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RESEARCH ARTICLE

How High School Mathematics Teachers Use New Textbook : A Case Study from China

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Abstract

In this paper, we propose a theoretical framework for Chinese high school mathematics teachers use new textbooks based on the work of Remillard (1999) and Chau (2014). Based on this framework, a multiple case approach was used to investigate how two high school mathematics teachers from Shanghai use new textbooks. The results suggest that in the curriculum mapping arena, both the novice teacher and the expert teacher often planned to appropriate the unit content, and sometimes planned to add supplemental content. When organizing the unit content, novice teacher always planned to follow the new textbook in sequence, while expert teacher often would follow the new textbook in sequence, but sometimes planned to rearrange the unit content. In the design arena, both the novice teacher and the expert teacher tended to appropriate the introduced tasks and definitions. The novice teacher often planned to appropriate the example problems and exercise problems, while the expert teacher often intended to flexibly use the example problems and exercise problems. In the construction arena, the novice teacher seldom adjusted the planned tasks; in contrast, the expert teacher adjusted the planned tasks more frequently. In the reflection arena, the novice teacher often thought she should improve the mathematics tasks, while the expert teacher almost always thought he needed to improve the mathematics tasks. The framework shown in this paper provides a tool to investigate how mathematics teachers use textbooks.

Keywords: Chinese mathematics teachers, new textbook use, theoretical framework, case study

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I. INTRODUCTION

Mathematics textbooks play an important role in the teaching and learning of mathematics (Valverde et al., 2002). As noted by researchers affiliated with the Trends in International Mathematics and Science Study (TIMSS), "Perhaps only students and teachers themselves are a more ubiquitous element of schooling than textbooks.… They represent school disciplines to students. They translate a country's curriculum policies into such representations" (Valverde et al., 2002, p. 1). Mathematics textbooks influence not only students' learning but also teachers' selection of content and teaching strategies (Cai, Ni, & Lester, 2011; Fan & Kaeley, 2000; Glasnović Gracin & Jukić Matić, 2021; Nicol & Crespo, 2006; Reys, Reys, & Chávez, 2004; Son & Kim, 2015; Stein, Remillard, & Smith, 2007; Tarr et al., 2008). In China, textbooks are the main resource used by mathematics teachers in their classroom teaching. In particular, textbooks are the most important source for teachers when making decisions about what and how to teach (Fan, Chen, Zhu, Qiu, & Hu, 2004).

Despite the importance of textbooks in everyday mathematics practice, it is also clear that no textbooks can predetermine the quality of instruction associated with its use. Teachers play a crucial role in the classroom use of textbooks (Stylianides, 2016). As Pepin and Haggarty (2001) noted ''Teachers decide … which sections of the textbook to use; the sequencing of topics in the textbook; the ways in which pupils engage with the text; the level and type of teacher intervention between pupil and text'' (p. 165). In other word, how mathematics teachers use textbooks is very important.

Despite the importance of textbook use by mathematics teachers, in a review of international textbook research, Fan, Zhu, and Miao (2013) found that many studies focused on textbook analysis and comparison but fewer focused on textbook use. Overall, among the limited number of studies on textbook use in teaching and learning, most were conducted in Western educational contexts (Fan, Chen, Zhu, Qiu, & Hu, 2004). As Zhu and Fan (2002) noted, few such studies have been conducted in Asian countries, particularly in the Chinese school context. Chen and Ding (2018) also pointed out that how Chinese teachers actually use textbooks in mathematics classrooms has seldom been studied. Therefore, it is essential to pay attention to Chinese mathematics teachers' use of textbooks.

In China, The High School Mathematical Curriculum Standard (2017 Edition) was issued in January 2018 (Ministry of Education of China, 2018). New mathematics textbooks have been used since the autumn of 2020 in Shanghai, old textbooks are being phased out from classroom instruction. Mathematics textbooks have played a significant role in curriculum reform and are often key to the implementation of the new curriculum (Howson, 2013; Obara & Sloan, 2009). Because these mathematics textbooks are fairly new, few researchers have explored how Chinese high school mathematics teachers use them. This study aims to investigate how Chinese high school mathematics teachers use new textbooks.

II. LITERATURE REVIEW

Mathematics Teachers' Use of Textbooks

How mathematics teachers use curriculum materials is an area of research that has grown significantly in the past two decades (Lloyd, Remillard, & Herbel-Eisenmann, 2009). Many researchers have focused on the interaction between mathematics teachers and textbooks (e.g., Lloyd, 2009; Remillard, 1999, 2000; Sherin & Drake, 2009). For example, Remillard (1999, 2000) studied how two-fourth grade teachers interacted with the same textbook in different ways to construct contrasting opportunities for student learning. The analysis highlighted the ways the teachers read the textbook and explored the factors that contributed to their different approaches to reading.

Some researchers have focused on implementation fidelity (e.g., Freeman & Porter, 1989; McNaught et al., 2010; Tarr, Reys, Reys, Chavez, Shih, & Osterlind, 2008). For example, using the concept of "implementation fidelity", McNaught et al. (2010) studied the use of two types of mathematics textbooks from the perspectives of both teachers and researchers in the US context. They found that teachers tended to assign fewer problems to students than the textbook authors recommended and covered less than 70% of textbook content on average.

A few researchers have focused on the use of specific topics in mathematics textbooks (e.g., Bieda, 2010; Eisenmann & Even, 2011; Sears & Chávez, 2014). For example, Sears and Chávez (2014) examined how mathematics teachers used proof tasks in two geometry textbooks, and found that even though geometry textbooks may have proof tasks with a higher-level of cognitive demand, there is no guarantee that such tasks would be assigned, or that the levels of cognitive demand in tasks would be maintained between the written and the enacted curriculum.

Still others have focused on the variations in the use of textbook. For instance, Thompson and Senk (2014) examined variations in teachers' use of a geometry textbook during instruction on congruence. They found that teachers taught between 60% and 100% of the lessons on congruence but often skipped content focused on unique applications. Overall, most of the studies were carried out on a small scale by individual researchers, and were exploratory in nature (Fan, Zhu & Miao, 2013).

Chinese Mathematics Teachers' Use of Textbooks

Several studies have been conducted in China to examine how mathematics teachers use textbooks (Fan et al., 2004). In their pioneering work, Fan et al. (2004) investigated how Chinese mathematics teachers use textbooks within and beyond classrooms. This study found that teachers treated textbooks as a main source but not the only source for their teaching. Moreover, there was no significant difference in teachers' use of textbooks across different genders, experiences, schools and regions.

Using a model of teachers' textbook use consisting of five levels (misuse, mechanical use, routine use, refined use and creative use) in four aspects (comprehending and studying textbooks, integrating textbooks, applying textbooks and making judgements on textbooks), Kong and Shi (2009) measured the level of five primary teachers' textbook use in China before and after an intervention in which the researchers provided teachers with professional guidance on how to prepare, implement and reflect on their use of textbooks. From the results, they claimed that the level of textbook use model was valid and accurate, and could help improve teachers' textbook-use skills and professional development.

Chen and Ding (2018) investigated how an expert mathematics teacher used textbooks in China and found that both the textbook and the enacted teaching included only one worked example; however, the teacher engaged students in unpacking the example in great depth. Both the textbook and the enacted teaching showed "concreteness fading" in students' use of representations. However, the Chinese teacher incorporated students' self-generated representations and facilitated students' active modeling of quantitative relationship.

To date, the use of textbooks by Chinese mathematics teachers has been mainly studied by quantitative approaches-mainly questionnaire surveys and interviews. As Chen and Ding (2018) noted prior studies on the use of textbooks by Chinese mathematics teachers have been based on teacher surveys and interviews rather than actual classroom observations. In addition, the recent editions of mathematics textbooks are relatively new, and few researchers have explored how high school mathematics teachers use them. The study therefore aims to investigate how high school mathematics teachers use new textbooks in China based on a qualitative approach-a case study.

Theoretical Framework

In her pioneering work, Remillard (1999) studied how two elementary teachers' use textbooks and constructed a model of teacher's role in curriculum development based on the findings. The model includes three arenas: the design arena, the construction arena, and the curriculum mapping arena. The design arena involves selecting and designing tasks for students. The construction arena involves enacting these tasks in the classroom and responding to students' encounters with them. A central activity of the construction arena is task adaptation, including the unrehearsed adapting and adjusting of tasks in order to facilitate students' work with them. Regardless of how the teachers used the textbook to select tasks, enacting them required teachers to make on-the-spot decisions about how to adapt them in response to classroom events. The curriculum mapping arena involves making choices that determine the organization and content of the curriculum over the year.

Remillard (2005) offered a framework of the teacher-curriculum relationship. The framework included four constructs: the teacher, the curriculum, the participatory relationship between them, and the resulting planned and enacted curriculum. The framework accentuates as the participatory relationship, the interaction between the teacher and curriculum resource. The left-hand circle of the framework represented the resources, stances, and perspective that the teacher brings to the participatory relationship with curriculum resources. The right-hand circle represented the particular curriculum resource or text being used. The participatory relationship between the teacher and curriculum consisted of interactions in which both the teacher and the curriculum are significant and active participants. The planned curriculum is the outgrowth of the participatory interaction relationship between the teacher and the curriculum. The enacted curriculum captures these plans as they unfold in a particular context with particular students. In other words, the enacted curriculum is co-constructed by teachers and students in a particular context.

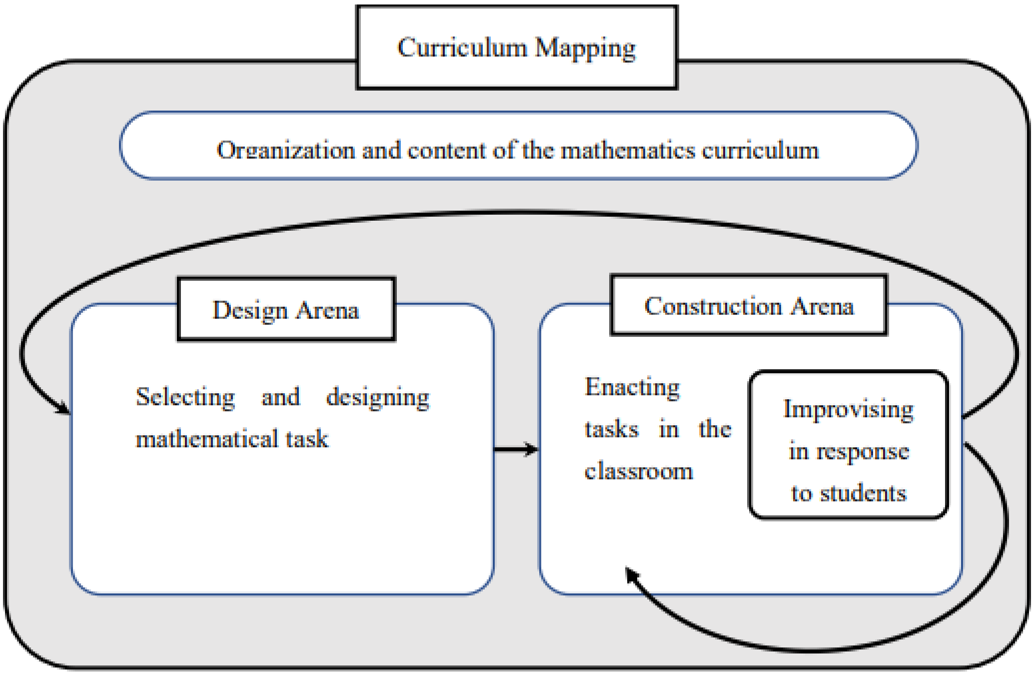

Figure 1. The model of the teacher's role in curriculum development (Remillard, 1999, p. 322)

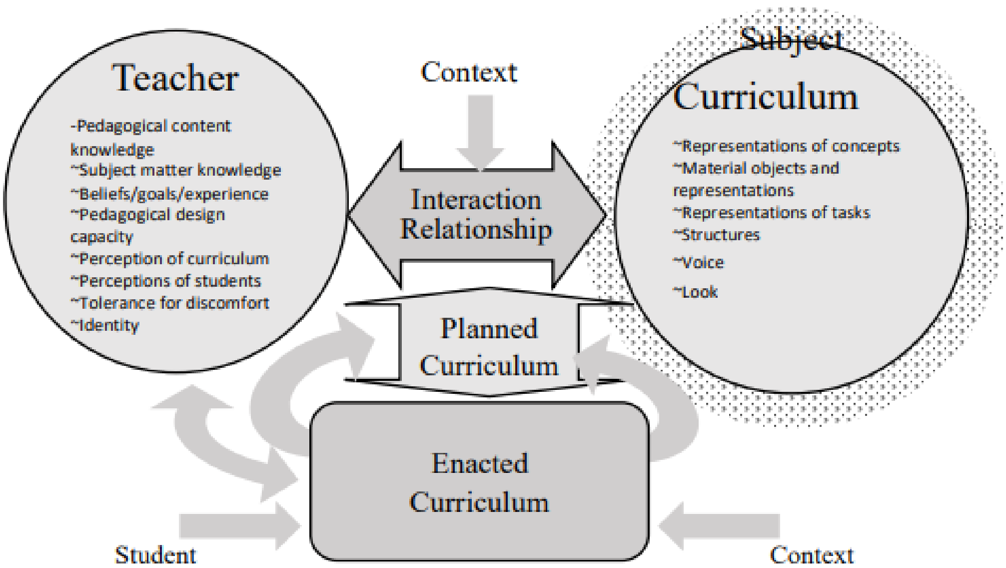

Figure 2. Framework of components of the teacher–curriculum relationship (Remillard, 2005, p. 235)

Chau (2014) established a framework of teacher' use of textbooks based on Remillard's (1999, 2005) work. Chau (2014) combined both Remillard's (1999) model of teacher's role in curriculum development and Remillard's (2005) framework of teacher-curriculum relationship.

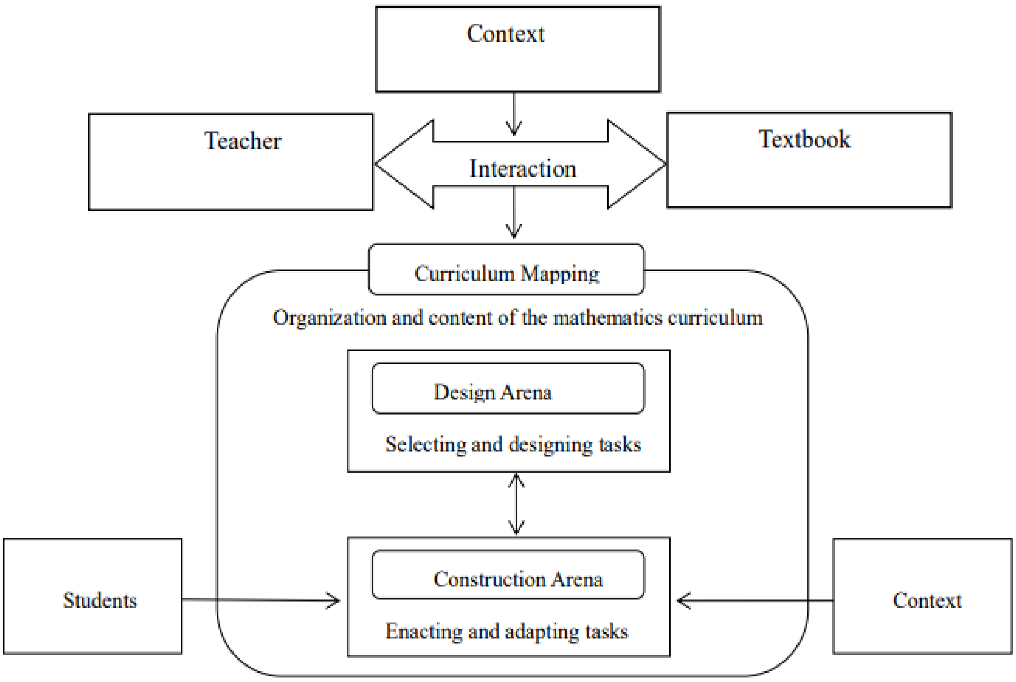

Figure 3. Framework of teacher' use of textbooks (Chau, 2014, p. 69)

The theoretical framework we propose seeks to adapt Chau's (2014) framework of teacher' use of textbooks to the mainland Chinese context. We add the reflection arena to Chau's (2014) framework. The design arena involves selecting and designing tasks for students before instruction. The construction arena involves enacting and adapting these tasks during instruction. Chau's (2014) framework does not include the reflection arena. However, textbooks are most commonly involved in the entire process of teaching in China, that is, the activities before, during, and after class, such as lesson planning, in-class questioning, and after-class evaluation and reflection, which suggests the need for a framework systematically covering all activities throughout the entire process (Fan, Cheng, Xie, Luo, Wang, & Sun, 2021). Furthermore, Sherin and Drake's curriculum strategy framework included teacher' reflection on what transpired and adapted after instruction. Thus, we add the reflection arena to Chau's (2014) framework. The reflection arena involves reflecting and improving tasks after instruction.

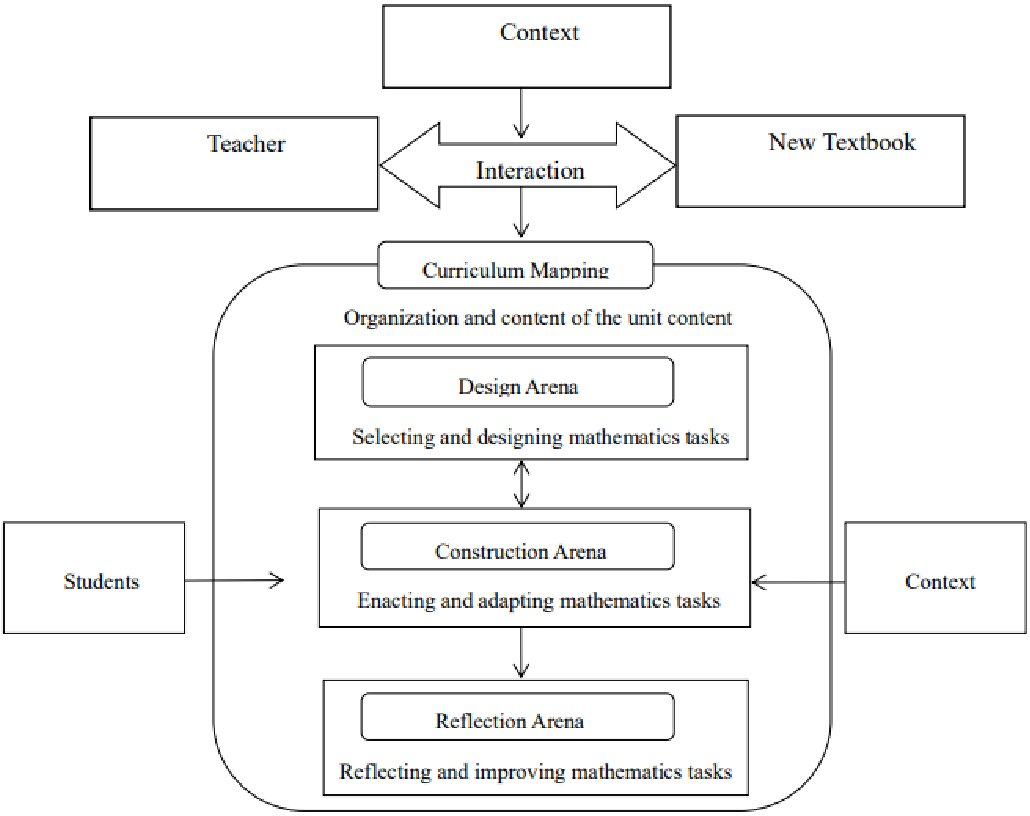

Figure 4. Framework for Chinese high school mathematics teachers use new textbook

Research Questions

Based on the framework, the following research questions were formulated for the present study:

(1) How do mathematics teachers select, design and organize unit content?
(2) How do mathematics teachers select and design mathematics tasks?
(3) How do mathematics teachers enact and adjust mathematics tasks?
(4) How do mathematics teachers reflect and improve mathematics tasks?

III. METHOD

This study employed a case study (Stake, 2013). By focusing on Chapter 3 (Logarithms); Chapter 4 (Power Functions, Exponential Functions and Logarithmic Functions); and Chapter 5 (Concept and Properties of Function) in the new textbook, we aim to examine how Chinese high school mathematics teachers use a new textbook.

Participants

The study investigated two teachers: Zhao, and Qian (pseudonyms). Ms. Zhao and Mr. Qian taught at the same district key high school in Shanghai. Ms. Zhao and Mr. Qian were the members of the same lesson preparation group (LPG) for the teachers of the same subject to the same grade, an organizational group which has existed in China's school system for a long time and serves the purpose of enabling joint lesson preparation (Li, Chen & Kulm, 2009). Ms. Zhao was a novice teacher who had two years of teaching experience at the time of data collection. Ms. Zhao received a bachelor's degree in Mathematics and Applied Mathematics from a prestigious normal university in Shanghai. Normal universities in China are traditionally teacher-preparation institutions where undergraduate mathematics programs offer courses related to both mathematics and pedagogy (Lu, Leung & Li, 2021, P. 31).

Mr. Qian was an expert teacher with twenty-eight years of teaching experience when the data were collected. Mr. Qian had gained his Bachelor of Science degrees on mathematics programmes at a normal university in Anhui Province. He was the mentor of the master teacher workstation, which is a professional learning community that is led by a master teacher who is officially recognized and financially supported by the local education department or government. Overall, the teaching experience of the two participants was quite different.

School

High schools in Shanghai can be classified into three categories: municipal key high schools, district key high schools and ordinary high schools. Municipal key high schools are intended for top-quality students, as designated by the local educational administration. The administration designates district key high schools as intended for average-quality students and ordinary high schools as catchalls for the remaining students.

New Textbook

A variety of mathematics textbook series are used for classroom instruction in mainland China (Xu, 2013). In China, all regions except for Shanghai City and Zhejiang Province have been required since the late 1980s had been requires to follow the national syllabus (now called the standard) (Zhu & Fan, 2006; Fan & Zhu, 2007). In the latest curriculum reform, Shanghai started to revert to the 2018 national standard. When the study was conducted, there were, in total, six series of new mathematics textbooks in use in China. The series of mathematics textbooks we selected was published by Shanghai Education Publishing House; we selected this series because it was used by the school districts in which the teachers were observed. New mathematics textbooks have been used since the autumn of 2020 in Shanghai and old textbooks were phased out from classroom instruction.

Data Collection

Data were collected from October 2020 to December 2020. Interviews, video-taped classroom observations and related documents were the main sources of data. The interviews were conducted before and after their daily classroom teaching. The interviews focus on how the teachers select, design and organize unit content in the curriculum mapping arena, select and design mathematics tasks in the design arena, and reflect and improve mathematics tasks in the reflection arena. All interviews were audio-recorded and then transcribed. The classroom observation focused on how the teachers enacted and adjusted mathematics tasks in the construction arena. The first author observed 20 40-min lessons (Zhao and Qian 10 lessons each). The related documents focused on the teachers' selection and design of mathematics tasks in the design arena.

Data Analysis

Analyzing the Selection, Design of Unit Content in Curriculum Mapping Arena. In order to analyze mathematics teachers' selection and design of unit content, we coded the teachers' selection and design of unit content. Relevant codes include appropriate omit, and add.

Analyzing Organize Unit Content in Curriculum Mapping Arena. Relevant codes include follow and rearrange.

Analyzing Selection and Design of Mathematics Tasks in the Design Arena. In order to analyze how mathematics teachers select and design mathematics tasks, we coded the teachers' selected and designed mathematics tasks as guided by Remillard's work (1999, 2018). Remillard 's two approaches of task selection include appropriation and invention. We extended Remillard's classification to include appropriation, extension, replacement, rearrangement, addition, and omission.

Analyzing the Enactment and Adjustment of Mathematics Tasks in the Construction Arena. In order to analyze enacted and adjusted mathematics tasks in lesson construction arena, we coded the teachers' enacted tasks as guided by Remillard's work (1999). In our research, relevant codes were adjusting planned tasks and not adjusting planned tasks.

Analyzing Reflection and Improvement of Mathematics Tasks in the Reflection Arena. The general strategy for this analysis was a constant comparative style (Strauss & Corbin, 1994), which permitted the categories to emerge from the data. Three categories emerged from this analysis: no reflection, reflection without improvement, and reflection with improvement.

IV. RESULTS

Selecting, Designing and Organizing Unit Content in The Curriculum Mapping Arena

Both Ms. Zhao and Mr. Qian often planned to appropriate the unit content of the new textbooks. Sometimes, both Ms. Zhao and Mr. Qian intended to add supplementary unit content. For example, regarding the monotonicity of function, when Ms. Zhao interacted with Mr. Qian, Mr. Qian believed that the Nike function (function $y = x + \frac{1}{x}$

is called the Nike function by Chinese high school mathematics teachers) was very important, but the new textbook did not cover this function. Thus, Mr. Qian intended to replace the example problem that proved function $y = x^2 - 2x$ is strictly decreasing in $(-\infty, 1]$ with a proof that the function $y = x + \frac{4}{x}$ is strictly increasing in $[2, +\infty)$ and strictly decreasing in $(0, 2]$. Influenced by Mr. Qian, Ms. Zhao planned to add the monotonicity of function. Similarly, Mr. Qian planned to add the operation on function (the monotonicity of the product of two strictly increasing function, or multiplying function).

Ms. Zhao always planned to follow the unit content organization of these three chapters. Mr. Qian often intended to follow the unit content organization of the new textbooks. Sometimes, Mr. Qian planned to rearrange the new textbook sequence. For example, in Chapter 3 (logarithms), the original order presented the definitions for logarithms, properties of logarithms, and base changing formula of logarithms. Mr. Qian planned to rearrange the order by putting the properties of logarithms and base changing formula of logarithms together and putting the exercises for these two parts together, so that he taught could help the students build a more complete knowledge structure.

Selecting and Designing Mathematics Tasks in The Design Arena

In new textbooks, a regular topic usually consisted of several parts: introductory activities, definitions, example problems and exercise problems. For most of the introductory activities and definitions, Ms. Zhao almost always planned to select or appropriate almost all introductory activities and definitions from the new textbook. For example, Ms. Zhao intended to appropriate the introductory activities in the new textbook, which were introduced by induction the common features of the three functions $y = x$, $y = x^2$, and $y = \frac{1}{x} = x^{-1}$.

Ms. Zhao often planned to select or appropriate example problems as well as exercise problems from the new textbook. She seldom planned to expand, add, rearrange, or replace the example problems or the exercise problems from the new textbook. For example, in the properties of logarithms lesson, Ms. Zhao planned to add the following example problem (1) $\ln x = \ln a + \ln b$ （2）$\lg x = 3\lg n - \lg m$ （3）$\log_a x = \frac{1}{2}\log_a b - \log_a c$. She explained that “I got these exercise problems from some other references. They are more difficult than those from the textbook. They are more challenging for the students”.

Ms. Zhao occasionally planned to replace the example problems and exercise problems from the new textbook with the tasks designed by the other teachers in the LPG. For example, when Ms. Zhao interacted with a teacher in the LPG, the teacher believed the example problem in new textbook was too easy, so he intended to replace the example problem if $f(x)=\log_3 x$, solve $f^{-1}(2)$, $f^{-1}(a)$ with if $f(x)=\frac{1}{1-x^2}(x<-1)$, solve $f^{-1}\left(-\frac{1}{3}\right)$. Influenced by this colleague, Ms. Zhao also planned to replace the example problem if $f(x)=\log_3 x$, solve $f^{-1}(2)$, $f^{-1}(a)$ with if $f(x)=\frac{1}{1-x^2}(x<-1)$, solve $f^{-1}\left(-\frac{1}{3}\right)$.

For many of the introductory activities and definitions, Mr. Qian often intended to select or appropriate introductory activities and definitions from the new textbook. Sometimes, he intended to replace introductory activities. For example, he planned to replace the introductory activities covering bank deposits in the new textbook with some old knowledge to lead into logarithms. He explained that "I think students may not understand the meaning of compound interest in the introductory activities. If I appropriate the introductory activities, I will spend some time explaining the meaning of compound interest and why after n years, the sum of the deposit and interest is 1.05^n. In this way, the introductory part will use too much time, and it will confuse the students. Thus, I did not plan to appropriate the introductory activities in the new textbook. I planned to review the previous lesson on exponent to lead into logarithms."

Mr. Qian often intended to expand, replace or rearrange the example problem as well as the exercise problems from the new textbook. Sometimes, he intended to expand example problem as well as exercise problem from the new textbook. For example, an example problem in the new textbook asked students to find the domain of function $y=\frac{1}{2^x+1}$. Mr. Qian intended to add alternative solutions after the textbook. After that, he would change the plus sign in the denominator into a minus sign. This means that he would change $y=\frac{1}{2^x+1}$ into $y=\frac{1}{2^x-1}$. He explained that "The solutions in the new textbooks had some limitations. Students easily omit $\frac{1}{2^x+1}>0$, so I planned to provide students with a better solution after explaining the solution in the textbook. Afterwards, I planned to turn $y=\frac{1}{2^x+1}$ into $y=\frac{1}{2^x-1}$ from the perspective of using variation teaching, which was more difficult than that in the textbook. At the same time, finding the domain of function $y=\frac{1}{2^x-1}$ can reflect the advantages of my solutions."

Sometimes, he intended to replace the example problems as well as the exercise problems from the new textbook. For example, Mr. Qian intended to replace the example problem prove function $y=x^2-2x$ is strictly decreasing in $(-\infty,1]$ with prove function $y = x + \frac{4}{x}$ is strictly increasing in $[2, +\infty)$ and strictly decreasing in $(0,2]$. He explained, "I think Nike function is very important, but the new textbook omits this function. Therefore, I planned to replace the example problem in new textbook with the Nike function".

Enacting and Adjusting Mathematics Tasks in The Construction Arena

Ms. Zhao seldom adjusted the planned tasks during instruction. For example, in the concept of function lesson on November 12, Ms. Zhao predicted that students would first use inductive reasoning to conclude that independent variable x had a range of values (取值范围), and then that independent variable x had domain (定义域). However, in her actual lesson, a student induced domain directly, and Mr. Zhao did not adjust the planned tasks. She still said, "Too fast. It is far away from the domain. In general, independent variable x has a range of values. x cannot be any number".

Unlike Ms. Zhao, Mr. Qian adjusted the planned tasks more frequently. He often adjusted the planned tasks when he confronted unanticipated students' idea, difficulties. On most occasions, when he confronted unanticipated students' ideas, he could flexibly adjust the planned tasks. For example, in the properties of logarithms lesson on October 22, when he enlighted students prove quotient properties of logarithms using product properties of logarithms, one student said "Add negative（加负的）" Mr. Qian asked "Add negative. What do you mean? $\log_a M - \log_a N$ regard as $\log_a M + (-\log_a N)$, then how would you prove it?" The student explained that "let $\log_a N = q$, then $-\log_a N = -q$," Mr. Qian said, "I cannot understand your idea, write your idea on the blackboard ". After the student wrote his idea on the blackboard, Mr. Qian continued with proving quotient properties of logarithms using product properties of logarithms.

For example, in the concept of inverse function lesson on December 9, Mr. Qian intended to require students to graph $y = 1.8x + 32$ and $x = \frac{y - 32}{1.8}$ (the inverse function of $y = 1.8x + 32$) in the coordinate system. Mr. Qian predicted students' reactions, and intended to explain that we can graph $y = 1.8x + 32$ on the coordinate system. However, we cannot graph $x = \frac{y - 32}{1.8}$ in the coordinate system. To graph $x = \frac{y - 32}{1.8}$ on the coordinate system, we need to interchange x and y and replace $x = \frac{y - 32}{1.8}$ with $y = \frac{x - 32}{1.8}$. However, in his actual lesson, a student said "we can graph $x = \frac{y - 32}{1.8}$ in the coordinate system. The independent variable is y, and the dependent variable is x. If we regard the horizontal axis as the y axis and vertical axis as x axis, we can graph $x = \frac{y-32}{1.8}$ in the coordinate system." Mr. Qian said, "Good idea! However, we cannot regard the horizontal axis as y axis, or the vertical axis as x axis. We can only regard the horizontal axis as x axis and vertical axis as y axis. What can we do ? " The student said "we need to interchange x and y", thereby eliciting the expected answer that x and y should be interchanged.

When he noticed that his students struggled with the planned tasks, he usually adjusted his teaching plan to review some relevant knowledge. For example, in the definition, graph and properties of power functions lesson on October 28, he graphed $y = x^{-\frac{2}{3}}$, when $x = 2$, $y = \frac{1}{\sqrt[3]{4}}$. He asked students, "Is $\frac{1}{\sqrt[3]{4}}$ greater than 1 or less than 1?" No students responded. Therefore, he asked students another question: "First, look at the denominator, is $\sqrt[3]{4}$ greater than 1 or less than 1 ?" There was still no answer. Mr. Qian said It seems that you do not quite understand $\sqrt[n]{a}$, so let' s review some knowledge about $\sqrt[n]{a}$. If $a > 1$, then $\sqrt[n]{a} > 1$, for example, $\sqrt{1.21} = 1.1$; if $0 < a < 1$, then $0 < \sqrt[n]{a} < 1$, for example, $\sqrt{0.25} = 0.5$.

Reflecting and Improving Mathematics Tasks in The Reflection Arena

Some improvements they made were based on their own reflection. Ms. Zhao reflected on her mathematics tasks after almost every class. With reflection, she often thought she should improve the mathematics tasks. For example, in the interview of the definition, graph, and properties of power functions lesson on October 28, Ms. Zhao mentioned, "The students did not understand the meaning of $f(x)$ in the exercise book very well. They didn't know whether to write it as $f(x)$ or y, so I should talk about it in class."

With reflection, Ms. Zhao sometimes thought she did not need to improve mathematics tasks. For example, in the interview of the definition, graph, and properties of power functions lesson on October 29, Ms. Zhao said "I feel satisfied that this class was very successful. In addition, I made it very clear in class, but students struggled with the homework. Students still had difficulty graphing power function. I can only say that they don't have good memories."

Ms. Zhao's reflection is influenced by context factors such as exams and teaching progress in other school. Occasionally, she would be pushed to catch up with the progress and have no time to reflect on her teaching. For example, in the interview of the definition, graph, and properties of logarithmic functions lesson on November 3, Ms. Zhao said, "I do not think about which part I am satisfied with or not. Next week will be the mid-term exam in which the logarithmic function will be tested. Because the classes time are so limited, what I am considering now is how to finish the second class on logarithmic function as soon as possible. And then the students can start to review."

Mr. Qian reflected on his mathematics tasks after every class. With reflection, he almost always thought he should improve mathematics tasks. For example, in the interview of the definition of logarithm lesson on November 19, he mentioned "I did not mention enough about the history of logarithm. To be honest, we still have the pressure of helping students to enter a more highly ranked school. If it were otherwise, I might have introduced more history on the generation of logarithm, which can influence students with mathematics culture. However, in this way, I would have no time to talk about the exercises in this class. Therefore, we are affected by the pressure of students who need to enter a higher school."

Moreover, Mr. Qian often reflected on his mathematics tasks by using mathematics education theory. For example, in the interview of the monotonicity of function lesson on November 19, he reflected on use of the multiple representations of monotonicity of function by using multiple representations theory. He mentioned "I did not do well in the multiple representations in class. I only introduced the concept for any numbers, if $x_1 < x_2$, then $f(x_1) < f(x_2)$ which represents that function $f(x)$ is strictly increasing. Students may think that this is true only if $x_1 < x_2$. I should also introduce the concept that for any numbers, if $x_1 > x_2$, then $f(x_1) > f(x_2)$ also represents that function $f(x)$ is strictly increasing. Multiple representations of the monotonicity of function can enable students to understand the essence of mathematics（数学本质）in the monotonicity increasing function. The essence of monotonicity increasing function is that the change of function values is consistent with that of independent variables. Multiple representations of monotonicity of function can provide students with a better understanding of this concept."

Discussion

Based on this framework, this study examined how two high school mathematics teachers from Shanghai use new textbooks. Our results contribute to studies on textbook use in three ways. First, we presented a theoretical framework based on the work of Remillard (1999, 2005) and Chau (2014). Second, most studies on textbook use were conducted in Western educational contexts (Fan, Chen, Zhu, Qiu, & Hu, 2004), and this study extends our understanding of Chinese mathematics teachers' use of textbook. In addition, studies on mathematics teachers' use of textbooks have mainly focused on elementary mathematics teachers, and this study extends such work to the high school level.

Concerning selecting and designing unit content in curriculum mapping arena, both novice teacher and expert teacher often planned to appropriate the unit content of the new textbook, and sometimes planned to add supplemental unit content. This result is inconsistent with the findings of previous Western studies. Tarr et al. (2008) examined textbook fidelity. They found that, on average, teachers used 60 to 70% of the lessons in the curriculum guides, regardless of the type of curriculum. The differences might be caused in part by differences in textbooks. U.S. textbooks in mathematics and, especially, in science include material on far more topics than is typical in other countries. U.S. science and mathematics textbooks tend to be "a mile wide and an inch deep." That is, although U.S. textbooks include many more topics than other countries' books, the few most emphasized topics account for less content than is the case internationally (Schmidt, McKnight, & Raizen, 1997).

Concerning organizing unit content in the curriculum mapping arena, the novice teacher always planned to follow the new textbook in terms of sequence, while the expert teacher often planned to follow the new textbook in terms of sequence, and sometimes planned to rearrange the unit content. This result is consistent with the findings of some previous studies. Yang (2014) also found that Chinese expert teachers would rearrange the topics in the textbook when they found that the topics in the textbook were not reasonable or suitable for their students. In addition, Mr. Qian reflected the principles of the new mathematics curriculum in high schools in a good way. Teaching plans should not be designed by individual class or point by point. They should be designed by putting some points with logical relations together. In this way, we can realize the goal that students form and develop mathematics core literacy (Shi, 2018).

Concerning selecting and designing mathematics tasks in the design arena, both novice teacher and expert teacher tended to appropriate the introduced tasks and definitions. The novice teacher often planned to appropriate the example problems and exercise problems, while the expert teacher often intended to flexibly use (expand, replace, omit, rearrange and add) the example problems and exercise problems. This result is consistent with the finding of some previous studies. Fan et al. (2004) also found that the ways in the textbooks might not be best ones so that Chinese mathematics teachers often provided students with alternative ways to broaden students' minds and encourage students to think. Ma (1999) also found that Chinese mathematics teachers also frequently share their ideas and develop understandings of mathematical content they have taught or are about to teach with their colleagues in the subject-based Teaching Research Group.

Concerning enacting and adjusting mathematics tasks in the construction arena, the novice teacher seldom adjusted the planned tasks, in contrast, the expert teacher adjusted the planned tasks more frequently. This result is consistent with the findings of some previous studies. Borko and Livingston (1989) reported that in their study of mathematics teachers, expert teachers were able to use student responses and questions as springboards for further discussion and keep the lesson on track at the same time. They were able to maintain a balance between student-centeredness and conent-centeredness. They were able to generate on-the-spot examples and mathematical problems for illustration and clarification of concepts. In contrast, novice teachers had difficulties maintaining the direction of the lesson when responding to student questions. They also had problems with questions that were unplanned. Consequently, they decided to curtail questions so that they could get through what they had planned, despite the fact that they valued responsiveness to students. In other words, instead of modifying their plans to suit students' needs, novice teachers suited their own needs by ignoring the students. Similarly, Yang (2014) also reported that Chinese expert teachers were able to flexibly adjust their lesson plans once they encountered unexpected student reactions.

Concerning reflecting and improving mathematics tasks in reflection arena, our results echo those of Gu (2001), who also pointed out that compared with novice teachers and experienced teachers, expert teachers had not only principle knowledge and case knowledge, but also rich strategic knowledge, that is, the strategy of applying pedagogy and psychology principles to special cases, whose core is the reflection on teaching practice.

Limitations of The Study

There were two limitations present in this study. First, due to the limited number of cases included, there may be other possible types of teacher use of new textbooks that were missed by this study. For a comprehensive understanding of how Chinese mathematics teachers use new textbook, more cases involving teachers with different background experiences are needed.

Second, this study focused on how mathematics teachers use new textbooks at only the district key high school level. However, according to the literature, the use of textbooks may be influenced by the context of school (e.g., Keiser & Lambdin, 1996; Valencia et al., 2006; Van Zoest & Bohl, 2002). For instance, Valencia et al. (2006) found that two of the novice teachers in their study were bounded in their ability to skilfully adapt their curriculum materials by the lack of resources at their schools. In contrast, the two novice teachers with greater pedagogical content knowledge and better access to resources 'learned the most and were able to adapt instruction' (p. 114). Similarly, Fan et al. (2021) found that Shanghai schools' characteristics have a greater influence on the extent to which textbooks play a facilitator role in the teachers' teaching. More specifically, textbooks play a larger facilitation role in teachers' teaching in ordinary schools and public schools. The findings and conclusions in the present study should not be generalized to mathematics teachers at the municipal key high school level and ordinary high school level. There is a need to explore how mathematics teachers use new textbook at those two levels to obtain a deep understanding of how new textbooks are used at each level in mainland China.

Conflict of interest The author declare that he has no conflict of interest.

References

Bieda, K. N. (2010). Enacting proof-related tasks in middle school mathematics: Challenges and opportunities. *Journal for Research in Mathematics Education, 41*(4), 351–382. https://doi.org/10.5951/jresematheduc.41.4.0351

Borko, H., & Livingston, C. (1989). Cognition and improvisation: Differences in mathematics instruction by expert and novice teachers. *American Educational Research Journal, 26*(4), 473-498. https://doi.org/10.3102/00028312026004473

Cai, J., Ni, Y., & Lester, F. K. (2011). Curricular effect on the teaching and learning of mathematics: Findings from two longitudinal studies in China and the United States. *International Journal of Educational Research, 50*(2), 63–64.

https://doi.org/10.1016/j.ijer.2011.06.001

Chau, Y. (2014). *Teachers' use of senior secondary Geography textbooks in Hong Kong: Implications for meaningful learning* [Unpublished doctoral dissertation, University of Hong Kong].

Chen, W., & Ding, M. (2018). Transition from textbook to classroom instruction in Mathematics: The case of an expert Chinese teacher. *Frontiers of Education in China*, *13*(4), 601-632. https://doi.org/10.1007/s11516-018-0031-z

Eisenmann, T., & Even, R. (2011). Enacted types of algebraic activity in different classes taught by the same teacher. *International Journal of Science and Mathematics Education, 9*(4), 867–891. https://doi.org/10.1007/s10763-010-9241-4

Fan, L. (2013). Textbook research as scientific research: Towards a common ground on issues and methods of research on mathematics textbooks. *ZDM Mathematics Education, 45*(5), 765–777. https://doi.org/10.1007/s11858-013-0530-6

Fan, L., Chen, J., Zhu, Y., Qiu, X., & Hu, Q. (2004). Textbook use within and beyond Chinese mathematics classrooms: A study of 12 secondary schools in Kunming and Fuzhou of China. In L. Fan, N. Y. Wong, J. Cai, & S. Li (Eds.), *How Chinese learn mathematics: Perspectives from insiders* (pp. 228–261). World Scientific.

Fan, L., & Kaeley, G. S. (2000). The influence of textbooks on teaching strategies: An empirical study. *Mid-Western Educational Researcher, 13*(4), 2–9.

Fan, L., & Zhu, Y. (2007). Representation of problem-solving procedures: A comparative look at China, Singapore, and US mathematics textbooks. *Educational Studies in Mathematics, 66*(1), 61–75. https://doi.org/10.1007/s10649-006-9069-6

Fan, L., Zhu, Y., & Miao, Z. (2013). Textbook research in mathematics education, development status and directions. *ZDM The International Journal on Mathematics Education, 45*(5), 633–646. https://doi.org/10.1007/s11858-013-0539-x

Fan, L., Cheng, J., Xie, S., Luo, J., Wang, Y., & Sun, Y. (2021). Are textbooks facilitators or barriers for teachers' teaching and instructional change? An investigation of secondary mathematics teachers in Shanghai, China. *ZDM The International Journal on Mathematics Education, 53*(6), 1313–1330. https://doi.org/10.1007/s11858-021-01306-6

Freeman, D. J., & Porter, A. C. (1989). Do textbooks dictate the content of mathematics instruction in elementary schools? *American Educational Research Journal, 26*(3), 403–421. https://doi.org/10.2307/1162980

Glasnović Gracin, D., & Jukić Matić, L. (2021). Use of textbooks and other resources in curriculum reform. A longitudinal case study. *ZDM The International Journal on Mathematics Education, 53*(6), 1373–1385. https://doi.org/10.1007/s11858-021-01271-0

Gu, L. (2001). Reform in teaching task. *Research in Educational Development,* (10), 5–12.

Howson, G. (2013). The development of mathematics textbooks: Historical refections from a personal perspective. *ZDM Mathematics Education, 45*(5), 647–658. https://doi.org/10.1007/s11858-013-0511-9

Keiser, J. M., & Lambdin, D. V. (1996). The clock is ticking: Time constraint issues in mathematics teaching reform. *The Journal of Educational Research, 90*(1), 23–31.

https://doi.org/10.1080/00220671.1996.9944440
Kong, F., & Shi, N. (2009). Process analysis and level measurement of textbooks use by teachers. *Frontiers of Education in China,* 4(2), 268–285. https://doi.org/10.1007/s11516-009-0014-1
Li, Y., Chen, X., & Kulm, G. (2009). Mathematics teachers' practices and thinking in lesson plan development: A case of teaching fraction division. *ZDM Mathematics Education, 41*(6), 717–731. https://doi.org/10.1007/s11858-009-0174-8
Lloyd, G. M. (2009). School mathematics curriculum materials for teachers' learning: future elementary teachers' interactions with curriculum materials in a mathematics course in the United States. *ZDM Mathematics Education, 41*(6), 763–775. https://doi.org/10.1007/s11858-009-0206-4
Lloyd, G. M., Remillard, J. T., & Herbel-Eisenmann, B. A. (2009). Teachers' use of curriculum materials. In J. T. Remillard, B. A. Herbel-Eisenmann, & G. M. Lloyd (Eds.), *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 3–14). Routledge.
Lu, X., Leung, F. K. S., & Li, N. (2021). Teacher agency for integrating history into teaching mathematics in a performance-driven context: A case study of a beginning teacher in China. *Educational Studies in Mathematics, 106*(1), 25–44. https://doi.org/10.1007/s10649-020-10006-z
Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Lawrence Erlbaum.
McNaught, M. D., Tarr, J. E., & Sears, R. (2010). *Conceptualizing and measuring fidelity of implementation of secondary mathematics textbooks: Results of a three-year study.* Paper presented at the annual meeting of the American Educational Research Association, Denver, CO.
Ministry of Education of China. (2018). *Putong gaozhong shuxue kecheng biaozhun* (2017 nian ban) [Mathematics curriculum standard for high schools (2017 version)]. People's Education Press.
MOE (Ministry of Education of China). (2018). *High school mathematics curriculum standards* (2017 Edition). People's Education Press (in Chinese).
Nicol, C. C., & Crespo, S. M. (2006). Learning to teach with mathematics textbooks: How preservice teachers interpret and use curriculum materials. *Educational Studies in Mathematics, 62*(3), 331–355. https://doi.org/10.1007/s10649-006-5423-y
Obara, S., & Sloan, M. (2009). Classroom experiences with new curriculum materials during the implementation of performance standards in mathematics: A case study of teachers coping with change. *International Journal of Science and Mathematics Education, 8*(2), 349–372. https://doi.org/10.1007/s10763-009-9176-9
Pepin, B., & Haggarty, L. (2001). Mathematics textbooks and their use in English, French, and German classrooms. *ZDM Mathematics Education, 33*(5), 158–175. https://doi.org/10.1007/BF02656616
Remillard, J. T. (1999). Curriculum materials in mathematics education reform: A framework for examining teachers' curriculum development. *Curriculum Inquiry, 29*(3), 315–342. https://doi.org/10.1111/0362-6784.00130

Remillard, J. T. (2000). Can curriculum materials support teachers' learning? *Elementary School Journal, 100*(4), 331–350. https://doi.org/10.1086/499645

Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211–246. https://doi.org/10.3102/00346543075002211

Remillard, J. T. (2018). Mapping the relationship between written and enacted curriculum: examining teachers' decision making. In G. Kaiser (Ed.), *Invited Lectures from the 13th International Congress on Mathematical Education* (pp. 483–500). Springer.

Reys, B. J., Reys, R. E., & Chávez, O. (2004). Why mathematics textbooks matter. *Educational Leadership, 61*(5), 61-67.

Schmidt, W. H., McKnight, C. C., & Raizen, S. C. (1997). *A splintered vision: An investigation of US science and mathematics education*. Kluwer Acadmic Publishers.

Sears, R., & Chávez, O. (2014). Opportunities to engage with proof: The nature of proof tasks in two geometry textbooks and its influence on enacted lessons. *ZDM Mathematics Education, 46*(5), 767–780. https://doi.org/10.1007/s11858-014-0596-9

Sherin, M. G., & Drake, C. (2009). Curriculum strategy framework: Investigating patterns in teachers' use of a reform-based elementary mathematics curriculum. *Journal of Curriculum Studies, 41*(4), 467–500. https://doi.org/10.1080/00220270802696115

Shi, N. (2018). Key problems in the revision of high school mathematics curriculum standard. *Journal of Mathematics Education, 27*(1), 8–10.

Son, J. W., & Kim, O. K. (2015). Teachers' selection and enactment of mathematical problems from textbooks. *Mathematics Educational Research Journal, 27*(4), 491–518. https://doi.org/10.1007/s13394-015-0148-9

Stake, R. E. (2013). *Multiple case study analysis*. Guilford Press.

Stein, M. K., Remillard, J., & Smith, M. S. (2007). How curriculum influences student learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 319–369). Information Age.

Strauss, A., & Corbin, J. (1994). Grounded theory methodology: An overview. In N. K. Denzin & Y. Lincoln (Eds.), *Handbook of qualitative research* (pp. 273–285). Sage.

Stylianides, G. J. (2016). *Curricular resources and classroom use: The case of mathematics*. Oxford University Press.

Tarr, J. E., Reys, R. E., Reys, B. J., Chávez, O., Shih, J., & Osterlind, S. J. (2008). The impact of middle grades mathematics curricula on student achievement and the classroom learning environment. *Journal for Research in Mathematics Education, 39*(3), 247–280.

Thompson, D. R., & Senk, S. L. (2014). The same geometry textbook does not mean the same classroom enactment. *ZDM Mathematics Education, 46*(5) ,781–795. https://doi.org/10.1007/s11858-014-0622-y

Valencia, S. W., Place, N. A., Martin, S. D., & Grossman, P. L. (2006). Curriculum materials for elementary reading: Shackles and scaffolds for four beginning teachers. *The Elementary School Journal, 107*(1), 93–120.

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the Book: Using TIMSS to Investigate the Translation of Policy into*

Practice Through the World of Textbooks. Kluwer.

Van Zoest, L. R., & Bohl, J. V. (2002). The role of reform curricular materials in an internship: The case of Alice and Gregory. *Journal of Mathematics Teacher Education, 5*(3), 265–288. https://doi.org/10.1023/A:1019816329185

Xu, B. (2013). The development of school mathematics textbooks in China since 1950. *ZDM Mathematics Education, 45*(5), 725–736. https://doi.org/10.1007/s11858-013-0538-y

Yang, X. (2014). Conception and characteristics of expert mathematics teachers in China. Springer Spektrum.

Zhu, Y., & Fan, L. (2006). Focus on the representation of problem types in intended curriculum: A comparison of selected mathematics textbooks from Mainland China and the United States. *International Journal of Science and Mathematics Education, 4*(4), 609–626. https://doi.org/10.1007/s10763-006-9036-9