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RESEARCH ARTICLE

Mathematics Teacher Educators' Collective Noticing on Microteaching

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Abstract

This article explores how mathematics teacher educators (MTEs) engaged in collaborative inquiry into the microteaching experiences of preservice teachers (PSTs), ultimately developing a noticing framework through collective MTE inquiry. We delve into the specifics of what MTEs notice focusing on three emerging categories of noticing on PST's microteaching videos—lesson structure, task quality, and teaching practices. Each category, along with MTEs' noticing within these components, is elaborated through vignettes. This approach positions MTEs' noticing as a crucial element in the overarching vision to enhance the teaching practices of PSTs.

Keywords: mathematics teacher educator, noticing, microteaching, preservice teachers

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I. INTRODUCTION

Goodwin's study (1994) describes professional vision as "ways of seeing and understanding events that are answerable to the distinctive interests of a particular social group" (p. 606). Applying this notion to teacher's professional vision, Sherin and her colleague (Sherin, 2001, 2007; Sherin et al., 2008) define a teacher's professional vision as "the ability to notice and interpret significant interactions in the classroom" (Sherin et al., 2008, p. 2). In this line of studies, researchers studied *noticing* (Mason, 2002; Stockero, 2014) or *professional noticing*[1] (Jacobs et al., 2010) as the ability to attend to particular aspects involving instruction and/or students' mathematical thinking in response. "Mathematics teachers need noticing expertise to detect and make sense of instructionally important features in complex classroom environments" (Jacobs & Spangler, 2017, p. 771). Based on this belief that noticing is an ability to be learned, previous studies have focused on professional noticing of preservice teachers (Krupa et al., 2017; Jacobs et al., 2010; Stockero, 2014) and practicing teachers (Jacobs et al, 2010; Sherin, 2007; Sherin et al., 2008). However little study has paid attention to mathematics teacher educators' (MTEs) noticing that might influence pre- or in-service teachers' development of noticing skills.

Addressing MTE's professional noticing is a challenging task due to the various situations that MTEs have experience in their education setting. Some MTEs teach only mathematical content courses, some do only methods courses and others teach both content and methods courses. In addition, due to the dual roles of MTEs being a teacher and a teacher educator, MTEs' professional vision is considered multi-layered, in contrast to the single layered teacher vision. According to Sherin and her colleagues (2008), a teacher's professional vision is related to classroom interactions with students. Similarly, MTEs' professional vision is linked to prospective or practicing teachers' knowledge and skills for teaching and learning. Furthermore, it includes how MTEs notice and interpret the interactions between a teacher and students. MTEs' professional vision, then encompasses the capability to notice and interpret both teachers' and students' mathematical thinking in a mathematical activity such as professional development program and teacher education program (Jacobs & Spangler, 2017). As stated in Jacobs and Spangler (2017), MTEs should be able to notice two layers of interactions by not only considering their own students (pre- or in-service teachers), but their students' students to support both teachers and students. In this paper, we narrow down our focus on one aspect of the multi-layered MTEs professional noticing in preservice teacher education programs, in particular, during their microteaching. Instead of teaching actual students, MTEs often have preservice teachers (PSTs) simulate a mathematics classroom among themselves—which is a form of microteaching. Microteaching means a teacher training technique that includes a real teaching situation with the reduction of teaching complexities such as number of students, scope of content, timeframe, etc.

In this study, we discuss MTEs' professional noticing on designing and implementing microteaching for the mathematics education program. In order to identify

[1] In this paper, these terms such as noticing, professional noticing, and professional vision have the same meaning.

MTE's noticing in microteaching practice and to find ways of improving mathematics teacher education programs, we present important noticing from microteaching that emerged through collaborative efforts among a group of MTEs by answering this research question: What do mathematics teacher educators notice and learn from preservice teachers' microteaching? MTEs can benefit from reflecting upon their professional visions as a means of improving the quality of teacher education (e.g., Carr, 1989; National Research Council, 2001). Such noticing can help MTEs build their vision on microteaching and ultimately improve the quality of teacher education programs.

II. THEORETICAL BACKGROUND

Professional Vision

We go back to Goodwin's study (1994) that has played a seminal role in the literature to conceptualize professional vision. Goodwin claimed the importance of non-observable knowledge for certain professions, which might not be observable to other groups. For example, mathematics teachers would emphasize the mathematical argumentation or justification during writing in mathematics lessons, while literacy teachers might focus on the grammatical components or coherence of the writing. Sherin (2001, 2007) defined that professional vision was relevant to selective attention and knowledge-based reasoning. For example, a teacher's professional vision enables the teachers to make sense of phenomena of classroom interactions. The concept of Sherin's professional vision is useful to frame a teacher's ability to notice and interpret complex classroom situations as an important professional skill.

During several decades, the major focus of professional vision in education has been centralized on teacher's professional noticing of children's mathematical thinking (Jacobs et al., 2010) or learning to notice (van Es & Sherin, 2008). The former unpacked mathematics teacher's decision-making process (attending, interpreting, and deciding how to respond) to advance students' mathematical thinking. On the other hand, the latter consists of three aspects: identifying what is important in a teaching situation, using what one knows about the context to reason about a situation, and making connections between specific events and broader principles of teaching and learning.

Noticing as an MTE's Professional Vision

In this study, the noticing of MTEs is related to Sherin's (2007) professional vision that notices and acts on an important pedagogical moment, as learning to notice. This view of MTEs toward PSTs' teaching motivated our study to explore existing data about PSTs' microteaching from the unique viewpoint of teacher educators. Although MTEs' professional visions may vary depending on a context and individuals' backgrounds, we focus on establishing commonly agreed components for effective microteaching. We chose the context of microteaching in a real classroom setting because MTEs can notice more on real classroom teaching with insight from interactions with students. That being said, we contemplate what the MTEs noticed by observing preservice teachers' microteaching and discussing what components of microteaching practices should be considered.

As the study explores what the MTEs notice when analyzing the PSTs' microteaching, it is worth noting how the MTE's noticing is different from teacher's or researcher's. Concerning the difference, Sherin's (2001) study explains that researchers use theoretical lenses to analyze classroom practices and draw findings while teachers draw on their own experiences to interpret classroom events. Ho and Tan (2013) added that the researcher's professional vision is "more observational tending to produce and articulate material representations such as academic papers or reports" (p. 416) whilst "teachers do develop an implicit professional vision of their classroom practices" (p. 416). As with Ho and Tan, we also view a microteaching situation as an object of observation and intend to make research in the future.

To conceptualize MTE's professional noticing, we first refer to Ho and Tan's (2013) professional vision that "is taken as a way to view classroom events as they are seen through the lens of a researcher and a teacher, both taken as professionals within the broad field of education" (p. 415). A key professional goal of MTEs is to provide PSTs with the knowledge of teaching and learning mathematics and to support them to develop effective teaching methods. Therefore, MTEs identify effective mathematics teaching practices with a consideration of the nature of schools, classrooms, and students. MTEs also carefully study the process of PST's learning to teach, since MTEs are responsible for providing PSTs with research-based knowledge within a productive learning environment. MTEs analyze elements of teaching by putting on a researcher's hat and interpret PSTs' (students') teaching practices (students' learning) with a teacher's hat on. Despite the meta nature of MTE's noticing, the primary noticing of MTEs is related to developing an explicit professional vision of teaching practices (Ho & Tan, 2013). In sum, MTE's professional noticing has their own unique ways to notice, examine, and interpret the elements of PSTs' teaching practices. Therefore, in this study, we would like to investigate a group of the MTEs' noticing when observing the PSTs' microteaching practices and to identify their distinctive perspectives for microteaching in contrast to teachers' one.

Microteaching in Preservice Teacher Education

Many teacher education programs have used a microteaching in consideration of developing PSTs' pedagogical content knowledge, teaching skills, and professional attitudes, since Dwight Allen and his colleagues to help PSTs develop teaching skills in a low-risk, simulated classroom environment (Allen, 1966; Grossman, 2009) introduced it to the teacher education field in the 1960s. MTEs today use a three-phase microteaching model: planning phase, teaching phase, and reflecting phase (See Figure 1) (Bell, 2007; Diana, 2013).

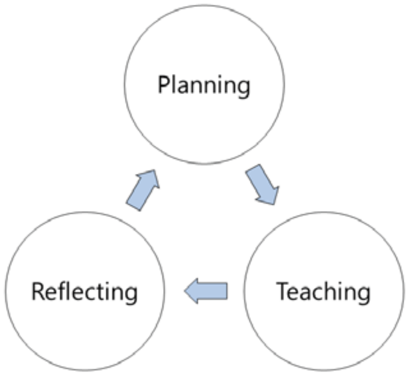

Figure 1. Cycle of microteaching

In the process of the microteaching cycle, a PST (or a group of PSTs) plans and teaches a lesson to their peers or a small group of students instead of a whole classroom of actual students. The following phase of reflections would consider the teaching recorded using video cameras. The MTE provides feedback that focuses on the performance of the PST's teaching. Then the PST reflects his/her own teaching, revises the lesson plan, and re-teaches it.

MTEs have found the contribution of microteaching to the PSTs' development of a variety of pedagogical skills such as classroom management, setting up appropriate teaching goals, and teaching practices (Abdurraham, 2010). Microteaching supports the development of PSTs' self-efficacy (Arsal, 2014) as well as the skill of critiquing/encouraging the peer PSTs to grow as reflective practitioners (Mergler & Tangen, 2010). Furthermore, microteaching benefits the PSTs in improving their ability to connect teaching theory to practice.

Instructional Triangle and Microteaching

Cohen and Ball (1999) used a triangle of interactions between teacher, students, and content in teaching. We adapt this instructional triangle in microteaching situations. We define microteaching as a learning environment being a series of interactions among PST, students, and mathematical content (see Figure 2). Although PSTs could be different from in-service teachers in many aspects, in microteaching, a PST must consider the learning environment as being composed of interactions between all three of these interactions. In particular, PSTs' teaching provides opportunities for insight into how PSTs view their students and the mathematical content in the context of their microteaching. In this study, we focused on PST's microteaching within the series of interactions by investigating it through MTEs' collective noticing. The Cohen and Ball's triangle provides one way to analyze microteaching because it brings attention to how PSTs view each element or interactions between elements in the triangle.

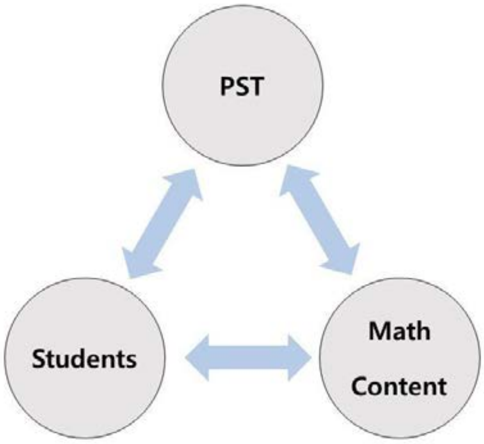

Figure 2. Cycle of microteaching *Instructional Triangle in Microteaching (Adapted from Cohen & Ball, 1999)*

III. METHODOLOGY

We analyzed the video recordings of PSTs' teaching demonstrations as a tool for developing noticing as a professional vision. By watching, reflecting, and discussing the PSTs' microteaching in the video recordings, we had opportunities to communicate thereby building and co-constructing MTEs' shared professional visions.

Participants

In this study, six MTEs participated: three worked in South Korea and three worked in the U.S.A. All six MTEs had K-12 education in South Korea and earned their doctoral degrees in mathematics education from research-centered institutions in the U.S. At the beginning of our discussions, the members did not know each other and did not have much in common other than their professional goals of improving mathematics education in either country. One spontaneous proposal to start meetings has led to an MTE learning community with shared goals and regularly scheduled meetings. These six MTEs have been working as university faculty members in mathematics teacher education or mathematics department. Therefore, the MTEs naturally shared issues of teacher education including microteaching. While the participating MTEs in this study may have experienced cultural differences in education systems due to working in different countries, the focus was less on observing such cultural distinctions. Instead, the emphasis was placed on what MTEs prioritize in microteaching. Due to the physical distance, all meetings were held online using Skype or Zoom.

Table 1. MTEs' backgrounds

Participant	Country	Gender	MTE experience
A	U.S.A	Male	11 years
B	U.S.A	Female	7 years
C	U.S.A	Male	6 years
D	South Korea	Male	16 years
E	South Korea	Female	14 years
F	South Korea	Male	12 years

Videotaped Lessons of Microteaching

Many teacher education programs have found teaching videos as a useful medium reflective of the present reality concerning PSTs' teaching knowledge and practices (Blomberg et al., 2011; Hamel & Viau-Guay, 2019). Our primary source of data is the video recordings of the participating PSTs' microteaching. In this study, researchers analyzed aspects that MTEs prominently noticed in microteaching through available microteaching videos. Therefore, the video selection was not based on the content of microteaching but rather on the availability of microteaching videos for analysis. The microteaching was implemented in a secondary mathematics methods course of a university-based teacher education program in South Korea. A total of 38 PSTs were enrolled in this methods course in their second year of the teacher education program. All participating PSTs had taken at least one methods course and six content courses in mathematics (mathematics education). Drawing upon Fernandez's (2010) model of Lesson Study, the PSTs had two teaching sessions in the process of microteaching that is, prepared, planned, taught, revised and re-taught, and reflected their own microteaching. We used the videos of PSTs' teaching in the first teaching session. The PSTs were assigned to nine groups (four to five PSTs in a group). Each group wrote a lesson plan on the Circle unit in 9th-grade Geometry. One representative PST from each group taught a lesson from a set of nine sequential lessons about the circle. Each microteaching involved an approximate 30-minute mini-lesson[2] and every lesson was videotaped (Table 1).

Table 2. PST's geometry microteaching

Lesson (Duration, min: sec)	#1 (32:07)	#2 (21:52)	#3 (24:53)	#4 (27:15)	#5 (25:19)	#6 (27:22)	#7 (23:27)	#8 (28:03)	#9 (17:34)
Preservice Teacher	A	B	C	D	E	F	G	H	I
Topics	Properties of the chords and tangents of a circle				Properties of the inscribed angles of a circle				

[2] The playing time of the 9th lesson video was cut short because of technical difficulties.

The microteaching took place in an after-school mathematics program at a local middle school with four 9th grade students after the regular school hours. Among the volunteered students who signed up for this after-school program, four students were selected based on their low academic performance and their homeroom teachers' recommendations. Microteaching in this study described a special circumstance in that PSTs taught four middle school students in the students' classroom for 30 minutes. The situation of teaching with real students can help MTEs see how classroom culture is united by a teacher's actions and students' reactions.

Data Collection

In order to identify each MTE's noticing on the microteaching videos, this study documented individual MTEs' thoughts including interpretations, theorizations, and implications about important teaching moments in the PSTs' teaching episodes. The main data we used for this study was the notes that individual MTEs made to respond to the following prompts: When watching the lesson video, please note any interesting instances with timestamps, describe the reasons why the instances are selected, and provide your interpretations, analysis or any implications. Prior to each meeting, each member reviewed the PSTs' lesson plans and their reflections, then took notes while watching video-recorded lessons. We individually analyzed nine PSTs' microteaching of geometry lessons and had recurrent discussions of what we had noticed.

We also collected recorded regular meetings using the video-conferencing software (i.e., Zoom and Skype). During online meetings, each member took turns to provide a description of what one has noticed in a video, and the discussion expanded to related issues, questions, and viewpoints including personal or institutional visions, the theory practice gap (Korthagen & Kessels, 1999), and cultural differences in PST education. With a growing sense of familiarity and trust over time, we found our conversations about the PSTs' microteaching had developed specific themes and the areas of analysis converged to yield a list of common interests, similar viewpoints, and concerns about microteaching in mathematics of PSTs.

Data analysis

We analyzed forementioned data through the constant comparative method (Corbin & Strauss, 2015) to develop an analytic framework to identify the MTE's noticing. First, due to the exploratory nature of the analysis, we conducted open coding (Saldaña, 2013). The open coding highlights the moments that are indicative of the ways the individual MTEs paid attention to the PST's microteaching practices as evidenced in the MTEs' own notes and discussion data. The preliminary codes were categorized and double checked for validity, we reviewed field notes again and debated until a consensus of shared codes was made. During this process, the codes were refined and further clarified through conversations with each member. The final codes were then organized into three main themes that were in congruence with the way this study conceptualized PST's classroom practices. To be specific, the MTE's noticing consisted of three categories-lesson structure, task quality, and teaching practices (Table 2). Each category is illustrated in the following sections.

Table 3. MTE's noticing on microteaching

Categories	Components	Key Question
Lesson Structure	• Structured lesson	• What format does a teacher show when implementing a lesson?
	• Learning goals for lessons	• How does a teacher set up learning goals?
Task Quality	• Cognitive complexity	• What level of cognitive demand tasks does a teacher provide?
	• Accessibility of task	• How does a teacher provide entry points, strategies, and/or exit points tasks?
Teaching Practice	• Mathematical tools	• What and how does a teacher utilize mathematical tools?
	• Mathematical discourse	• How does a teacher engage students in a productive mathematical discourse?
	• Classroom culture	• How does a teacher establish a classroom environment?

IV. WHAT WE LEARN FROM MTE'S NOTICING ON PST MICROTEACHING

Lesson Structure

An effective lesson structure enables teachers to facilitate students' mathematics problem solving and reasoning (Sullivan, et al., 2015). The MTEs viewed a good mathematics lesson as making harmony in mathematical thoughts and interactions between the teacher and students around mathematical tasks. Hence, a lesson structure involves the ways of launching tasks and creating social interactions among the teacher and students about mathematics through actions and reactions. We discuss two components of structured lessons and learning goals in this category.

Structured Lesson. The MTEs focused on the structure or format of the lessons that the PSTs performed during the microteaching. Across all nine microteaching lesson videos, the MTEs discussed the structure of the lessons consistently. All PSTs implemented quite similar lesson plan structures which included three phases: launching, developing, and summarizing a lesson. As one period of microteaching was limited to about 30 minutes, some PSTs could not complete their planned lessons. However, we noticed that the PSTs moved through various stages of their lessons despite the time constraints. In a PST's microteaching, the general structure of launching - developing - summarizing was successfully implemented within 30 minutes. Instead of just evaluating the PSTs' teaching quality, the MTEs focused on discussing the flow of class. For example, in the video lesson #5, PST *E* demonstrated the following flow: (i) *motivating* students by using educational websites and presenting the lesson objectives, (ii) *developing* tasks and *facilitating* discussions, and (iii) *summarizing* the topic with a quiz. In a meeting, one MTE mentioned, "Maintaining the flow of the lesson means that the teacher has prepared a good mathematics task implemented through smooth transitions and effective instructional pacing."

In addition, the PSTs in microteaching videos generally tried to implement structured lessons in which a teacher directly instructed students what and how to solve problems. In most microteaching videos, the PSTs seemed to play the role of a deliverer of knowledge (Munter, 2014) rather than a facilitator of learning. For example, in the video lesson #3, PST *C* taught Two Tangent Theorem in which two tangent lines drawn from a point outside of a circle are the same length. He asked the questions to check students' prior knowledge (e.g., what are tangent lines?), provided a task to draw tangent lines on a given circle, and asked the students to justify two congruent triangles and to solve the practice problems. However, the PST did not leverage student responses. He asked questions and provided answers with little to no wait time. Some MTEs identified this lesson as a "teacher-centered" instruction and the teacher role as an authority (cf. "sage on the stage vs. guide on the side" in King, 1993). Although not all microteaching had a general structure that the MTEs expected, the lesson structure arose as an important component of successful microteaching for several lesson videos.

Learning Goals for Lessons. The MTEs noticed how a PST set up learning goals in microteaching. Most PSTs in our study provided clear goals for their lessons. At the beginning of their lessons, the PSTs stated brief, clear learning objectives of what learners were expected to do and know at the end of the lessons. However, the MTEs have found that these learning goals did not necessarily motivate students in learning the new concepts. For example, in the video lesson #1, PST *A* asked students to read learning goals out loud. He neither permitted time for students to ask questions about the learning goal, nor explained what activities would support them to achieve the goal. Some MTEs working in Korea mentioned that they often saw this kind of demonstration that PST let students read learning goals. However, the MTEs agreed that it was difficult to convince students of learning goals just by simply reading them aloud.

After reading the objectives in the beginning of the lessons, the PSTs did not remind the students of the learning objectives during any activity or problem solving. On the video lesson #7, PST *G* had never mentioned why the students were making inscribed angles using the origami paper. It appeared that students did what they were told to do without thinking about why they were doing it. Two MTEs mentioned they recognized the importance of writing learning objectives on the board or chanting them out loud in microteaching. One of the MTEs insisted that PSTs should set learning objectives and remind their students of the objectives during the lesson not only to clarify the activities but also to motivate them to be engaged in learning. Although all MTEs did not agree that learning objectives should be presented to students explicitly, the MTEs commonly put emphasis on clear goals for lessons.

Task Quality

The central role of mathematical tasks is to "*promote mathematical reasoning and problem-solvin*g and allow *accessible entry points* and varied solution strategies" (NCTM, 2014, p. 17). While reviewing the microteaching videos, the MTEs commonly paid attention to the quality of the tasks in the lessons. For their microteaching demonstrations, the PSTs collaborated to develop the lesson plans and to design the mathematical tasks. Regarding task quality, this study found two components of microteaching: cognitive complexity and accessible entry points. We describe each component with some examples from the data.

Cognitive Complexity. The MTEs paid attention to the level of cognitive demand of tasks that the PSTs provided. It has been widely accepted that tasks with a high level of cognitive demand can impact the way students interact with mathematical content to build their conceptual understanding (Henningsen & Stein, 1997; Herbst, 2003; Smith & Stein, 1998). Tasks with lower levels of cognitive demand require students to memorize or reproduce facts without making connections to fundamental mathematical ideas. On the other hand, tasks with higher levels of cognitive demand require students to connect mathematical concepts and/or procedures and enable students to engage in higher order thinking such as explanation, justification, meta-cognition, and generalization.

The MTEs noticed the PSTs had implemented lower levels of cognitive demands across microteaching videos. They provided few opportunities for students to be engaged in complex and non-algorithmic thinking. For example, in the video lesson #4, PST *D* reactivated students' prior knowledge of circles such as the terminology and definition. The students only recalled the meaning of some terminologies such as radius, center, and chord. Then, the PST *D* presented the first main task about the property of tangent lines from a point to circle. Then he provided a triangle ABC with an inscribed circle (Figure 3) and prompted, "Find a pair of line segments with the same length."

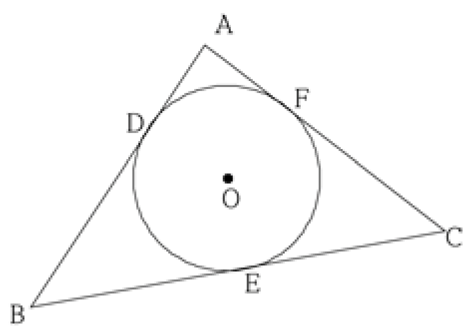

Figure 3. A task of finding segments in the video lesson #4

Most students wrote down the answers, "$\overline{AD} \simeq \overline{AF}$, $\overline{CF} \simeq \overline{CE}$, $\overline{BD} \simeq \overline{BE}$", then PST *D* moved on to the next task without going deeper in this relationship of the segments that students answered. The MTEs found this task failed to elicit students' mathematical problem-solving ability and mathematical reasoning. In fact, one MTE indicated that students had to be given an opportunity to explore the relationships between the pairs of segments they had answered. Another mentioned that having students conjecture their own ideas would be helpful for them to exert considerable cognitive effort rather than just identifying the sameness between segments.

Accessibility of tasks. The MTEs put importance on how a PST provided entry points, strategies, and/or exit points in dealing with tasks. Since students have a wide range of experiences in mathematics and in everyday life, it is important to use mathematical tasks that have flexible and accessible entry points. In other words, mathematical tasks should have various solution paths with some challenges along the way. This quality can be characterized as the openness (Yeo et al., 2022; Yeo, 2017). Frobisher (1994) described the quality of tasks as the methods of the solution rather than the solution itself. Multiple entry points allow diverse students to accommodate in learning situations and relieve anxiety to make them choose their preferred strategies. Therefore, students are expected to engage in the task in a way that makes sense to them.

In our analysis of videos, the MTEs took notice of the beginning phase of the task implementation. There was a wide spectrum of provision of entry points in the PSTs' microteaching. For example, in the video lesson #6, PST *F* elicited different strategies to solve a task about the property of inscribed angles. First, he showed the class a comic strip that they had a quest to partition a magic stone with exactly the same angles to escape the trap (Figure 4[3], left). The goal of this task was to figure out three congruent angles. Then, he provided the questions in the worksheet: "What information is given?" and "what mathematical concepts or properties can you use to complete the quest?" Lastly, the PST provided a paper-cut magic stone to each student so that they can find the solution more concretely (Figure 4, right).

[3] This figure was provided in the lesson #6 plan. The cartoon was illustrated by PST *F*.

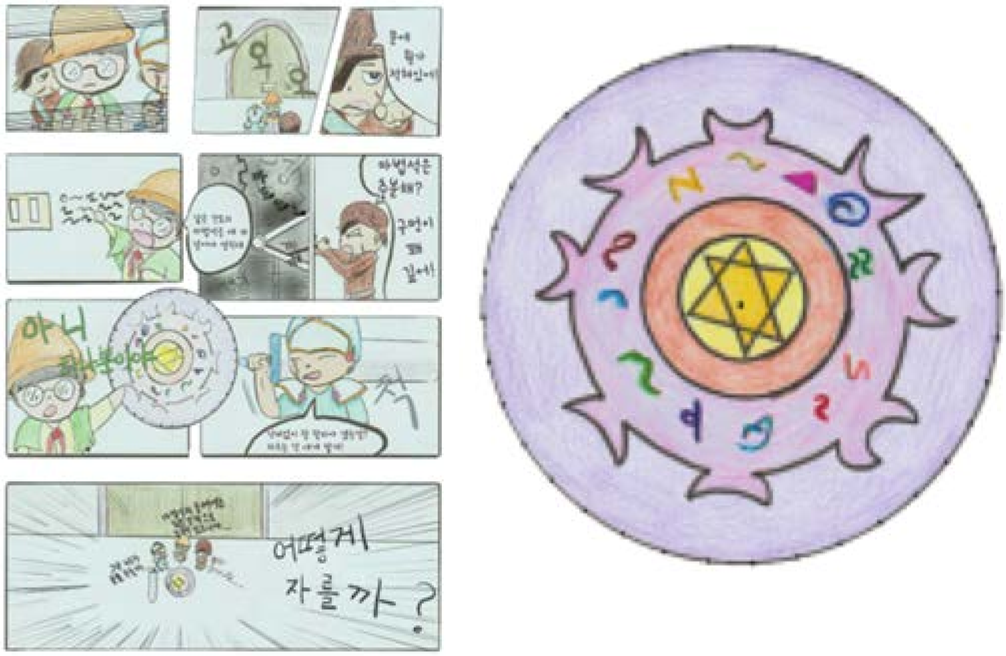

Figure 4. Magic stone task with a cartoon in the video lesson #6

In this task, every student had a chance to investigate the context of the task and review some related mathematical topics. To solve the task, the students partitioned the magic stone in their own ways. Even though the students knew the property of inscribed angles, each student had the freedom to select the size of the angle for splitting. One of the MTEs pointed out the potential of such a task to support students to access targeted mathematical concepts and skills meaningfully. Although some MTEs worried about the degree of freedom, all agreed that accessibility of tasks helped students take a different approach and think of various strategies to get a solution.

Teaching Practice

For this category, the MTEs examined how the PSTs orchestrated their instructions to make students actively engage in the lessons. First, we confer the teaching practices that emerged during the analysis. These practices include whether the PSTs used various mathematical tools, facilitated meaningful discourse, and built classroom cultures. Next, we explain what the MTEs have noticed about the PSTs' teaching practices with illustrative examples.

Mathematical Tools. One of the microteaching assignments required in the methods course was to develop mathematical tasks that use mathematical tools. The use of tools in the mathematics classroom is to promote participation in which students touch and move objects while they explore and develop mathematical ideas. These tools include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Most PSTs used tools such as a pencil and a paper, a ruler, a protractor, a compass, and/or a wax paper. For example, in the video lesson #2, PST *B* gave the students some sheets of wax paper to explore why the length of the chords did not change. However, he did not use the tool to clarify how the dots on the wax paper have the same intervals, when the number of dots between the chord was the same. When the MTEs watched the video clip, one of the MTEs pointed out hardly any PST encouraging students to select tools and to think about mathematics behind the effects of mathematical tool use. In the MTEs' notes after watching the video clips, most stated the use of mathematical tools. They mentioned that most PSTs asked students to use tools, but could hardly support students to use them effectively in a way that students make connections between mathematics and the experience of using the tools.

Mathematical Discourse. Participating in mathematical discourse has long been identified as an essential component of students' mathematics learning (NCTM, 1991, 2000, 2007, 2014). The benefits of engaging students in meaningful discourse include clarifying understandings, constructing convincing arguments, developing the language to express mathematical ideas, and learning to see things from other perspectives. Discourse about meaningful mathematics content is a fundamental way to develop deep conceptual understanding (NCTM, 2014). According to Staples and King (2017), three key functions of the teacher's role in engaging students in meaningful mathematical discourse are (i) eliciting student teaching including providing opportunities for students to generate ideas and then share their ideas with the class; (ii) supporting student-to-student exchanges about mathematical ideas including establishing a common knowledge base from which to work and helping students make sense of one another's ideas; and (iii) guiding and extending the mathematics including guiding the development of mathematical ideas, pursuing common misconceptions and ensuring appropriate disciplinary norms to advance the learning of the class.

The MTEs focused on the following three key discussion functions that PSTs should consider: (i) engaging students in explaining their mathematical reasoning in small groups and/or classroom situations, (ii) facilitating discussions among students that support making sense of a variety of strategies and approaches, and (iii) scaffolding classroom discussions so that connections between representations and mathematical ideas take place. The MTEs noticed that the PSTs had little experience in mathematics lessons with rich discourse. As mentioned earlier, most lessons were teacher-centered in which students were not given time to think about the questions, explain their ideas, critique other's thinking, and argue with one another. One MTE mentioned that the PSTs might not have enough experiences of group/peer discussions in their mathematics classrooms in the grade school or even in college. Moreover, the MTEs chorused the effectiveness of providing PSTs the opportunity to conduct microteaching with the actual students. Since the conversations with peer PSTs instead of actual students during the microteaching limits the expected student responses or reactions, microteaching with actual students would be desirable for the PSTs' more effective discourse experience in teaching situations.

Classroom Culture. The classroom cultures, norms, or environment is made by teachers and students as a community. We believe a desirable classroom environment is where students freely ask questions, discuss with other students, and respect each other's ideas. Such classroom environments should support students' problem-solving ideas (Hiebert et al., 1996), students' ownership of their own learning (Lampert, 1990), and the shared classroom norms to respect everyone's idea (Van de Walle, et al., 2009). The patterns of interactions are closely related to the classroom environment. The MTEs have observed that the interactions—between teacher and students, among students, between tasks and students—can be changed by the classroom environment.

While observing the microteaching videos of nine PSTs, the MTEs have noticed some patterns of PSTs' teaching practices related to establishing classroom cultures. Although they acknowledged PSTs might not have enough time for establishing relationships with students, the MTEs identified some collective patterns from most of the PSTs in establishing the classroom culture as follows. First, the PSTs hardly encourage students to ask questions. Most questions and answers came from the PSTs. Throughout the nine microteaching lessons, the MTEs have not observed any student asking the teachers a challenging question. Every question was dominantly asked by the PSTs during lessons. Second, the PSTs seemed not to expect students to answer their questions. Instead, the PSTs answered their own questions right away without waiting. For example, PST *A* on the video lesson #1 continuously asked the same questions, "Are you following?" repeatedly, without waiting for students' responses. He did not look at the students after asking 7 questions. Third, the PSTs in our data showed little wait time for their students' contemplation of the given questions by the PSTs. As Chapin and her colleagues (2009) asserted, wait time is critical not only for a student to contribute to the discussion but also to make their own conceptual learning. Regardless of the importance, it is not easy for teachers to adopt the wait time in their classrooms. During our sequential learning community meetings, wait time was continuously mentioned. Lastly, even if this microteaching setting with four students is optimized for small group discussion, the MTEs could not see a PST conducting a small group or partner discussion. Once PST *D* told the students to, "Try to solve it with your partner" but the students worked by themselves quietly.

The MTEs discussed that the patterns described above made it difficult for students to freely ask questions or to create an atmosphere of respect for each other's opinions in class. Although some PSTs try to build rapport with students (e.g., remembering students' names), the PSTs need support to build a desirable classroom environment in general. Since PSTs respond to their peers for their evaluation in microteaching with peers, MTEs may not clearly confirm whether classroom culture is created with appropriate teaching practices.

V. DISCUSSIONS AND CONCLUSIONS

For this study, six MTEs from regionally and culturally diverse institutions have engaged in professional discussions on PSTs' microteaching. They watched and analyzed the microteaching videos, discussed what they had noticed from the secondary PSTs' teaching practices with pupils during microteaching, and shared a professional vision for preservice teacher education. The MTEs' collaborative investigations of the PSTs' teaching practices contributed to characterize the MTEs' noticing as a professional vision for mathematics microteaching.

A way to investigate MTEs' noticing is found to be similar to noticing teachers' professional vision as in Sherin (2001). MTEs noticed PSTs' teaching as teachers observed and interpreted students in the class. This process is similar to studying a teacher's professional vision. Teachers focused on the subject, students, and environment. However, since MTEs should create the opportunity to improve teaching, MTEs had an interest in what makes an effective (clinical) environment for practicing teaching by PSTs. For teachers, teaching is praxis, whereas teaching is another subject to study for teacher educators. Professional development of teaching is a shared interest among teachers, researchers, and MTEs.

As Ho and Tan (2013) stated, *researchers* tend to "see notable aspects of the teacher's practices by way of coding and highlighting and organize seemingly disparate classroom events into a common analytical framework" whereas *teachers* focused on viewing "classroom practices and interactions in terms of pedagogy and praxis." The findings of this study confirm that *MTEs* communicated diverse perspectives and expertise as both researchers and teachers. As researchers, the MTEs focused on their research interest in PST education. Nonetheless it is worth noticing that some MTEs took mathematical tasks seriously. Some were interested in teacher questions or teaching decisions while others focused on interactions between PSTs and their students. We were able to learn these differences from self-reports. Despite these different interests, the MTEs in this study shared their thoughts about PSTs' practices from the microteaching videos and formed shared visions for PST's microteaching.

The MTEs in this study have assumed the combined role of a researcher and a teacher educator (a teacher of preservice teachers). For example, the MTEs were teacher educators when they wrote down the instances of noticeable teaching moments that (1) serve as examples to illustrate the best or emergent teaching practices for preservice teachers and (2) serve as evidence to evaluate PSTs' teaching qualities. As researchers, the MTEs were naturally attracted to the task of finding the convergent themes that connect multiple ideas in MTEs' analysis to the noticing on microteaching. The work described in this study is part of the large vision in teacher education and aims to understand what teacher educators notice, connect, and learn from microteaching as opposed to regular classroom teaching events. So it does not mean to represent a whole professional vision or does not substitute for an effective method of assessing PSTs' instructions.

The MTEs discussed MTE's noticing on PSTs' microteaching considering naturally drawn on three subjects of instruction– such as teacher, students, and mathematics contents shown in instructional triangle model (see Cohen and Ball, 1999). In this study, thanks to the nature of microteaching involving both the PSTs and the actual students provided while teaching and learning mathematics, the MTEs had the opportunity to examine the PSTs' teaching concerning the relationships of three aspects holistically. In sum, our findings suggest three categories for PSTs' microteaching drawn upon the MTEs' noticing: lesson structure, task quality, and teaching practices. We schematized the MTE's noticing around (see Figure 5) PST, students, and mathematics by examining teaching practices, lesson structure, and task quality as the relationships between the subjects (cf. Figure 2 Instructional Triangle in Microteaching). The MTEs noticed the category of lesson structure as focusing on interaction between PST and math contents, the category of task quality as focusing on interaction between students and math contents, and the category of teaching practices as focusing on interaction between PST and students in the PSTs' microteaching.

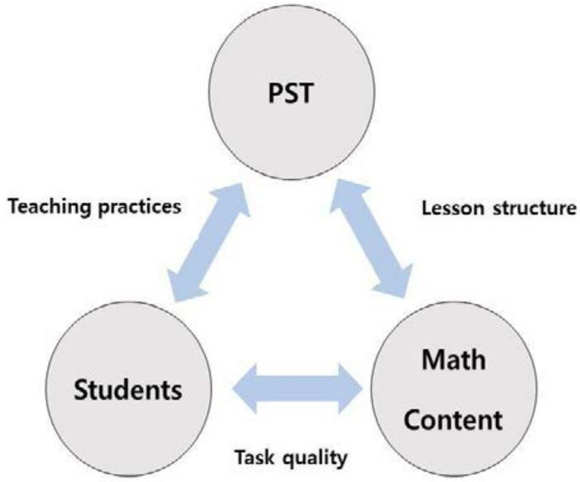

Figure 5. MTEs' noticing on microteaching using instructional triangle

From the MTEs' noticing on the PSTs' microteaching practices, we, as MTEs, had in-depth discussion on three categories–lesson structure, task quality, and teaching practices. In fact, PST, students, and mathematical content are intertwined and affect each other in teaching; however, we identified that the PSTs' microteaching was partially related to each subject in the instructional triangle. We noticed that the PSTs were unbalanced presenting three categories (teaching practices, lesson structure, and task quality) in their microteaching. In particular, the PSTs did not successfully connect the students and mathematical content in their microteaching– their focus was more on delivering mathematical content disregarding students' engagement to the content. We think that this is because the PSTs do not take all relevant factors (teacher, students, and math content) into account when teaching. Therefore, teacher education programs should provide PSTs comprehensive experience responding to the PST's unbalanced and disconnected understanding of the instructional triangle. In addition, acknowledging that microteaching could be a one-time teaching opportunity for PSTs, MTEs need a shared vision to make microteaching more systematic in regards with three subjects of instruction and their relationships.

Many teacher education programs require PSTs' microteaching, often with their peers, not with real students. PSTs with strong content knowledge may be able to practice structured lessons without students. However, there may be some limitations to improve their pedagogical content knowledge such as ability to teach *in response to* students' reactions or classroom situations or ability to change tasks *in response to* students' understanding. This type of microteaching could not give authentic teaching experience. For example, the PSTs in this study had taken more than 7 mathematics content courses before their microteaching and might hold a strong mathematics content knowledge. However, most of the PSTs did not show high quality of tasks due to the lack of focus on the interaction between students and math contents or between the PST themselves and the students. These results imply the need for more rigorous and coherent microteaching experiences, in which the PSTs plan and implement the practices by considering the relationships between/among teacher, students, and mathematical contents as a whole.

As for the emerging question why so many microteaching videos that we viewed showed teacher-centered lessons, the MTEs posited that when microteaching was conducted in front of peers in university classrooms instead of actual students, the PSTs could choose to focus on the teacher's actions and became teacher-centered. However, the MTEs could not conclude the causes of a teacher-centered lesson in this microteaching, because they also noticed some teacher-centered instructions during the microteaching with actual middle school students as well. The MTEs in this study only could assess whether microteaching performed by PSTs was teacher-centered or student-centered. What is more important is that MTEs should create a space where PSTs' microteaching is conducive for student-centered instruction.

It is a limitation of our study that different noticing may appear depending on the environment of microteaching. Since our study is in the context of one kind of microteaching, this framework can be used in other contexts, but the same results with this study cannot be guaranteed. Also, depending on what requirements a MTE sets for microteaching, the enacted teaching may be different.

Professional vision might not be easily accessible by teachers due to rapidity and unconsciousness (Sherin et al., 2008). MTEs' professional vision also has complexity in that an MTE plays the perennial role of the bridge of theory and practice. This study afforded the opportunity to better understand how individual views combine to shape the shared MTEs' noticing for a meaningful microteaching practice in methods. We have only taken the first step to develop MTEs' vision. Future research can investigate how MTEs implement new vision found in a learning community (i.e., intervention), reform microteaching in their programs, and how the online learning community supports or hinders the work.

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