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RESEARCH ARTICLE

Elementary School Students' Mathematical Metaphors for Line Segments, Straight Lines, and Rays

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Abstract

This research investigates the development of elementary students' concepts of line segments, straight lines, and rays, employing metaphor analysis as a research methodology. By analyzing metaphorical expressions, the research aims to explore how elementary students form these geometric concepts line segments, straight lines, and lays and evolve their understanding of them across different grades. Surveys were conducted with elementary school students in grades three to six, focusing on metaphorical expressions and corresponding their reasons associated with line segments, straight lines, and rays. The data were analyzed through coding and categorization to identify the types in students' metaphorical expressions. The analysis of metaphorical expressions identified five types: straightness, infinity or direction, connections of another geometric concepts, shape and symbols, and terminology.

Keywords: line segment, straight line, ray, mathematical metaphor, learning geometry, elementary school student

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I. INTRODUCTION

In the realm of elementary school mathematics, the predominant focus lies on shapes within the framework of Euclidean geometry. According to the axioms of Euclidean geometry, points and lines are conceptualized as entities devoid of magnitude, requiring a mental visualization process. “Geometric objects are things that exist in our mind. Many of them are idealizations if things that also exits in the physical words.”(Goldenberg & Clements, 2014, p. 30). The manner in which points and lines are introduced to elementary students, preceding exposure to the axiom system, involves a process of concretization or visualization. The concepts of lines, rays, and line segments, which are integral components of elementary mathematics, assume a pivotal role as fundamental shapes with profound implications for geometric reasoning and problem-solving across the entire geometric domain. Despite the recent surge in emphasis on geometric thinking, research in geometry learning remains notably sparse compared to the extensive literature on number and operation studies (Goldenberg & Clements, 2014). Despite its scarcity, there have been studies in Korean mathematics education related to points and lines, such as research on teachers’ perceptions of points and lines (Choi et al., 2011) and an analysis of points or lines in textbooks (Kim, 2018; Yi, 2021). Choi et al. (2011) analyzed elementary school teachers’ perceptions on line segments and angles, revealing their proclivity not to recognize them as distinct shapes. The analysis identified causal factors and proposed remedies, highlighting the tendency of teachers to restrict shapes to planar figures with area based on learning experiences and everyday surroundings. It also underscored the curriculum’s insufficient explicit reference to points, line segments, and angles, and the inclination to consider them merely as constituents of shapes.

This research aims to focus on the process of concept formation based on the assumption that elementary students’ geometric concepts undergo transformation and deepening. Specifically, it investigates how students develop concepts of line segments, straight lines, and rays and examines how these conceptualizations evolve as students progress through grades. The research employs metaphor analysis of elementary students to explore the formation of concepts related to line segments, straight lines, and rays. Metaphors link two experiential domains, combining the abstract with the concrete, the linguistic with the perceptual, and the conceptual with the factual (Kövecses, 2010), forming a network of relationships between concepts. While traditionally treated as a literary device in linguistic studies, recent conceptual metaphor theories leverage metaphors as clues to understanding human inference. Through the metaphorical expressions of elementary students, this study seeks to comprehend not isolated individual concepts but the interconnected web of concepts. Expressing a specific concept metaphorically reveals how it is cognitively associated with other concepts.

Introducing metaphor analysis as a research methodology in mathematics education, this study aims to analyze how elementary students form concepts of line segments, straight lines, and rays. Following Lakoff and Núñez (2000), the study plans to extract and discuss metaphors by mapping the target domain and the source domain. Exploring the metaphorical expressions of elementary students, the study seeks to

understand how they are forming concepts related to line segments, straight lines, and rays as part of elementary mathematics learning. By extracting approaches to "straightness" and "finiteness and infiniteness" in metaphorical expressions and categorizing them based on how students perceive spatial dimensions in 0-dimensional points, 1-dimensional lines, 2-dimensional planes, and 3-dimensional spatial solids, the study aims to discuss how elementary students are forming concepts of line segments, straight lines, and rays. Additionally, the study analyzes how these concepts evolve as students progress through different grades. The research questions are as follows:

1. Within the framework of metaphorical expressions used by elementary school students, how do they conceptualize line segments, straight lines, and lays?
2. How does the conceptualizations of these geometric elements evolve as they progress through different grade levels?

II. RELATED LITERATURE

Line Segment, Straight Line, and Ray in Geometry

School mathematics predominantly engages with the Euclidean geometric framework. Particularly in elementary mathematics, shapes are approached through the lens of Euclidean geometry. In the realm of Euclidean geometry, the terms "point" and "straight line" are referred to as "undefined terms" (Henderson & Taimina, 2010, p. 62), and Euclidean geometry consists of five axioms. The first axiom is "A point is that which has no part.", and second axiom is "A line is breadthless length" (Joyce, 1996). In Euclidean geometry, a point denotes only a position without any parts, embodying indivisibility, while a line signifies length without breadth.

In Euclidean geometry, points and lines are conceived as entities devoid of size; nevertheless, within the realm of school mathematics, they are visualized and brought to light through diverse representations. Furthermore, even when a concept is rigorously defined, what emerges is not merely the definition of concepts but the accompanying conceptual imagery. A conceptual image, an essentially non-verbal entity conjoined with the nomenclature of the concept, is intertwined with visual representation, mental imagery, impressions, and experiences, exhibiting a degree of variability contingent upon the given situation (Tall & Vinner, 1981; Vinner, 1991). The conceptualization of shapes involves both a process of idealization and simplification in mind, along with practical applications in the physical world. For example, even when thickness is imperceptible or when a line has irregularities, it is distilled and treated as a straight line. The process of learning lines in elementary education leads students to formulate a conceptual image of a "line with thickness." This conceptualization is more appropriately perceived as a "nonstandard conception" (Ely, 2010; Lee, 2021) within the framework of set theory, rather than an error arising from a lack of understanding.

Within the context of lines, the term "straight" assumes a pivotal significance. Geometrically, it conveys the notion of the "shortest distance" and this geometric essence finds practical application in activities such as extending a string or a rubber band to

delineate a “straight line.” A line, in this geometric framework, demonstrates “symmetry” across multiple dimensions, encompassing reflection at every point, rotation by half a turn at every point, and parallel translation along the line (Henderson & Taimina, 2010, p. 63). In the curricula of elementary schools and mathematics textbooks, a line segment is defined as connecting two points straightforwardly, a line is expounded by extending a line segment straight in both directions, and a ray is characterized as a straight line emanating from a single point (Ministry of Education, 2015, 2018, 2022). Notably, the instructional content relating to line segments and rays in elementary mathematics predominantly treats them as integral components of plane figures, conceptualizing for instance, a line segment as “sides of a polygon” and a ray as “sides of an angle.”

Line Segment, Straight Line, and Ray in CCSSM

By examining insights from the Common Core State Standards for Mathematics (CCSSM, 2010), this study explores the directions and focal points of geometry instruction. The CCSSM explicitly incorporates concepts such as line segments, lines, and rays as substantial learning objectives, notably highlighted in the 4th-grade standard: “Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.” For instance, preceding the 4th grade, concepts such as “understanding the properties of shapes,” “composition and decomposition of shapes,” and “sides and angles of polygons” underscore the significance of line segments, lines, and rays as fundamental geometric entities. Table 1 describes an overview of the geometry standards outlined by Common Core State Standards Initiative.

Mathematical Metaphor

“Mathematical objects emerge through negotiations between metaphor and rigor” (Sfard, 2000, p. 324). Metaphors, transcending the interplay between the concrete and the abstract, language and perception, and concepts and facts, entail a philosophical engagement with the very essence of the subject. When delving into mathematical metaphors, their narratives become intertwined with the philosophy of mathematical identity, exerting an influence on mathematical practices (Park, 2017). However, the dualistic nature of metaphors, oscillating between two realms, has sparked both acclaim as wellsprings of imagination and critique for obscuring judgment and inducing confusion in human cognition (Kim, 2005). Especially for proponents of mathematics as a precise and lucid discipline, the ambiguity introduced by metaphors may seem an impediment to be eradicated. Conversely, from the perspective of recent paradigms viewing mathematics as an uncertain and evolving domain, the inherent ambiguity of metaphors can be regarded as a resource fostering mathematical growth.

“Where mathematics comes from: How the embodied mind brings mathematics into being” (Lakoff and Núñez, 2000) challenges the conventional assumption that mathematics is detached from the human mind, advocating for mind-based mathematics. By scrutinizing mathematical metaphors ingrained in unconscious and everyday language, the book investigates the origins of abstract mathematical concepts. For example, the authors conceptualize “set theory” (target domain) as “containers” (source domain),

denoting the "container metaphor" (cf. "conduit metaphor", Reddy, 1993). According to this metaphor, the interior of the container signifies a set, objects inside the container are elements of the set, the container within the container is a subset, and the exterior of the container is the complement set, among other aspects. This mapping of the (typically "abstract") target domain to the (often "concrete") source domain facilitates a renewed perspective on concepts across different domains. Other prominent mathematical metaphors include interpreting functions as machines ("machine metaphor") and viewing equality as a balance scales ("balance scales metaphor").

Table 1. Mathematics standards in geometry domain

	Geometry
Kindergarten	Identify and describe shapes. Analyze, compare, create, and compose shapes.
Grade 1	Reason with shapes and their attributes.
Grade 2	Reason with shapes and their attributes..
Grade 3	Reason with shapes and their attributes..
Grade 4	Draw and identify lines and angles, and classify shapes by properties of their lines and angles. 4.G.A.1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. 4.G.A.2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. 4.G.A.3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
Grade 5	Graph points on the coordinate plane to solve real-world and mathematical problems. Classify two-dimensional figures into categories based on their properties.
Grade 6	Solve real-world and mathematical problems involving area, surface area, and volume.

Adopted from CCSSI (2010)

The analysis of mathematical metaphors entails extracting metaphors by mapping the target domain and source domain. For example, arithmetic (target domain) is mapped onto motion (source domain), conceptualizing arithmetic as "motion along a path," as elucidated in the "motion metaphor" (Chui, 1994, p. 37). According to the motion metaphor, arithmetic operations involve traversing along a number line, with the origin 0 serving as the starting point. Rightward movement from the origin represents positive integers N, leftward movement represents negative integers -N, and the distance from the origin denotes the absolute value.

Researching metaphor has manifested in diverse areas of educational research and mathematics education, encompassing studies on metaphor as an educational discourse (Cameron, 2003), metaphors pertinent to mathematical perspectives and mathematics education (Presmeg, 1997; Sfard, 1994, 1997, 1998, 2000), metaphors inherent in students' conceptualizations (Armstrong, 2008), students' metaphors regarding mathematical concepts (Chui, 1994; Ju & Kwon, 2003; Kim, 2023; Kim & Shin, 2007), and metaphors embedded in mathematics teachers' conceptualizations and practices (Cassel & Vincent, 2011; Chapman, 1997; Heaton, 2000; Kim, 2005; Krussel, Edward, & Springer, 2004). The analysis of teacher' perceptions and practices through metaphors (Bullough, 1991; Cassel & Vincent, 2011; Clandinin, 1986), and metaphor as a research methodology in education (Schmitt, 2005) and mathematics education (Danesi, 2007), has surfaced across varied domains.

III. METHODS

Participants

This inquiry employs a mathematical metaphor analysis to comprehensively examine how elementary school students perceive and internalize the geometric concept of "lines" (i.e. line segments, straight lines, and rays). The study aims to unveil the understanding of elementary school students as it evolves across different grade levels. The research centers on the concepts after the learning at school, specifically the introduction of on curricula of elementary mathematics on line segments, rays, and straight lines. Therefore, the participants selected for this research were students who had completed the learning of line segments, straight lines, and rays during their third grade.

Survey data were collected from elementary school students in grades three, four, five, and six having completed the learning contents in mathematics curriculum's relevant section (i. e. line, segments, straight lines, and rays). The surveys were administered across three classrooms per grade level, totaling 98 participants in the third grade, 62 in the fourth grade, 86 in the fifth grade, and 64 in the sixth grade, resulting in a comprehensive sample of 310 respondents.

Data Collection

This research conducted a metaphor analysis using data collected through surveys to examine the metaphors associated with each geometric concept (i.e., line segments, straight lines, and rays). The survey questions and analysis procedures for metaphor analysis were informed by previous studies (Cassel & Vincent, 2011; Chapman, 1997; Danesi, 2007; Kim, 2023; Kim & Shin, 1997; Reeder, Utley, & Cassel, 2009; Schmitt, 2005). To facilitate metaphor analysis, the survey questions were designed to elicit metaphorical expressions and provide a rationale for each response. Participants were instructed to metaphorically describe a single word that comes to mind when thinking about each term, such as line segments, straight lines, and rays, and were asked to describe

explanations or rationales behind the words they chose. For example, a survey question took the form of "To me, 'line segments' is ____. This is because ___". Survey data were collected over three weeks in September 2022.

Data Analysis

The procedural framework of this study unfolded in the following four phases (cf. Kim, 2023).

First was the phase of data collection and organization. Survey questions were formulated based on the learning content stipulated in elementary school mathematics curricula, specifically addressing line segments, straight lines, and rays. Students in grades 3, 4, 5, and 6 were prompted to provide metaphorical expressions and rationales. Only responses that contained both metaphorical expressions and rationales were included in the analysis. The responses were then paired, associating each student with their metaphorical expression and its corresponding rationale.

The second phase involved coding the collected data according to predetermined categories. Types of metaphorical expressions related to lines were categorized from pairs of collected metaphorical expressions and their corresponding reasons. One aspect focused on the recognition of characteristics inherent to line segments, straight lines, and rays as learning content, such as expressions of "straight" and representations of "infinity" or "direction." The other aspect involved representing lines in connection with spatial shapes of different dimensions. These 2-dimensional plane figures, and composition within 3-dimensional spatial shapes, respectively. Recognizing that spatial dimensions of line perception are not mutually exclusive, this study acknowledged that elementary students might express multiple perspectives simultaneously. In such cases, the focus was placed on the rationale provided by the students in their descriptions, and the most relevant aspect was coded accordingly. Additionally, even if the metaphorical expression was the same, coding was done based on the rationale provided. Responses unrelated to the survey questions or those where the metaphorical expression and its rationale were not related each other were excluded from coding. The coded data formed the foundational material for categorization and qualitative analysis of metaphorical expressions, facilitating frequency analysis for each geometric concept and distribution analysis across grade levels. Five types of metaphorical expressions related to the concept of lines were identified:

(i) Metaphorical expressions based on the notion of "straightness"
(ii) Metaphorical expressions based on the notion of "infinity" or "direction"
(iii) Metaphorical expressions based on the connection to another geometric concepts
(iv) Metaphorical expressions based on the shapes and symbols
(v) Metaphorical expressions based on the mathematical terminology in Korean.

The third phase is to nvolved categorizing and qualitatively analyzing metaphorical expressions. Common categories appearing across metaphorically expressed facets were identified. Conceptual Metaphor Theory (Kovecses, 2010; Lakoff & Johnson,

1980), which posits that understanding concepts in one (abstract) experiential domain is achieved through metaphors from another (concrete) experiential domain, was employed as the theoretical underpinning for metaphor analysis. Each concept was designated as "the target domain", and metaphorical expressions associated with "the source domain" were categorized. Analyzing the categories of metaphorical expressions shed light on how elementary students conceptualize geometric concepts related to lines, such as line segments, straight lines, and rays.

The fourth phase involved analyzing the distribution patterns of metaphorical expressions. A comparative analysis was conducted to examine how students in different grades form the concept of lines and how this conceptual formation evolves across grades. The study discussed the characteristics of forming concepts related to line segments, straight lines, and rays, analyzing how the metaphorical patterns associated with these concepts change across grade levels.

IV. RESULTS

Types of Metaphorical Expressions about Lines by Students

In analyzing metaphorical expressions related to lines, five types of metaphorical expressions were identified. The analysis of elementary school students' concepts of lines was conducted based on the types of metaphorical expressions.

(i) Metaphorical expressions based on the notion of "straightness" are fundamental notions contrasting curves, found in the concepts of lines, line segments, and rays. The current mathematics curriculum does not clearly distinguish between lines and curves. In this context, an analysis was conducted on how students connect concepts related to lines.

(ii) Metaphorical expressions based on the notions of "infinity" or "direction" involve discussing the differences between lines, line segments, and rays. Students recognize that a line extends infinitely in both directions, a ray extends infinitely in one direction, while a line segment has a finite length. The analysis focused on how students relate these lines.

(iii) Metaphorical expressions based on the dimension of space have traditionally treated lines as "sides" of two-dimensional plane figures or described line segments as connecting two "points" (i. e. zero-dimensional figures). An analysis was conducted on how students form concepts related to lines in this context.

(iv) Metaphorical expressions based on shape and symbols include cases where students metaphorically express straight lines or lays using "arrow" symbols (↔, →, ←) and relate line segment to their "endpoints" symbol(•).

(v) Metaphorical expressions based on terminology involve linguistic connotations associated with each term. In Korean, the term "Jik-sun" (straight line) is derived from the word "Jik," meaning "straight" or "direct." It is read as a line that is straight. Similarly, the term "Ban-jik-sun" (half-line or lay) is composed of "ban," signifying "semi" or "half."

The term "Sun-bun" (line segment) is interpreted as a line that has been divided, with "bun" conveying the notion of "division" or "separation."

Students' Metaphors for Line Segments

The present section delineates instances exemplifying five distinct types derived from students' metaphorical expressions related to line segments. Table 2 provides illustrative cases exhibited by students across different grades. Upon prompting elementary school students to conceptualize a line segment, a survey was conducted to document the words evoked and the underlying rationales. The categorization into types was accomplished through coding, where even identical words found distinct classifications based on their associated reasons.

Table 2. Students' metaphorical expressions for line segments

Type	Grade			
	Grade 3	Grade 4	Grade 5	Grade 6
Straightness	arm, road, age, tunnel	(physical) leg, line, rubber band, highway, friend, thread, jelly	confidence, standing in line	time, road
Finiteness	field, room, traffic light, stop, stick, future, drop, life, age, corridor	honesty, life, length, death, utility pole, destination, honesty, life, length, death, utility pole, destination, impossibility	person, holiday, rope, shield, pencil lead, limitation, control, goal	life, friend, age, destruction, pencil, grades, rules, fate
Connection with Geometric Concepts	road, robot, beginning and end, prison, shape, two points, line and line	jump rope, homework, regret, running, delivery, floor (basic shape), square, friends (connection of lines)	part of a straight line, friendship (connection of points), running (from start to finish), element (part of a shape), canvas (basic shape), surface (composite of line segments), material, clay (collecting line segments into shapes)	life (beginning and end), wall, bridge (connection of line segments), body (collection of line segments), shuttle run (two points), encounter (point and line), ladder, treasure map (starting point and destination)
Shape / Symbol	pencil cap, eye, needle, jump rope, tightrope	needle, pencil, parallel	character, drawing, ruler	book, fluorescent lamp
Line Segment Terminology	rule (distinguishing between line and ray)	/	line (letter line of a line segment)	/

Table 3 presents both the quantity and percentage of cases about line segment based on each type and corresponding grade level. As seen in Table 3, the predominant type, observed in 48.4% of the overall instances, involved associating a line segment with various geometric shapes. Subsequently, 37.1% of the total cases reflected upon the finiteness of a line segment, while the fundamental notion of "straightness" accounted for a mere 3.2% of the entire instances. Notably, in the cases of third-grade students, the type associated with "finiteness" was the most prevalent, comprising 33.3% of the third-grade instances. In the cases of fourth, fifth, and sixth-grade students, the most prevalent type was the association of the line segment with different geometric shapes, constituting 39.3%, 49.3%, and 48.4% for each respective grade.

Table 3. Frequency of metaphoric expressions for line segments (Percentage within each grade)

Type	Grade				Total
	Grade 3 70	Grade 4 56	Grade 5 67	Grade 6 62	 255
Straightness	8 (11.6%)	11 (19.6%)	3 (4.5%)	2 (3.2%)	24 (3.2%)
Finiteness	27 (37.7%)	17 (30.4%)	24 (35.8%)	23 (37.1%)	91 (37.1%)
Connection with Geometric Concepts	23 (33.3%)	22 (39.3%)	33 (49.3%)	30 (48.4%)	108 (48.4%)
Shape / Symbol	11 (16.0%)	6 (10.7%)	6 (9.0%)	7 (11.3%)	30 (11.3%)
Line Segment Terminology	1 (1.4%)	0 (0%)	1 (1.5%)	0 (0%)	2 (0%)

Students' Metaphors for Straight Lines

A straight line refers to an infinite straight path extending in both directions. In the elementary school mathematics curriculum, the concept of a line segment is initially introduced, followed by an explanation of extending in both directions from the endpoints of a line segment.

Table 4 provides metaphorical representations employed by elementary school students in relation to the straight line. Metaphorically, the notion of "straightness" about a straight line is depicted as progressing without deviating, symbolized by terms such as "success," "happiness," "challenge," or metaphorical expressions like "highway" and "ruler." Metaphoric expressions linking the notion "infinity" of a line most frequently featured the term "universe." In the context of relationships with other geometric shapes, references were predominantly made to two-dimensional "plane figures," and connections were drawn to the concept of "line segments" or gatherings of "points." Arrows indicating the infinite extension of a line prompted associations with concepts such as "glasses," "scale," "characters," "precipice," and "death." Additionally, an explanation was provided for the term "Jik" in Korean, where "Jik-Sun" translates to "straight-line," with "Jik" signifying straightness.

Table 4. Students' metaphorical expressions for straight lines

Type	Grade			
	Grade 3	Grade 4	Grade 5	Grade 6
Straightness	road, front, highway, challenge	road, life, ruler, family	life, time, role model, success, happiness	challenge, train, long jump, ground, highway, corridor
Infinity	cosmos, road, height, question, future, cosmos, family, study, sea, tunnel	future, homework, highway, Earth, cosmos, death, happiness, rubber band, dream	life, cosmos, world, study, love, number	past, imagination, hope, challenge, effort, love, cosmos, study, horizon, family, history, galaxy, horizon, Earth, Möbius strip
Connection with Geometric Concepts	running, (polygon's) side, road, collection of points	(shape's) side, helper (composition of shapes), link (connecting line segments)	exceeding limits, freedom (going beyond points)	/
Shape / Symbol	angry expression, scale, subtraction symbol, equal sign, glasses, balance scale	characters, death, floor	number 1, cliff	turning point, regret, underline
Straight Line Terminology	(straight line's) straight	/	/	/

Table 5 presents instances of metaphorical expressions related to a straight line, indicating the number of cases for each type and the corresponding percentages by grade. Metaphorical representations associated with the notion "infinity" of a straight line were notably prominent, constituting 57.7% of the total. Specifically, across grades, the percentage increased gradually from the 3rd grade (54.7%) to the 4th grade (62.9%), and 6th grade (70.3%), excluding 5th grade (45.9%). However, the notion "straightness" of a straight line accounted for 24.5% of the total instances, demonstrating a particularly elevated percentage of the 5th grade (40.5%) when compared to other grades. Expressing a straight line in relation to other geometric shapes decreased gradually as grades advanced, with cases dropping from 11.6% in the 3rd grade. Notably, in the 6th grade, there were no

instances where a straight line was metaphorically associated with other shapes.

Table 5. Frequency of metaphoric expressions for straight lines (Percentage within each grade)

Type	Grade				Total
	Grade 3 86	Grade 4 62	Grade 5 74	Grade 6 64	286
Straightness	15 (17.4%)	11 (17.7%)	30 (40.5%)	14 (21.9%)	70 (24.5%)
Infinity	47 (54.7%)	39 (62.9%)	34 (45.9%)	45 (70.3%)	165 (57.7%)
Connection with Geometric Concepts	10 (11.6%)	5 (80.6%)	5 (6.8%)	0 (0%)	20 (7.0%)
Shape / Symbol	13 (15.1%)	7 (11.3%)	5 (6.8%)	5 (7.8%)	30 (10.5%)
Straight Line Terminology	1 (1.2%)	0 (0%)	0 (0%)	0 (0%)	1 (0.3%)

Students' Metaphors for Lays

The concept of a ray was initially included in the early stages of South Korea's elementary school mathematics curriculum but was later removed as part of a curriculum reduction. It was reintroduced in the 2009 Revised Mathematics Curriculum. The reintroduction aimed to utilize the concept of a ray as a component of angle or as angle's sides when mathematically defining angles (Kim, 2018). Table 6 presents metaphorical expressions of rays by elementary school students.

Although a ray is straight not curved such as straight lines or line segments, the type of metaphorical expressions denoting the notion of "straightness" were limited when compared to other lines. Conversely, associations with continuous motion in one direction, such as "time," "running," "train," and "marathon," were invoked. In relation to other geometric shapes, the metaphorical expressions like "meeting" or "path" were evoked when thinking about moving in one direction from a point, and the idea of cutting a line was associated with "knife." However, in the elementary mathematics curriculum in South Korea, the interpretation of a ray as a side of an angle was emphasized, with only one example, the "nutrient" metaphor, highlighting its role in generating angles, being presented to underscore the understanding and connection of angles. A notable distinction from other lines was the significant occurrence of cases where participants directly associated the term "half" (the term "Ban" in Korean) with the term "half-line"(the term "Ban-Jiksun" in Korean). The Korean term "Ban" (half) was interpreted as signifying division, breakage, or insufficiency. Additionally, a case used the word containing the term "Ban" in Korean (e.g., "Ban-friends, meaning "classmates" in Korean) to express his or her understanding of term "Ban".

Table 6. Students' metaphorical expressions for lays

Type	Grade			
	Grade 3	Grade 4	Grade 5	Grade 6
Straightness	effort, corridor	life, apartment	Running	/
Infinity / Direction	spring, midline, scale, light, wall, development, running, roller coaster, hair, tree	greed, perseverance, laser, honesty, study, dream, love, imagination, clock, rubber band, playground, alley	road, time, sharp, choice, obstacle, challenge, speaking (irreversible), surrender, fainting	time, unfairness, prejudice, regret, attraction, loneliness, sky, train, life, slave, dream, road, natural number, marathon
Connection with Geometric Concepts	meeting (point and line), stone on the road (point and line), intersecting lines (meeting of straight lines)	stubbornness (passing through a point), adventure (only a starting point), divided road (dividing a straight line)	knife (cutting a straight line), midpoint (half of a straight line), nutrient (creating angles), capricious	failure (half of a straight line), starting point (starting from a point)
Shape / Symbol	toothbrush, snake, flower stalk, finger, eraser (shape cut like half a straight line)	ice cream, pizza (shape divided into pieces)	discomfort, stop, kimbap	scissors
Line Segment Terminology	half (from the name of a half-line)	broken item (straight line divided in half), Ban-friends (from the name of a Ban in Korean)	half (half of a half-line)	shortage (half of a half-line)

Table 7 presents the number of cases and the percentage by grade based on metaphorical expression types related to rays. The notion of "unidirectionality" of rays emerged as the highest, constituting 58.6% of the total cases. Across grades, the percentage increased gradually from the 3rd grade (36.4%) to the 4th grade (55.8%), 5th grade (70.0%), and 6th grade (75.8%). In comparison to other lines such as straight lines or line segments, the notion of "straightness" accounted for only 3.1% of the total cases, decreasing with higher grades. Notably, no cases expressing the "straightness" of rays were observed in the 6th grade. The type based on the shape of rays amounted to 9.6% of the total, with

percentages decreasing from the 3rd grade (18.2%) to the 4th grade (11.5%), 5th grade (4.7%), and 6th grade (1.6%). Moreover, in contrast to other types of lines, the term-based type for rays was substantial, constituting 11.5% of the total cases and appearing in all grades

Table 7. Frequency of metaphoric expressions for lays (Percentage within each grade)

Type	Grade				Total
	Grade 3 77	Grade 4 52	Grade 5 70	Grade 6 62	261
Straightness	4 (52.0%)	3 (5.8%)	1 (1.4%)	0 (0%)	8 (3.1%)
Infinity / Direction	28 (36.4%)	29 (55.8%)	49 (70.0%)	47 (75.8%)	153 (58.6%)
Connection with Geometric Concepts	23 (29.9%)	6 (11.5%)	11 (15.7%)	5 (8.0%)	45 (17.2%)
Shape / Symbol	14 (18.2%)	6 (11.5%)	4 (5.7%)	1 (1.6%)	25 (9.6%)
Lay Terminology	8 (10.4%)	8 (15.4%)	5 (7.1%)	9 (14.5%)	30 (11.5%)

V. DISCUSSION AND CONCLUSION

The foundation of elementary mathematics mostly lies in Euclidean geometry, where points and lines, considered undefined terms, are essential. The conceptualization of these elements involves both idealization and simplification, and emphasizes the role of conceptual imagery intertwined with visual representation. Despite recent attention to geometric thinking, research on geometry learning remains sparse compared to number and operation studies. This research investigates the development of elementary students' concepts of line segments, straight lines, and rays, employing metaphor analysis as a research methodology. By analyzing metaphorical expressions, the research aims to explore how elementary students form these geometric concepts related to lines and evolve their understanding of them across different grades.

Surveys were conducted with elementary school students in grades three to six, focusing on metaphoric expressions and corresponding their reasons associated with line segments, straight lines, and rays. The data were analyzed through coding and categorization, applying Conceptual Metaphor Theory to identify the types in students' metaphoric expressions. The analysis of metaphoric expressions identified five types: straightness, infinity or direction, connections of another geometric concepts, shape and symbols, and terminology.

The study extends to students' metaphors for line segments, with types including straightness, finiteness, connection with geometric concepts, shape/symbol, and terminology. The analysis reveals prevalent associations with geometric shapes (48.4% of

instances) and a focus on finiteness (37.1% of instances). Straightness accounts for only 3.2% of cases. Line segments take precedence in the realm of mathematical education within the Korean curriculum, particularly as the foremost concept in the study of lines. The introduction of line segments is predominantly associated with the exploration of the sides of triangles and quadrilaterals. In this context, the consideration of line segments extends beyond their inherent straightness or finiteness, emphasizing their role as fundamental components of polygons. Comparing straight and curved lines, the notion of the "straightness" of line segments prompts the recognition of their property as the "shortest distance." This concept finds resonance with the idea of "geodesics" in non-Euclidean geometry. It is imperative for elementary students to engage in diverse real-world experiences that allow them to perceive line segments not only as the "sides" of polygons or "edges" of polyhedra but also as one-dimensional entities. Such activities are essential for enriching the experience of line segments and enhancing geometric thinking, extending beyond the confines of two-dimensional and three-dimensional shapes.

For straight lines, students use metaphors such as road, challenge, infinity, and shapes like arrows. Notably, 57.7% of instances relate to the infinity of straight lines, with increasing percentages across grades. The study finds that students' understanding of straightness increases in the 5th grade (40.5%). Associations with geometric concepts decrease with grade advancement. The concept of "infinity" associated with straight lines exhibited a consistently high frequency across all grade levels, with percentages increasing as the grade level advanced. However, in the fifth grade, the notion of the "straightness" of a straight line surpasses the prevalence of the "infinity" concept. The percentage in the fourth grade is higher than that in the fifth grade, and when compared to the significant increase in the prevalence of the "infinity" concept in the sixth grade, one may speculate about the implications related to the fifth-grade mathematics curriculum. In the fourth grade, the geometric content involves discussions of parallel lines and their relationships, which could contribute to the prevalence of the concept of the "infinity" of straight lines. In the fifth grade, geometric learning shifts towards the examination of polyhedra, focusing on their faces and edges. This observation suggests a correlation between the geometric content and the varying emphasis on different aspects of straight lines at different grade levels. Geometric concepts are dynamic and, far from being fixed, are revisited and reinforced in conjunction with other learning materials.

The research also explores students' metaphors for rays, emphasizing the notion of "unidirectionality" (58.6% of instances). Straightness is less emphasized, constituting only 3.1% of cases, with a decline in higher grades. Terminology-based expressions are substantial, constituting 11.5% of instances. With the reintroduction of the semi-line in the revised 2009 curriculum, attention has been drawn to its role in defining angles. While only one metaphorical representation has been suggested in connection to this, elementary students predominantly associate the semi-line with "unidirectionality." In contrast to line segments, which are most frequently conceptualized as the "sides" of polygons, semi-lines are not commonly perceived as components of planar shapes. Notably, the everyday meaning of the term "half-line" in the Korean language strongly influences its interpretation. It becomes apparent that guidance in linking geometric terminology with its everyday meaning is essential for a comprehensive and nuanced understanding, especially

considering the prevalent association of "half" ("Ban" meaning "half" or "semi" in Korean) in the term "half line"("Ban-Jiksun"). Facilitating such connections can contribute to a broader and more meaningful interpretation of geometric concepts in alignment with their everyday implications.

This research provides insights into students' metaphoric thinking about line segments, straight lines, and rays, shedding light on their conceptualizations and highlighting shifts in understanding across different grades. Geometric concepts rely on the formation of abstract ideas through concrete images, and metaphors play a crucial role in this process. To develop a profound and comprehensive understanding of these concepts, it is imperative to engage students in experiencing concreteness in the real world, facilitating abstraction, and exploring a variety of metaphors.

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