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The Tradeoff of Bullwhip Effect with Inventory Costs in a Supply Chain

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공급사슬에서 채찍효과와 재고비용 사이의 상충

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In this paper, an alternative inventory policy that trades off the bullwhip effect at an upstream facility with cost minimization at a current facility, with the goal of reducing system wide total expected inventory costs, when external demand distributjon is autocorrelated, is considered. The alternative inventory policy has a form that is somewhere between one that completely neglects the autocorrleation and one that actively utilizes the autocorrelation. For this purpose, a mathematical model that allows us to evaluate system wide total expected inventory costs for a periodic review system is developed. This model enables us to identify an optimal inventory policy at a current facility that minimizes system wide total expected inventory costs by the best tradeoff of the bullwhip effect at an upstream facility with cost minimization at a current facility. From numerical experiments, it has been found that (i) when the autocorrelation is negative, the optimal policy is one that actively utilizes the autocorrelation, (ii) when the autocorrelation is small and positive, the optimal policy is one that neglects the autocorrelation, and (iii) when the autocorrelation is large and positive, the optimal policy is somewhere between one that actively utilizes the autocorrelation and one that neglect the autocorrelation.

Keywords : Inventory Policy, Bullwhip Effect, Inventory Costs, Correlated Demand

1. Introduction

In industry, actual demand distribution of many consumer goods have often been found to be autocorrelated [1, 2, 4]. However, industry practitioners are not aware of accurate inventory control models since most inventory control models are based on the assumption that the demand is independent from one period to the next. Therefore, they do not make use of accurate inventory control models [6].

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Urban [6] determined accurate reorder levels for a continuous review system and Zinn et al. [7] conducted simulation studies to evaluate the impact of autocorrelation on safety stocks. Recently, Kim [2] developed a mathematical model that, for a periodic review system, compared accurate inventory control models with traditional inventory control models under autocorrelated demand in a supply chain in which there are two participants, a retailer and a manufacturer. From which, Kim [2] found that knowledge of the actual demand process is always beneficial to the retailer in terms of reducing long run average total inventory costs per period, given that the retailer knows how to use it, and that the retailer's knowledge of the actual demand process may induce bullwhip effect, thus increasing the long run average total inventory costs per period at the manufacturer. Therefore, it is possible that sometimes a supply chain in which the retailer uses traditional inventory control models and the manufacturer uses accurate inventory control models has a lower system wide long run average total costs than a supply chain in which both the retailer and the manufacturer use accurate inventory control models.

The above observation naturally leads us to consider a policy for the retailer that trades off reduction of the bullwhip effect at the manufacturer with cost minimization at the retailer, which is the goal of this paper. Therefore, this paper can be regarded as a sequel to Kim [2].

The remainder of this paper is organized as follows: Section 2 reviews the supply chain model of Kim [2] and Section 3 develops a policy for the retailer that trades off reduction of the bullwhip effect at the manufacturer with cost minimization at the retailer. Then, in Section 4, through numerical studies, managerial insights gleaned are explained. Final remarks are addressed in Section 5.

2. Model Review

As mentioned in Section 1, the model developed by Kim [2], is used in this paper. For the clear description of this paper, a special case of the model, where the review period (lead time) of the retailer's is 1 (0) and the review period (lead time) of the manufacturer's is 1 (1), is reviewed in this section. For general cases, see Kim [2].

2.1 Model Structure

A supply chain consisting of a single retailer and a single manufacture is considered where external demand distribution is an autocorrelated one. If we let d_t be the demand faced by the retailer during period t, $t \in \{1, 2, 3, ...\}$, then we can write

$$d_t = \mu + \rho d_{t-1} + \epsilon_t \tag{1}$$

where $\mu > 0$, $-1 < \rho < 1$, and $\epsilon_t \sim N(0, \sigma^2)$.

The retailer, at the start of every review period t, observes the inventory level and the previous demands, and calculates the order-up-to level $y_{j,t}$, $j = s, n$, from which the retailer determines the order quantity $q_{j,t}$, $j = s, n$, to place to the manufacturer, and the shipment of which the retailer receives right away. The order the retailer receives, in turn, is used to meet the previous backlogged demands and the current period's demand. Here the subscript 's', short for 'smart', refers to the retailer who is aware that the demand distribution is an autocorrelated one and thus takes advantage of this knowledge to determine the order-up-to level and the subscript 'n', short for 'naïve', refers to the retailer who is not aware that the demand distribution is an autocorrelated one and thus resort to an inventory policy based on the assumption of i.i.d. demand to determine the order-up-to level.

The manufacturer, at the start of period t, receives and ships the order quantity $q_{j,t}$, $j = s, n$, to the retailer. If the manufacturer does not have enough stock on hand to fill the order quantity, the manufacturer can always find an alternative source to borrow from and that the borrowed items are returned to the source when the next replenishment arrives. The manufacturer places an order at the start of period t, right before the retailer orders. The order arrives at the start of period $t+1$. The supplier from which the manufacturer orders is assumed to have infinite capacity so that the manufacturer's order is always satisfied. As for the manufacturer, whether the retailer acts smart or naïve $(j = s, n)$ determines the demand stream faced by the manufacturer. In addition, it is assumed that the manufacturer acts smart and knows the demand stream faced by the retailer, which is made possible by information sharing with the retailer.

2.2 The Retailer

Smart Retailer From Equation (1), the actual effective lead time (=1) demand follows a normal distribution with mean

$$E[d_t \mid d_{t-1}] = \mu + \rho d_{t-1}$$

and variance

$$V(d_t \mid d_{t-1}) = \sigma^2.$$

Next, the order-up-to level for period t, $y_{s,t}$ will be calculated as

$$y_{s,t} = \mu + \rho d_{t-1} + z\sigma \tag{2}$$

where z is a constant, whose value will be described later.

Next, to calculate the expected holding cost per period, an approximation for the average inventory level, which is $y_{s,t}-E[d_t \mid d_{t-1}]+\frac{E[d_t \mid d_{t-1}]}{2}$, given by Silver and Peterson [5], is used. Then the average inventory level over the period t is approximated as follows:

$$inv_s = z\sigma + \frac{\mu+\rho d_{t-1}}{2}.$$

Therefore, the expected average inventory level per period can be represented as follows:

$$inv_s = z\sigma + \frac{1}{2}\left(\frac{\mu}{1-\rho}\right) \quad (3)$$

Next, notice that the average number of stockouts for period t for an order-up-to level of $y_{s,t}$ can be written as

$$\int_{y_{s,t}}^{\infty}(d_t \mid d_{t-1}-y_{s,t})dF(d_t \mid d_{t-1}),$$

where $F(d_t \mid d_{t-1})$ is the cumulative distribution function of demand for the period t, given d_{t-1}. Since $d_t \mid d_{t-1}$ follows a normal distribution, the above formula can be simplified to

$$\sigma\int_{z_s}^{\infty}(x-z_s)\phi(x)dx = \sigma[\phi(z_s)-z_s(1-\Phi(z_s))] \quad (4)$$

where x is a standard normal random variable, $z_s = \frac{y_{s,t}-(\mu+\rho d_{t-1})}{\sigma}(=z)$ is the standardized value of the order-up-to level, $\phi(\bullet)$ is the probability distribution for the standard normal random variable, and $\Phi(\bullet)$ is the cumulative distribution function for the standard normal random variable.

Therefore, if we denote the holding cost per unit per unit time by h and the penalty cost per unit associated with backlogged demand by p, the long run average inventory cost per period, g_s, can be written as

$$g_s = h\, inv_s + p\sigma[\phi(z_s)-z_s(1-\Phi(z_s))], \quad (5)$$

where $z=\Phi^{-1}\left(\frac{p-h}{p}\right)$.

Finally, the order quantity for period t, which becomes the manufacturer's demand for that period, can be written as

$$q_{s,t} = y_{s,t}-y_{s,t-1}+d_{t-1} = (1+\rho)d_{t-1}-\rho d_{t-2} \quad (6)$$

Naive Retailer A naïve retailer believes that the effective lead time demand is i.i.d. from a normal distribution with a mean $E[d_t]=\frac{\mu}{1-\rho}$ and a variance $V[d_t]=\frac{\sigma^2}{1-\rho^2}$. which becomes the perceived effective lead time demand,

Next, the order-up-to level for period t, $y_{n,t}$, which is constant from period to period, will be calculated as

$$y_{n,t} = \frac{\mu}{1-\rho}+z\frac{\sigma}{\sqrt{1-\rho^2}} \quad (7)$$

Notice that the expected average inventory level per period can be represented as follows:

$$inv_n = z\frac{\sigma}{\sqrt{1-\rho^2}}+\frac{1}{2}\left(\frac{\mu}{1-\rho}\right) \quad (8)$$

Next, since $z_n = \frac{y_{n,t}-(\mu+\rho d_{t-1})}{\sigma}$ is a random variable due to the dependence of d_{t-1}, the long run average number of stockouts per period can be written as

$$E[\sigma[\phi(z_n)-z_n(1-\Phi(z_n))]],$$

where the expectation is taken over z_n.

Since z_n follows a normal distribution with mean $\frac{z}{\sqrt{1-\rho^2}}$ and variance $\frac{\rho^2}{1-\rho^2}$, the above expression for the long run average number of stockouts per period can be simplified as

$$\sigma[h(E[z_n])(1+V(z_n))-E[z_n](1-H(E[z_n]))], \quad 9)$$

where $h(\bullet)$ and $H(\bullet)$ are the pdf and cdf for a normal distribution with mean 0 and variance $1+V(z_n)$. Fore derivation, see Kim and Ryan [3].

Therefore, the long run average inventory cost per period, g_s, can be written as

$$g_n = h\,inv_n + p\left[h(E[z_n])(1+V(z_n)) - E[z_n](1-H(E[z_n]))\right]. \tag{10}$$

Finally, the order quantity for period t, which becomes the manufacturer's demand for that period, can be written as

$$q_{n,t} = y_{n,t} - y_{n,t-1} + d_{t-1} = d_{t-1} \tag{11}$$

The long run average total inventory costs per period for a naive retailer, g_s, will be always larger than or equal to the long run average total inventory costs per period for a smart retailer, g_s. For details, see Kim [2].

2.3 The Manufacturer

With Smart Retailer First, notice that the actual effective lead time (=2) demand faced by the manufacturer with smart retailer can be written as

$$D_{s,t} = \sum_{u=0}^{1} q_{s,t+u} \mid d_{t-1}, d_{t-2}, \ldots = (1+\rho)\mu + (1+\rho+\rho^2)d_{t-1} - \rho d_{t-2} + (1+\rho)\epsilon_t$$

Therefore, the actual effective lead time demand faced by the manufacturer with the smart retailer follows a normal distribution with mean

$$E[D_{s,t}] = (1+\rho)\mu + (1+\rho+\rho^2)d_{t-1} - \rho d_{t-2},$$

and variance

$$V(D_{s,t}) = (1+\rho)^2\sigma^2.$$

Next, the order-up-to level at the start of period t, $Y_{s,t}$ will be calculated as

$$Y_{s,t} = (1+\rho)\mu + (1+\rho+\rho^2)d_{t-1} - \rho d_{t-2} + Z(1+\rho)\sigma \tag{12}$$

where Z is a constant, whose value will be described later.

Notice that the expected average inventory level per period can be represented as follows:

$$INV_s = Z(1+\rho)\sigma + \frac{1}{2}\left(\frac{\mu}{1-\rho}\right). \tag{13}$$

Next, notice that, since $Z_s = \dfrac{Y_{s,t} - \{(1+\rho)\mu + (1+\rho+\rho^2)d_{t-1} - \rho d_{t-2}\}}{(1+\rho)\sigma}$ $(=Z)$ is the standardized value of the order-up-to level, the average number of stockouts for period t for an order-up-to level of $Y_{s,t}$ can be written as

$$(1+\rho)\sigma\left[\phi(Z_s) - Z_s(1-\Phi(Z_s))\right] \tag{14}$$

Therefore, if we denote the holding cost per unit per unit time, by H, and the penalty cost per unit associated with backlogged demand by P, the long run average inventory cost per period, G_s, can be written as

$$G_s = HINV_s + P(1+\rho)\sigma\left[\phi(Z) - Z(1-\Phi(Z))\right], \tag{15}$$

where $Z = \Phi^{-1}\left(\dfrac{P-H}{P}\right)$.

With Naive Retailer First, notice that the actual effective lead time demand faced by the manufacturer with naive retailer can be shown as

$$D_{n,t} = \sum_{u=0}^{1} q_{n,t+u} \mid d_{t-1}, d_{t-2}, \ldots = \mu + (1+\rho)d_{t-1} + \epsilon_t$$

Therefore, the actual effective lead time demand faced by the manufacturer with the naive retailer follows a normal distribution with mean

$$E[D_{n,t}] = \mu + (1+\rho)d_{t-1},$$

and variance

$$V(D_{n,t}) = \sigma^2.$$

Next, the order-up-to level at the start of period t, $Y_{n,t}$ will be calculated as

$$Y_{n,t} = \mu + (1+\rho)d_{t-1} + Z\sigma. \tag{16}$$

Notice that the expected average inventory level per period can be represented as follows:

$$INV_n = Z\sigma + \frac{1}{2}\left(\frac{\mu}{1-\rho}\right). \tag{17}$$

Next, notice that, since $Z_n = \frac{Y_{n,t} - \{\mu + (1+\rho)d_{t-1}\}}{\sigma}$ $(=Z)$ is the standardized value of the order-up-to level, the average number of stockouts for period t for a fixed value of $Y_{n,t}$ can be written as

$$\sigma[\phi(Z_n) - Z_n(1-\Phi(Z_n))]. \tag{18}$$

Therefore, the long run average inventory cost per period, G_n, can be written as

$$G_n = HINV_n + P\sigma[\phi(Z) - Z(1-\Phi(Z))], \tag{19}$$

where $Z = \Phi^{-1}\left(\frac{P-H}{P}\right)$.

Notice that when, $\rho > 0$ and the retailer is navie, the manufacturer faces less severe bullwhip effect, i.e., the variance of the orders placed by the naive retailer, $V(D_{n,t})$, is less than that of the orders placed by the smart retailer, $V(D_{s,t})$. Therefore, when $\rho > 0$, the manufacturer is better off with a naive retailer than with a smart retailer. That is, when $\rho > 0$, $G_s > G_n$.

Now that we have briefly reviewed the model in Kim [2] for calculating the long run average inventory cost per period incurred at each participant in the supply chain, we are now ready to consider an alternative policy for the retailer that trades off reduction of the bullwhip effect at the manufacturer with cost minimization at the retailer.

3. An Alternative Policy at the Retailer

3.1 Relationship between Retailer and Manufacturer

Since we are interested in system wide expected total inventory costs, we define the following function:

$$TC_j = g_j + G_j,\ j = s, n, \tag{20}$$

where j, $j = s, n$, denotes a form of the retailer's order-up-to level.

As demonstrated in Section 2, when $\rho > 0$, we have $V(D_{s,t}) > V(D_{n,t})$, which causes $G_s > G_n$, which, in turn, sometimes leads us to observe $TC_s > TC_n$. In other words, the smart retailer causes more bullwhip effect at the manufacturer, which leads to higher costs at the manufacturer. Thus, sometimes, depending on the system parameter values, we have a system with naive retailer perform better than a system with smart retailer.

This observation leads us to the following question: "What is the form for an order-up-to level at the retailer that trades off reduction of the bullwhip effect at the manufacturer with cost minimization at the retailer, with the goal of reducing system wide expected total inventory costs?" In other words, we want to find an order-up-to policy for the retailer that provides lower system wide expected total inventory costs than TC_s or TC_n, i.e., one that provides lower system wide expected total inventory costs than a system with a smart or a system with a niave retailer.

As noted above, the policy we seek will trade off reduction of the bullwhip effect at the manufacturer with cost minimization at the retailer. Notice that order-up-to level for period t, which we denote by $y_{\alpha,t}$, of form

$$y_{\alpha,t} = \alpha y_{n,t} + (1-\alpha) y_{s,t}, \tag{21}$$

where $0 \le \alpha \le 1$, allows us to make this trade off.

From Equation (21), if α is close to 0 and ρ is positive, then $y_{\alpha,t} \approx y_{s,t}$. Therefore, minimizing costs at the retailer becomes a priority. If α is close to 1, then $y_{\alpha,t} \approx y_{n,t}$. Therefore, reducing the bullwhip effect becomes a priority. Notice that α is also a measure of the amount of neglect to the most recent demand since $y_{s,t}$ (when $\alpha = 0$) refers to an order-up-to policy that reacts to the most recent demand information and $y_{n,t}$ (when $\alpha = 1$) refers to an order-up-to policy that completely neglects the most recent demand information. The smaller value of α indicates more reaction to the most recent demand information.

We define TC_α, i.e., the system wide expected total inventory costs, as follows:

$$TC_\alpha = g_\alpha + G_\alpha.$$

Next, we have a following proposition:

Proposition 1 $min_{\ \alpha} TC_\alpha \le \ min\ \{TC_n, TC_s\}$, where $0 \le \alpha \le 1$.

Proof. Proposition 1 is self-explanatory.

3.2 Optimal Value of α

Given a set of order-up-to levels of this form, we next consider the optimal value of α, i.e., the value of α that minimizes TC_α, as a function of the autocorrelation.

Retailer In this case, the order-up-to level, at the start of period t, for the retailer, becomes:

$$\begin{aligned} y_{\alpha,t} &= \alpha y_{n,t} + (1-\alpha) y_{s,t} \\ &\quad + z\left(\alpha \frac{\sigma}{\sqrt{1-\rho^2}} + (1-\alpha)\sigma\right), \\ &= \alpha\left(\frac{\mu}{1-\rho}\right) + (1-\alpha)(\mu + \rho d_{t-1}) \end{aligned} \tag{22}$$

where $z = \Phi^{-1}\left(\frac{p-h}{p}\right)$. In addition, the average inventory level per period, inv_α, becomes

$$inv_\alpha = z\left(\alpha\left(\frac{\sigma}{\sqrt{1-\rho^2}}\right) + (1-\alpha)\sigma\right) + \frac{1}{2}\left(\frac{\mu}{1-\rho}\right). \tag{23}$$

Therefore, the expected total inventory costs per period for the retailer, g_α, can be written as:

$$\begin{aligned} g_\alpha &= h\, inv_\alpha \\ &\quad + p\sigma\left[h(E[z_\alpha])(1+V(z_\alpha)) - E[z_\alpha](1 - H(E[z_\alpha]))\right], \end{aligned} \tag{24}$$

where $E[z_\alpha] = z\left(\alpha \frac{1}{\sqrt{1-\rho^2}} + (1-\alpha)\right)$, $V(z_\alpha) = \frac{\alpha^2\rho^2}{1-\rho^2}$, and $h(\bullet)$ and $H(\bullet)$ are the pdf and cdf for a normal distribution with mean 0 and variance $1+V(z_\alpha)$. Finally, the order quantity placed by this retailer, at the start of period t, can be written as:

$$\begin{aligned} q_{\alpha,t} &= y_{\alpha,t} - y_{\alpha,t-1} + d_{t-1} \\ &= (1-\alpha)\rho(d_{t-1} - d_{t-2}) + d_{t-1}. \end{aligned} \tag{25}$$

Notice that $q_{\alpha,t}$ is a function of the decision variable, α, in addition to the previous demands at the retailer.

Manufacturer Next, the actual effective lead time demand faced by the manufacturer at the start of period t, becomes:

$$\begin{aligned} D_{\alpha,t} &= \sum_{u=0}^{1} q_{\alpha,t+u} \mid d_{t-1}, d_{t-2},\ldots \\ &= q_{\alpha,t} \mid d_{t-1}, d_{t-2} + q_{\alpha,t+1} \mid d_{t-1}, d_{t-2} \\ &= (1+(1-\alpha)\rho)\mu + (1+\rho+(1-\alpha)\rho^2)d_{t-1} \\ &\quad - (1-\alpha)\rho d_{t-2} + (1+(1-\alpha))\epsilon_t \end{aligned} \tag{26}$$

From Equation (26), we have

$$\begin{aligned} E[D_{\alpha,t}] &= (1+(1-\alpha)\rho)\mu + (1+\rho+(1-\alpha)\rho^2)d_{t-1} \\ &\quad - (1-\alpha)\rho d_{t-2} \end{aligned}$$

and

$$V(D_{\alpha,t}) = (1+(1-\alpha))^2\sigma^2.$$

Notice that, if $\alpha = 0$ ($\alpha = 1$), these expressions are identical to those presented in Section 2 for the smart (naive) retailer. Therefore, we have

$$\begin{aligned} Y_{\alpha,t} &= E[D_{\alpha,t}] + Z\sqrt{V(D_{\alpha,t})} \\ &= (1+(1-\alpha)\rho)\mu + (1+\rho+(1-\alpha)\rho^2)d_{t-1} \\ &\quad - (1-\alpha)\rho d_{t-1} + Z(1+(1-\alpha)\rho)\sigma, \end{aligned}$$

where $Z = \Phi^{-1}\left(\frac{P-H}{P}\right)$.

Now that we have written the order-up-to level for the manufacturer as a function of α, we have:

$$INV_\alpha = Z(1+(1-\alpha)\rho)\sigma + \frac{1}{2}\left(\frac{\mu}{1-\rho}\right). \tag{27}$$

Therefore, the expected total inventory costs per period for the manufacturer, given α, can be written as follows:

$$\begin{aligned} G_\alpha &= H\left(Z(1+(1-\alpha)\rho)\sigma + \frac{1}{2}\left(\frac{\mu}{1-\rho}\right)\right) \\ &\quad + P(1+(1-\alpha)\rho)\sigma\left[\phi(Z) - Z(1-\Phi(Z))\right], \end{aligned} \tag{28}$$

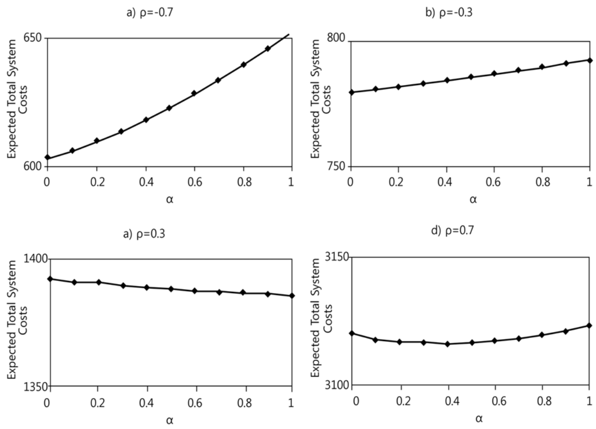
where $Z = \Phi^{-1}\left(\frac{P-H}{P}\right)$.

Thus we can write the system wide expected total inventory costs as $TC_\alpha = g_\alpha + G_\alpha$. Notice that this system wide expected total inventory costs is a function of α only, the decision variable, once the system parameters such as μ, ρ, σ, etc. are given.

4. Numerical Experiment

Now that we have developed expressions for the expected total inventory costs per period for the retailer and the manufacturer as a function of α, we are ready to determine the optimal value of α, the value of α that leads to the minimum system wide expected total inventory costs, either numerically or analytically.

We now present a numerical example to examine how the optimal value of α varies as a function of ρ. In this example, the demand process is specified by $\mu=600$, $\sigma=15$. The cost parameters for the retailer are $h=2$, $p=50$, and those for the manufacturer are $H=1$, $P=25$. In Figure 1 (a), (b), (c) and (d), the y-axis refers to the system wide expected total inventory costs per period whereas the x-axis refers to the values of α which vary between 0 and 1 when $\rho=-0.7,-0.3,0.3,0.7$, respectively.



<Figure 1> Costs as a Function of α

From <Figure 1>, some interesting results are found. They are:

1. When ρ is negative, the optimal value of α is always 0. This implies that $y_{s,t}$ is optimal, i.e., a system with a smart retailer is optimal.
2. When ρ is small and positive, the optimal value of α is close to 1. This implies that more weight should be placed on the form of order-up-to level that reduces bullwhip effect, which is $y_{n,t}$.
3. When ρ is large and positive, the optimal value of α is somewhere between 0 and 1. This implies that an appropriate balance is needed between reduction of bullwhip effect at the manufacturer and cost minimization at the retailer.

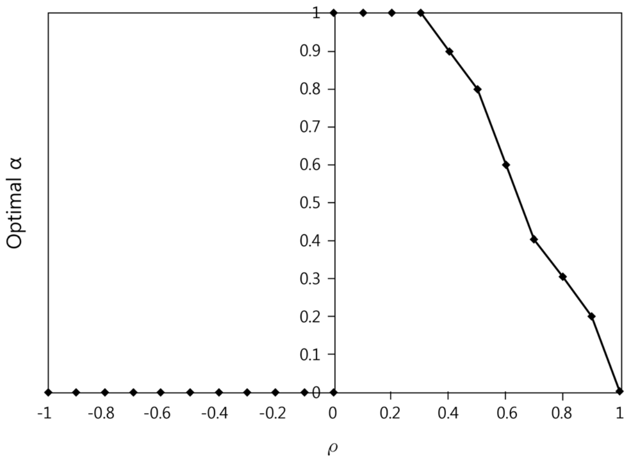
We can interpret these results as follows. Result 1 is due to the fact that, when ρ is negative, $y_{s,t}$ not only minimizes the retailer's costs but also causes less bullwhip effect. Notice that, when ρ is negative,

$$V(q_{s,t})=V((1+\rho)d_{t-1}-\rho d_{t-2})$$
$$=(1+2(1-\rho^2))\frac{\sigma^2}{1-\rho^2}<\frac{\sigma^2}{1-\rho^2}=V(d_t)=V(q_{n,t}).$$

Result 2 is due to the fact that, when ρ is small and positive, the previous demand, d_{t-1}, is not a good indicator for the current demand, d_t. Therefore, by adapting to the most recent demand information, the retailer causes more bullwhip effect without significantly improving its forecasts and its own costs. So, in this case, reducing the bullwhip effect is more critical than reducing costs at the retailer.

Finally, result 3 is due to the fact that, when ρ is large, the previous demand becomes a good predictor for the current demand. Therefore, by adapting the order-up-to level to the most recent demand information, the reduction in the expected total inventory cost at the retailer is great, but causes an increase in the expected total inventory cost at the manufacturer due to the bullwhip effect. In this case, reducing costs at the retailer is critical as well as reducing the bullwhip effect at the manufacturer.

<Figure 2> shows the optimal value of α as a function of ρ. In this figure, the system parameters are the same as in Figure 1. The x-axis refers to the value of ρ and the y-axis refers to the optimal value of α. Notice that when $\rho=0$, we have $y_{s,t}=y_{n,t}$. Therefore, when $\rho=0$, the optimal value of α becomes any number between 0 and 1.



<Figure 2> Optimal α as a Function of ρ

5. Final Remarks

The alternative order-up-to level developed in Section 3 can be used to demonstrate the fundamental tradeoff of the bullwhip effect at an upstream facility with cost minimization at a current facility, with the goal of reducing system wide total expected inventory costs. Notice that this policy allows us to trade off the costs/benefits of not reacting to the most recent demand information vs. reacting to the most recent demand information. That is, the optimal value of α measures the optimal degree of neglect. For instance, when the optimal value of α is small (large), more (less) reaction to the most recent demand information is better in terms of minimizing system wide expected total inventory costs.

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