employed methods to provide a starting solution (an initially complete tour) by iteratively extending a connected partial tour. Some particular constructive-heuristic methods that can be mentioned are Greedy Heuristic, Quick-Borůvka Heuristic, and Savings Heuristic [10][11][12][13]. However, the best practice to obtain an optimal route is to carry out two-phase techniques. A constructive method is firstly applied then the result is refined with an improvement algorithm; construction produces a feasible solution then it is improved by improvement heuristics that employ intelligent search techniques to find a high-quality solution, neighborhood solutions (λ -OPT exchanges[14]) for example. In fact, most of constructive heuristics have now been left in disuse, due to the robustness of current metaheuristics.

Current meta-heuristics for the VRP are basically classified into local search and population-based heuristics. Local search methods explore the solution space by moving from a solution to another neighbor solution at each iteration. Some famous algorithms are simulated annealing (Kirkpatrick, Gelatt, and Vecchi[15]), deterministic annealing (Dueck and Scheuer [16]), tabu search (Glover [17]), and neighborhood search variants. Population-based heuristics evolve a population of solutions which may be combined together in the hope of generating better ones. These include ant colony optimization, genetic algorithms, scatter search,... Notably, a wide variety of powerful hybrids of these algorithms have emerged. The most well-known family of hybridizations is Population-based search and Local Search. (Ho, Ang, & Lim, 2001), aim to minimize the total number of vehicles, total distance traveled and total waiting time of vehicles creating a hybrid heuristics method that combines the strengths of both tabu search (TS) and genetic algorithm (GA).

The exact methods are considered to have the most optimal results, especially beneficial for small and medium-scaled problems. In practice, ILP is a very useful tool regarding the ability to represent a number of real-world situations that involve multiple variables and constraints. Therefore, within the scope of our article, we consider ILP the most appropriate approach that solves our problem radically.

The remaining part of this paper is organized as follows. Section 2 introduces the ILP method's idea and points out its advantages and real-life applications. Our problem and the proposed model are presented in Section 3. In Section 4, we detail our experiment regarding the programming environment, collected dataset, and obtained results. We also present a comparison between the traditional heuristic method and our proposed method, indicating that the proposed ILP approach gives much less time to solving and fewer resources. Finally, we show our conclusions and suggest future work in Section 5.

2. Integer Linear Programming Model

Principle of ILP

In integer linear programming, an ILP in standard form is expressed as ^[17]:

$$\begin{array}{ll} \text{Maximize or Minimize} & c^T x \\ \text{Subject to} & Ax \leq b, \\ & X \geq 0, \\ \text{And} & x \in Z^n \end{array}$$

where x vector is decided variable; c and b are vectors and A is a matrix, where all entries are integers

This model allows solving problems with very large dimensions, from tens to hundreds of thousands of variables in acceptable time. Solving a linear programming problem to find the result is an optimal solution according to the set goal and satisfying the established constraints. This result will support decision making for the day-to-day business of logistics enterprises

Linear programming has many applications such as:

- Select the lowest cost input combination for the output product
- Determine the optimal budget
- Deciding the optimal portfolio (or asset allocation)
- Allocating advertising budgets to media
- Plan to use machines
- Decide on the lowest cost shipping method
- Plan fleets of vehicles
- Optimal distribution of manpower
- Choose the most suitable factory location

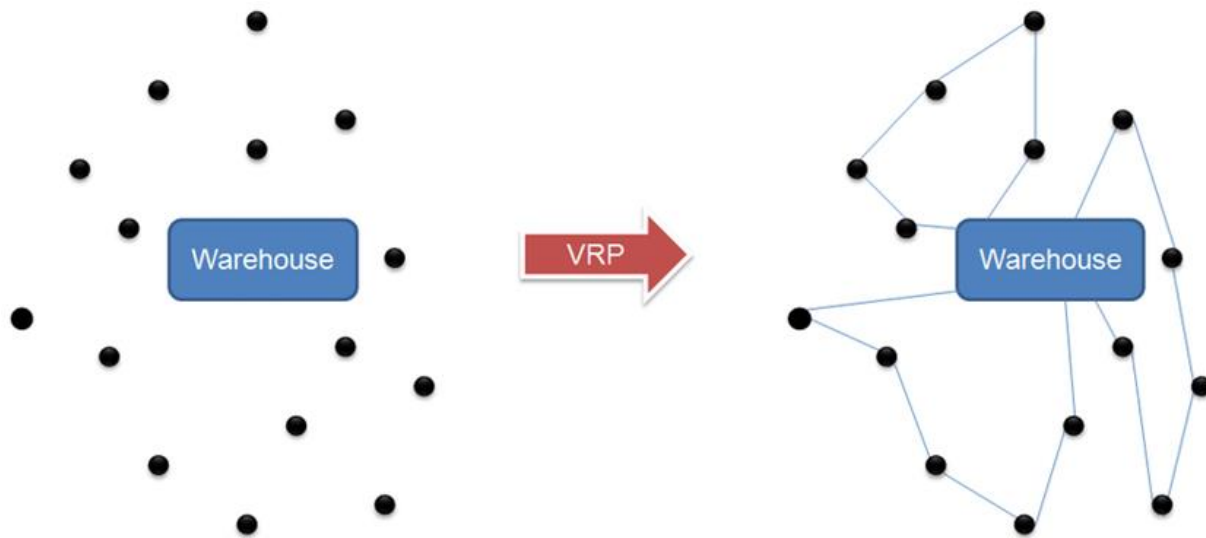


Figure 1. An example of fleet planning

3. The proposed Approach

The system framework is shown in Figure 2. Accordingly, the input to the system is customer information, warehouse and customer demand list; The output of the system is a report on the number of vehicles, the number of customers, information on each route, the list of customers on each route.

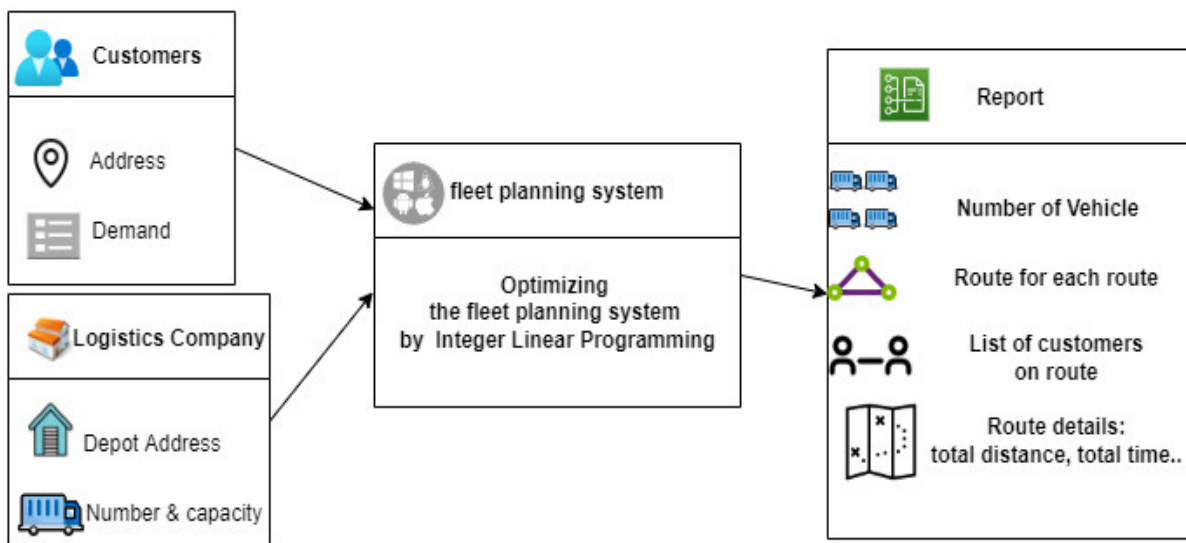


Figure 2. The proposed system framework

The vehicle routing problem (VRP) is a very important problem. It focuses on finding the route for vehicles while distributing commodities from warehouses of a supplier to its customers [18]. Mathematically, the VRP problem can be represented by a set N of customers and a directed graph G . Graph G has $|V|+2$ vertices, the customers are represented by vertices indexed from $1, 2, \dots, n$; vertex 0 represents the starting warehouse and vertex $n+1$ represents the ending warehouse. Each arc (i, j) has a value c_{ij} that describes the distance from vertex i to vertex j . Each vertex i ($0 < i < n+1$) has value d_i show that demand of customer i . A vehicle starts at the starting warehouse, it provides goods to several customers and ends at the ending

warehouse. Each customer is served by a vehicle. The objective of the problem is to find the routes (number of vehicles required) for the fleet of vehicles to satisfy the demand of all customers and minimize the total distance [19].

In order to describe the the mathematical model of the problem, we use the parameters as follows:

n: Total number of customers

N = {**1, 2, ..., n**}: Set of customers

V = {**0, 1, ..., n + 1**}: The set of vertices of the graph **G**

Q: capacity of a vehicle

c_{ij}: Distance from vertex *i* to vertex *j* $\forall \{i, j\} \in V$

d_i: demand at node customer *i* $0 < i < n+1$

Then we have the two variables in the model:

$x_{ij} \in \{0, 1\}$, $\forall \{i, j\} \in V$; $i \neq j$ represent a path from customer *i* to customer *j*

The objective function of the problem:

$$\min \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij}$$

Constraints of the problem:

$$\sum_{i \in V} x_{ij} = 1 \quad \forall j \in V, j > 0 \quad (1)$$

$$\sum_{j \in V} x_{ij} = 1 \quad \forall i \in V, i \leq n \quad (2)$$

The Miller-Tucker-Zemlin (MTZ) constraints [20]:

$$u_j = u_i + d_j \quad \forall x_{ij}=1 \quad \forall i, j \in V \quad (3)$$

$$d_j \leq u_j \leq Q \quad \forall j \in V \quad (4)$$

In there: **u_i** is an additional continuous variable which represents the load left in the vehicle after visiting customer *i*

(1), (2): make sure each customer gets exactly a vehicle go in and a vehicle go out.

(3), (4): make sure of the customer's cumulative demands at customer *j*.

The number of vehicles in the solution is the number of different vehicles that come from the warehouse

4. Experiment results

4.1. Environment

Below, the VRP problem-solving ILP model is implemented using the Python programming language. The program runs on a Core i7-E7440 4600U 2.7GHz 8G RAM personal computer. The program uses numpy, docplex, matplotlib libraries, especially the 64-bit Cplex Studio ver2210 library, which is used to increase memory during big data experiments.

4.2. Collected data

The data was collected from a dairy distribution business with a total of approximately 1350 customers. This business currently has 15 trucks for delivery, on average each vehicle will serve the needs of about 28

customers per day.

The data includes information about the coordinates of the warehouse, the coordinates of each customer, the list of needs of each customer. The input data after collection is described into 2 files with the storage structure as shown in Figure 3.

Nodos	Loc_x	Loc_y
0	18	27
1	178	78
2	84	144
...

(a) Geography location of customers

Nodos	Demand_i
1	26
2	68
3	14
...	...

(b) Customers' demand

Figure 3. Data structure of Input files

We have conducted a practical survey at the logistics company, analyzing the number of vehicles of the company, the number of customers taking care of each vehicle and the total number of customers being cared for by day of the week.

With the total number of customers of the company is 1,309 customers with monthly needs; With strong financial resources in February and December (the last month of the year and near the Lunar New Year), customer demand in these months is the highest, in addition, January, June, August, September: when The income of customers is reduced due to spending a lot in the summer, so the demand of customers is also reduced

Currently, the enterprise uses 15 trucks to delivery packs of milk to customers; On average, the business meets the needs of about 218 customers every day (chart 1 and table 1)

Cus. i	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
1	2	30	9	19	20	12	4	13	16	2	3	25
2	15	23	16	22	22	11	9	8	16	3	22	28
3	20	13	17	2	13	10	9	6	12	22	24	2
4	6	23	20	19	20	2	2	16	7	5	4	11
5	14	5	3	12	13	17	15	18	2	12	16	5
6	14	20	3	6	9	6	16	16	17	5	13	20
7	16	7	19	6	12	16	8	5	1	22	20	15
8	19	4	14	22	25	9	4	3	17	6	12	11
9	9	11	4	14	6	20	17	17	20	6	3	18
10
1290
1291	8	6	5	14	21	1	2	6	16	11	12	18
1292	18	19	17	16	2	20	9	15	5	13	25	30
1293	2	9	22	21	20	5	13	2	3	20	18	17
1294	19	11	7	9	20	4	17	20	1	1	16	21
1295	19	18	24	24	17	4	6	10	14	17	22	20
1296	19	13	17	22	19	19	8	7	5	4	11	1
1297	2	13	19	8	10	1	9	16	4	24	4	28
1298	11	14	15	6	7	15	13	6	4	6	9	6
1299	4	9	11	1	9	13	3	11	10	21	1	30
1300	20	29	23	23	16	7	8	3	7	18	12	16
Aver by month	12	15	14	14	15	10	9	10	9	11	13	17
Aver	13											

Table 1. Statistical table of customer demands by month

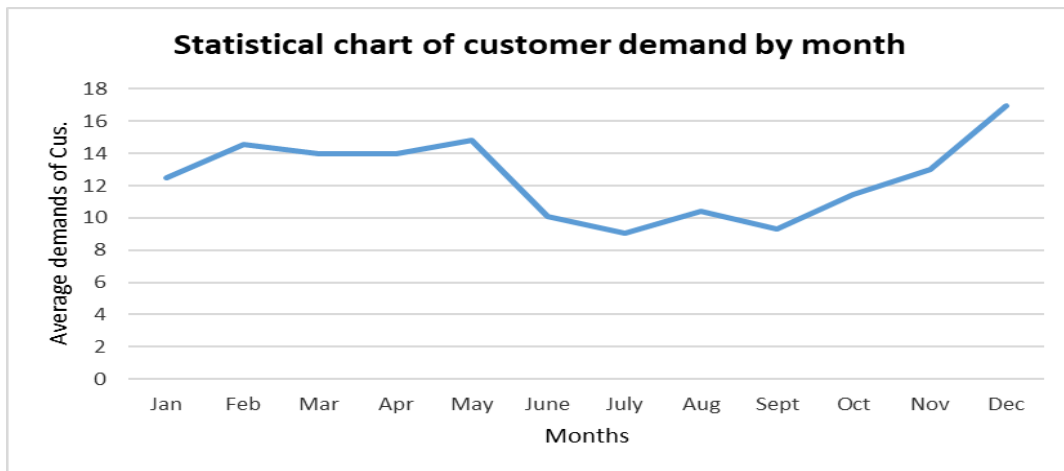


Chart 1: Statistical chart of customer demands by month

According to the survey results, with a total of 1300 customers, average monthly customer demand ranges from 9 to 16 boxes of goods, peak months are February, December (the high demand these months are the last months of the calendar year and the lunar calendar), the company uses 15 vehicles to serve the demands of about 218 customers, each car travels about 700 units of distance per day (*).

Several questions can be raised up here for optimizing this in order to save enterprise's cost consisting of

1. How many trucks can be used to solve the current need of the enterprise?
2. How many customers can be served with current numbers of trucks?
3. What is the average distance traveled by each car in a day?

4.3. Experiment

The VRP problem-solving ILP model is implemented using the Python programming language. Table 2 and Figure 4 shows the optimal routing with input data with respect to 218 and 250 customers. The company only needs to use 13 vehicles and 15 vehicles, respectively, to serve the needs of 218 and 250 customers, the total distance of the fleet is 7761 and 8727 units, respectively.

Num of customers	Total Distance	Num of vehicles	Time (s)	average distance per a vehicle
200	6880	12	5211	573
218	7761	13	5213	597
250	8727	15	5215	582
300	10422	18	5218	579

Table 2. Experimental results

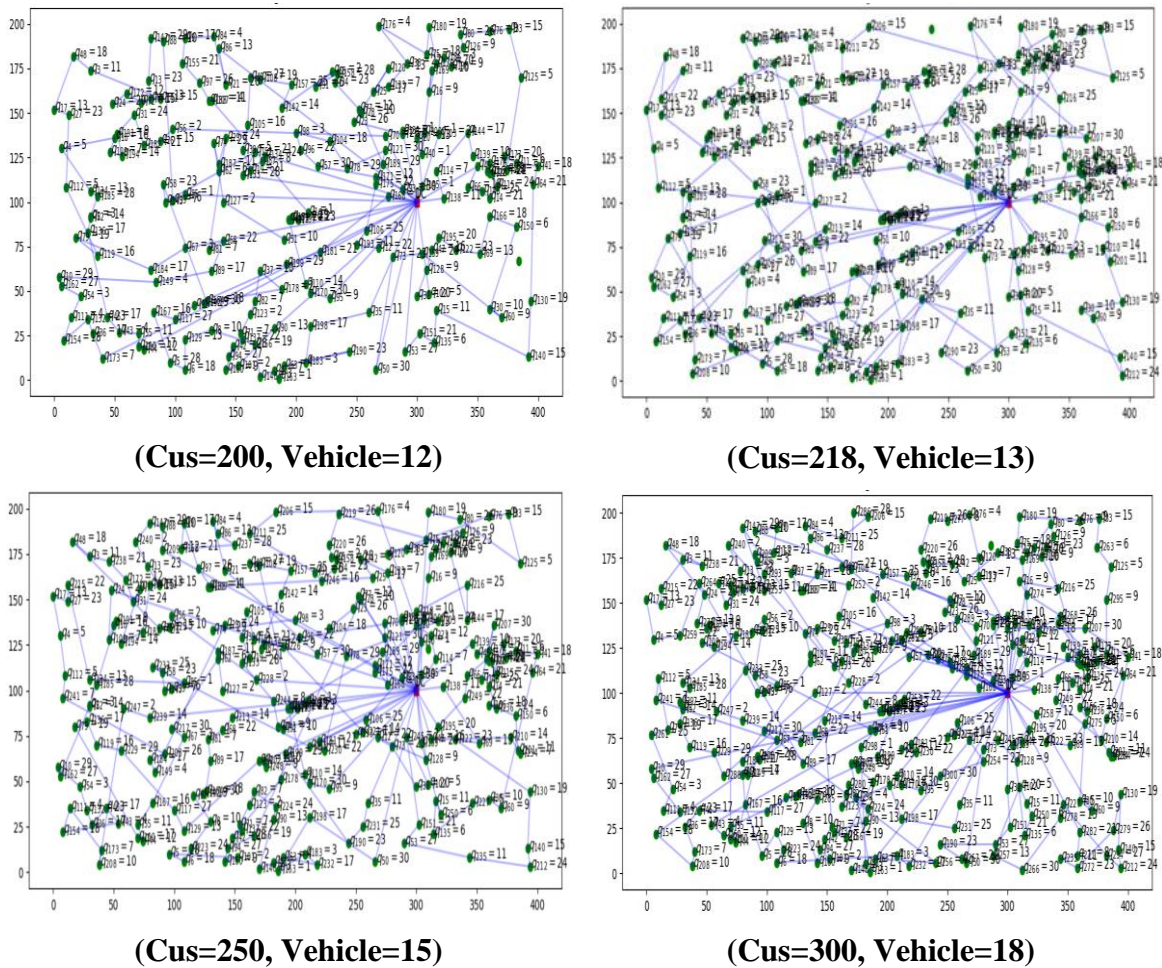


Figure 4. The optimal routing with different numbers of customers

These experimental results have shown that, with 218 customers, the system only needs 13 cars to meet all customers' needs, in addition, with 15 cars of the business can meet the needs of 250 customers, each vehicle only runs about 570 to 590 distance units. In addition, the research team also conducted an experiment with a larger number of customers (N=300) than previous research (N <=200). When compared with the survey results in section (*), the experimental results are better in terms of the number of vehicles, saving time and distance traveled by vehicles (table 3).

Number of Customers	Number of cars	
	Trad. Heuristic	ILP
200	15	12
218	16	13
250	18	15
300	20	18

Table 3. Table comparing experimental results with traditional data

Experimental results show that the system is more optimized when applying the ILP method in fleet scheduling

5. Conclusion

The integer linear programming model provides a very interesting approach to solving fleet planning problems for a small logistics company in Vietnam, which has a limited number of vehicles, number of customers and needs results. We presented the current status of the fleet at a Vietnamese logistics company. Then we tried to model a real-world problem by integer linear programming, and conducted experiments with collected data sets. Even the model could not completely solve the potential issues but some-how it can improve the heavy task of scheduling with a small extra cost. For the near future, we would like to try the current state of art method for VRP problem to deal with this problem.

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REFERENCES

- [1] Alexander Gutiérrez-Sánchez, Linda Bibiana Rocha-Medina, "VRP variants applicable to collecting donations and similar problems: A taxonomic review", *Computers & Industrial Engineering* 164 (2022)
- [2] Victor Pillac, Michel Gendreau, Christelle Guéret, Andrés L. Medaglia, "A Review of Dynamic Vehicle Routing Problems", *interuniversity Research center on enterprise network*, (2011).
- [3] Eray Demirel, Neslihan Demirel, Hadi Goken, "A mixed integer linear programming model to optimize reverse logistics activities of end-of-life vehicles in Turkey", *Journal of Cleaner Production* No. 112, pp 2101-2113, (2016).
- [4] László Kovács, Anita Agárdi and Tamás Bányai, "Fitness Landscape Analysis and Edge Weighting-Based Optimization of Vehicle Routing Problems", *Processes* 2020, 8, 1363. <https://doi.org/10.3390/pr8111363>
- [5] Bagaria, V.; Ding, J.; Tse, D.; Wu, Y.; Xu, J. "Hidden hamiltonian cycle recovery via linear programming". *Oper. Res.* 2020, 68, 53–70
- [6] Jiuxia Zhao; Minjia Mao; Xi Zhao; Jianhua Zou, "A Hybrid of Deep Reinforcement Learning and Local Search for the Vehicle Routing Problems", *IEEE Transactions on Intelligent Transportation Systems* (Volume: 22, Issue: 11, November 2021)
- [7] Zhao Wang; Yuusuke Nakano; Ken Nishimatsu, "The Vehicle Routing Problem with Time Windows and Time Costs", 2021 International Conference on Data Mining Workshops (ICDMW), Doi: 10.1109/ICDMW53433.2021.00042
- [8] Penglin Li; Chen Zhi; Wei Li, "An Algorithm to Solve Heterogeneous Vehicle Routing Problem With Second Trip", *IEEE Access* (Volume: 9), Doi: 10.1109/ACCESS.2021.3049510
- [9] Roberto Baldacci, Aristide Mingozzi, Eleni Hadjiconstantinou, "An Exact Algorithm for the Capacitated Vehicle Routing Problem Based on a Two-Commodity Network Flow Formulation", *Institute for Operations Research and the Management Sciences (INFORMS)*, 2004, Doi: 10.1287/opre.1040.0111
- [10] G. Clarke and J.W. Wright, Scheduling of vehicles from a central depot to a number of delivery points, *Operations Research*, 12 (1964), pp. 568–581
- [11] Li-Jiao Wu; Zhi-Hui Zhan; Sam Kwong; Jun Zhang, "Real Traffic Distance-Aware Logistics Scheduling", 2021 IEEE International Conference on Systems, Man, and Cybernetics (SMC), Doi: 10.1109/SMC52423.2021.9659167
- [12] Ashraf Elneima; Mohamed Salih, "Optimisation of Vehicle Routing Problem using Hyper-heuristics", 2020 International Conference on Computer, Control, Electrical, and Electronics Engineering (ICCEEE), Doi: 10.1109/ICCEEE49695.2021.9429682
- [13] Khadija Ait Mamoun; Lamia Hammadi; Antonio G.N Novaes; Abdessamad El Ballouti; Eduardo Souza De Cursi, "An optimisation solution of Capacitated Vehicle Routing Problem (CVRP)", 2022 11th International Symposium on Signal, Image, Video and Communications (ISIVC), Doi: 10.1109/ISIVC54825.2022.9800726
- [14] S. Lin, Computer solutions of the traveling salesman problem, *Bell System Technical Journal*, 44 (1965), pp. 2245–2269

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- [15] S. Kirkpatrick, C.D.Gelatt, and M.P.Vecchi, Optimization by simulatedannealing, *Science*, 220 (1983), pp. 671–680
 - [16] G. Dueck and T. Scheuer, Threshold accepting: A general purpose optimiza-tion algorithm appearing superior to simulated annealing, *Journal of ComputationalPhysics*, 90 (1990), pp. 161–175.
 - [17] Future paths for integer programming and links to artificial intelligence, *Com-puters & Operations Research*, 13 (1986), pp. 533–549
 - [18] Dantzing, G., Ramster, R., “The truck dispatching problem”, *Management Science* 6, page 80-91, (1995).
 - [19] G.Laporte, Y.Nobert, “A branch and bound algorithm for the capacitated vehicle routing problem”, *Operations Research Spektrum*, pp 77-85, (1983)
 - [20] Department of Industrial Engineering, Baskent University, Baglica Kampusu, “The multiple traveling sales man problem: an overview of formulations and solution procedures”, Eskisehir Yolu 20. km., 06530 Ankara, Turkey