J. Appl. Math. & Informatics Vol. **41***(2023), No. 2, pp. 265 - 272*
https://doi.org/10.14317/jami.2023.265

SOME EXTENSION ON HESITANT FUZZY MAXIMAL, MINIMAL OPEN AND CLOSED SETS

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Abstract. This article presents a novel notion of hesitant fuzzy cleanly covered in hesitant fuzzy topological spaces;moreover two strong hesitant fuzzy separation axioms are investigated. Based on fuzzy maximal open sets few properties of hesitant fuzzy cleanly covered are obtained. By dint of hesitant fuzzy minimal open and fuzzy maximal closed sets two strong hesitant fuzzy separation axioms are extended.

AMS Mathematics Subject Classification : 54A40, 03E72.
Key words and phrases : Hesitant fuzzy minimal open, hesitant fuzzy strongly regular, hesitant fuzzy strongly normal.

1. Introduction

Zadeh[9] established fuzzy set in 1965 and Chang[1] developed fuzzy topology in 1968. As an addendum to fuzzy sets, the notion hesitant fuzzy set introduced by Torra[8] in 2010. In 2019 Deepak et. al.[2] introduced hesitant fuzzy topological space and extended the study to hesitant connectedness and compactness in hesitant fuzzy topological space.

In this article, we obtain some properties and strong hesitant fuzzy separation axioms of hesitant fuzzy minimal and fuzzy maximal open sets.

Throughout this paper (X,τ) refers to hesitant fuzzy topological spaces.

2. Preliminaries

Definition 2.1. [7]*A proper nonzero hesitant fuzzy open set ξ of $\Psi$is called a*
(i)hesitant fuzzy minimal open set if ξ and h^0 are only hesitant fuzzy open sets contained in ξ.
(ii)hesitant fuzzy maximal open set if h^1 and ξ are only hesitant fuzzy open sets containing ξ.

Received January 12, 2022. Revised April 24, 2022. Accepted February 7, 2023.
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Theorem 2.2. [7] If ξ is a hesitant fuzzy maximal open set and υ be a hesitant fuzzy open subset, then either $\xi \vee \upsilon = h^1$ or $\upsilon < \xi$.

Theorem 2.3. [7] If ξ is a hesitant fuzzy minimal open set and υ be a hesitant fuzzy open subset, then either $\xi \wedge \upsilon = h^0$ or $\xi < \upsilon$.

Definition 2.4. [4] A hesitant fuzzy set h in X is a function $h : X \to P[0,1]$, where $P[0,1]$ represents the power set of [0,1].

We define the hesitant fuzzy empty set h^0(resp. whole set h^1) is a hesitant fuzzy set in X as follows: $h^0(x) = \phi$ (resp. $h^1(x) = [0,1]), \forall x \in X$. $HS(X)$ stands for collection of hesitant fuzzy set in X.

Definition 2.5. [3] Two hesitant fuzzy set $h_1, h_2 \in HS(X)$ such that $h_1(x) \subset h_2(x), \forall x \in X$, then h_1 is contained in h_2.

Definition 2.6. [3] Two hesitant fuzzy set h_1 and h_2 of X are said to be equal if $h_1 \subset h_2$ and $h_2 \subset h_1$.

Definition 2.7. [4] Let $h \in HS(X)$ for any nonempty set X. Then h^c is the complement of h which is hesitant fuzzy set in X such that $h^c(x) = [h(x)]^c = [0,1] \backslash h(x)$.

Definition 2.8. [5] Let (X, τ) be a hesitant fuzzy topological space. Let $x_\lambda \in H_p(X)$ and $N \in HS(\eta)$. Then the hesitant fuzzy neighbourhood N of x_λ is defined as if for an hestitant fuzzy set $U \in \tau$ such that $x_\lambda \in U \subset N$.

Definition 2.9. [5] Let X be a nonempty set. A hesitant fuzzy τ of subsets X is said to be hesitant fuzzy on X if

(i) $h^0, h^1 \in \tau$.

(ii) $\bigvee_{i \in J} h_i \in \tau$ for each $(h_i)_{i \in J} \subset \tau$.

(iii) $h_1 \cap h_2 \in \tau$ for any $h_1,\ h_2 \in \tau$.

The pair (X, τ) is called hesitant fuzzy topological space. The members of τ are called hesitant fuzzy open sets in X. A hesitant fuzzy set h in X is hesitant fuzzy closed set (in short hesitant fuzzy cover) in (X, τ) if $h^c \in \tau$.

Definition 2.10. [5] Let (X, τ) be a hesitant fuzzy topological space and $h \in X$. Then the hesitant fuzzy interior and closure are respectively defined as:

(i)$int_H(h) = \bigvee \{U \in X : U \subset h\}$.

(ii)$cl_H(h) = \bigwedge \{V \in X : h \subset V\}$.

3. Hesitant Fuzzy Maximal, Minimal Open and Closed Sets

Definition 3.1. *A hesitant fuzzy cover* $\mathbf{G}$ *of* X *is said to be a hesitant fuzzy minimal cover if for any* $\delta \in \mathbf{G}$, $\mathbf{G} - \{\delta\}$ *is not a hesitant fuzzy cover of* X*. Then* $\mathbf{G}$ *is called a hesitant fuzzy minimal open(resp.closed) cover if each member of* $\mathbf{G}$ *is hesitant fuzzy open(resp. closed).*

If $\mathbf{G}$ is a hesitant fuzzy minimal open cover of X, then there do not exist distinct $\delta, \gamma \in \mathbf{G}$ such that $\gamma < \delta$. Further if a hesitant fuzzy open cover $\mathbf{G}$ of

X having two distinct hesitant fuzzy open sets δ,γ such that $\gamma < \delta$, then $\mathbf{G}$ is not a hesitant fuzzy minimal open cover of X. Each cover with hesitant fuzzy minimal open of a space is finite and each cover with hesitant fuzzy open of a space having a finite hesitant fuzzy minimal open cover.

Definition 3.2. *A hesitant fuzzy cover* $\mathbf{G}$ *of* X *is said to be hesitant fuzzy disconnected if for each* $\delta \in \mathbf{G}$*, there exists a* $\gamma \in \mathbf{G}$ *such that* $\delta \wedge \gamma = h^0$*.*

Theorem 3.3. A hesitant fuzzy minimal open cover consists of a hesitant fuzzy minimal open set is hesitant fuzzy disconnected.

Proof. Let $\mathbf{G}$ be a hesitant fuzzy minimal open cover of X. Take $\delta \in \mathbf{G}$ be a hesitant fuzzy minimal open set. As δ is a proper nonzero hesitant fuzzy open set and $\mathbf{G}$ is a hesitant fuzzy cover of X, there could be a minimum of one more hesitant fuzzy open set $\gamma \in \mathbf{G}$. By Theorem 3.4, we could have $\delta \wedge \gamma = h^0$ or $\delta \gneqq \gamma$. $\mathbf{G}$ being a hesitant fuzzy minimal open cover, $\delta \gneqq \gamma$ is not possible. □

Corollary 3.4. A hesitant fuzzy minimal open cover having only hesitant fuzzy minimal open sets is hesitant fuzzy disconnected.

The succeeding definiton is required to have not less than of two nonzero hesitant fuzzy set in a hesitant fuzzy open cover of a hesitant fuzzy topological space.

Definition 3.5. *A hesitant fuzzy topological space* X *is said to be hesitant fuzzy cover if each hesitant fuzzy open cover of* X *has a hesitant fuzzy minimal open cover comprising of exactly two hesitant fuzzy open sets.*

Consequently a hesitant fuzzy cover topological space is a hesitant fuzzy compact space. The reverse implication is false.

Example 3.6. *Let* $X = \{a, b, c\}$ *and* $\tau = \{h^0, h_1, h_2, h_4, h^1\}$ *where*
$h_1(a) = [0, 0.3]$,$h_1(b) = [0, 0.5]$,$h_1(c) = [0, 0.7]$
$h_2(a) = [0, 0.3)$,$h_2(b) = [0, 0.5)$,$h_2(c) = [0, 0.7)$
$h_3(a) = (0.3, 1]$,$h_3(b) = (0.5, 1]$,$h_3(c) = (0.7, 1]$
$h_4(a) = [0, 0.3) \vee (0.3, 1]$,$h_4(b) = [0, 0.5) \vee (0.5, 1]$,$h_4(c) = [0, 0.7) \vee (0.7, 1]$
$h_5(a) = [0.3, 1]$,$h_5(b) = [0.5, 1)$,$h_5(c) = [0.7, 1]$ *and*
$h_6(a) = \{0.3\}$,$h_6(b) = \{0.5\}$,$h_6(c) = \{0.7\}$
Hence (X, τ) *is hesitant fuzzy cleanly covered and the space having no hesitant fuzzy maximal and minimal open sets.*

Example 3.7. From example 3.6, consider the hesitant fuzzy topology $\tau = \{h^0, h_1, h_2, h_3, h_4, h_5, h_6, h^1\}$. The hesitant fuzzy space (X, τ_1) is hesitant fuzzy compact but not hesitant fuzzy cleanly covered because it is having many hesitant fuzzy minimal open and maximal open sets which covers h^1.

Theorem 3.8. If each hesitant fuzzy open cover of X contains a hesitant fuzzy maximal open set, then X is hesitant fuzzy cleanly covered.

Proof. Let $\mathbf{G}$ be a hesitant fuzzy open cover of X and $\xi \in \mathbf{G}$ be a hesitant fuzzy maximal open set. Choose if another hesitant fuzzy maximal open set $\sigma \in \mathbf{G}$, $\sigma \neq \xi$, it could be $\xi \vee \sigma = h^1$ by Theorem 3.4 . Hence $\{\xi, \sigma\}$ forms hesitant fuzzy subcover of $\mathbf{G}$ for X. Let ξ be the only hesitant fuzzy maximal open set in $\mathbf{G}$. In order to cover X, ξ remains to be a proper nonzero hesitant fuzzy open set and minimum of one more hesitant fuzzy open set $\sigma \in \mathbf{G}$, $\sigma \neq \xi$. If $\sigma = h^1$, then $\mathbf{G}$ is a trivial hesitant fuzzy open cover of X. Hence we assume that $\sigma \neq h^1$. By Theorem 3.4, we could have $\xi \vee \sigma = h^1$ or $\sigma \lneqq \xi$. If for all $\Psi \in \mathbf{G}$, we could have $\Psi \lneqq \xi$, then $\mathbf{G}$ cannot be a hesitant fuzzy open cover of X. Since it implies that there exists a hesitant fuzzy open set $\sigma \in \mathbf{G}$, $\sigma \neq \xi$ such that $\xi \vee \sigma = h^1$.

Recall that a family $\mathbf{G}$ of hesitant fuzzy subsets of X is called hesitant fuzzy locally finite if each $x_v \in X$ has a hesitant fuzzy neighborhood meeting only hesitant fuzzy finitely many members of $\mathbf{G}$. Also if each $\{\Psi_s \mid s \in \Pi\}$ is a hesitant fuzzy locally finite family of hesitant fuzzy sets in X, then $Cl(\bigvee_{s\in\Pi} \Psi_s) = \bigvee_{s\in\Pi} Cl(\Psi_s)$. □

Theorem 3.9. If $\{\Psi_s \mid s \in \Pi\}$ is a family of distinct hesitant fuzzy minimal open sets in a hesitant fuzzy locally finite space X, then $Cl(\bigvee_{s\in\Pi} \Psi_s) = \bigvee_{s\in\Pi} Cl(\Psi_s)$.

Proof. As Ψ_s is a hesitant fuzzy minimal open set for each $s \in \Pi$, we have $\Psi_{s_1} \wedge \Psi_{s_2} = h^0$ for $s_1, s_2 \in \Pi$ with $s_1 \neq s_2$. For each $x_v \in X$, there exists a finite hesitant fuzzy open set Ψ such that $x_v \in \Psi$. Since Ψ is a finite hesitant fuzzy open set and $\Psi_{s_1} \wedge \Psi_{s_2} = h^0$ for $s_1, s_2 \in \Pi$ with $s_1 \neq s_2$, Ψ intersects only finitely many members of $\{\Psi_s \mid s \in \Pi\}$. Hence $\{\Psi_s \mid \Psi \in \Pi\}$ is hesitant fuzzy locally finite. □

Definition 3.10. *([7]) A proper nonzero hesitant fuzzy cover β of X is said to be a hesitant fuzzymic set if any hesitant fuzzy cover set which is contained in β is h^0 or β.*

Theorem 3.11. ([7]) If β is a hesitant fuzzymic set and ϱ is any hesitant fuzzy cover set, then either $\beta \wedge \varrho = h^0$ or $\beta < \varrho$.

Definition 3.12. *([7]) A proper nonzero hesitant fuzzy cover β of X is said to be a hesitant fuzzy maximal set if any hesitant fuzzy cover set which contains β is h^1 or β.*

Theorem 3.13. ([7]) If β is a hesitant fuzzy maximal set and ϱ is a hesitant fuzzy cover set, then either $\beta \vee \varrho = h^1$ or $\varrho < \beta$. xd

"As the proof of succeeding Theorem 3.14, Corollary 3.15, Theorem 3.16 and Theorem 3.17 are analogous to Theorem 3.3, Corollary 3.4, Theorem 3.8 and Theorem 3.9, then proofs are omitted."

Theorem 3.14. A hesitant fuzzymic cover having hesitant fuzzymic set is hesitant fuzzy disconnected.

Corollary 3.15. A hesitant fuzzymic cover having of only hesitant fuzzymic sets is hesitant fuzzy disconnected.

Theorem 3.16. If each hesitant fuzzy cover $\mathfrak{F}$ of X contains a hesitant fuzzy maximal set β, then there exists an $\varrho \in \mathfrak{F}$ such that $\beta \vee \varrho = h^1$.

Theorem 3.17. If $\{\Psi_s \mid s \in \Pi\}$ is a family of distinct hesitant fuzzymic sets in a fuzzy locally finite space X, then $Cl(\bigvee_{s\in\Pi} \Psi_s) = \bigvee_{s\in\Pi} Cl(\Psi_s)$.

Theorem 3.18. Let η and β be hesitant fuzzy closed set in X such that $\eta \wedge \beta \neq h^0$. Then $\eta \wedge \beta$ is a hesitant fuzzymic set in $(\eta, \mathfrak{F}_\eta)$ if β is a hesitant fuzzymic set in $(X, \mathfrak{F})$.

Proof. Analogous to "proof of Theorem 4.14 [6]". □

4. Two Strong Hesitant Fuzzy Separation Axioms

Suppose δ and γ are hesitant fuzzy minimal open sets in X, then $\delta \wedge \gamma = h^0$[7]. By dint of this notion the two succeeding strong hesitant fuzzy separation axioms is introduced.

Definition 4.1. *A hesitant fuzzy topological space X is said to be hesitant fuzzy strongly regular iff for every singleton in X and every hesitant fuzzy cover set ϱ in X such that $x_\upsilon \leq co\ \varrho$, there exists distinct hesitant fuzzy minimal open sets δ, γ such that $x_\upsilon \leq \delta$, $\varrho \leq \gamma$ and $\delta \leq co\ \gamma$.*

Clearly, a hesitant fuzzy strongly regular topological space is hesitant fuzzy regular.

Definition 4.2. *A hesitant fuzzy topological space X is said to be hesitant fuzzy strongly normal iff for any two hesitant fuzzy cover sets β, ϱ such that $\beta \leq co\ \varrho$, there exists two disjoint hesitant fuzzy minimal open sets δ, γ such that $\beta < \delta, \varrho \leq \gamma$ and $\delta \leq co\ \gamma$.*

For a hesitant fuzzy subset ξ of a hesitant fuzzy topological space X, we define

$$MinInt_H(\xi) = \begin{cases} h^0 \text{ if } \xi \text{ contains no hesitant fuzzy minimal open set} \\ \bigvee \{\upsilon \mid \upsilon \text{ is a hesitant fuzzy minimal open set contained in } \xi\} \end{cases}$$

$$MaxCl_H(\xi) = \begin{cases} h^1 \text{ if } \xi \text{ contained in no hesitant fuzzy maximal set} \\ \bigwedge \{\beta \mid \beta \text{ is a hesitant fuzzy maximal set containing } \xi\} \end{cases}$$

The union and intersection of finitely many distinct hesitant fuzzy minimal open and hesitant fuzzy maximal open sets is not hesitant fuzzy minimal open and hesitant fuzzy maximal open respectively. Since $MinInt_H(\xi)$ (resp. $MaxCl_H(\xi)$) may not be hesitant fuzzy minimal open (resp. hesitant fuzzy maximal).

Example 4.3. *Let $X = \{a, b, c\}$ and $\tau = \{h^0, h_1, h_2, h_4, h^1\}$ where*
$h_1(a) = \{0.2\}$,$h_6(b) = \{0.3\}$,$h_6(c) = \{0.4\}$
$h_2(a) = [0, 0.2)$,$h_2(b) = [0, 0.3)$,$h_2(c) = [0, 0.4)$

$h_3(a) = (0, 0.2]$,$h_3(b) = (0, 0.3]$,$h_3(c) = (0, 0.4]$
$h_4(a) = [0.2, 1]$,$h_4(b) = [0.3, 1]$,$h_4(c) = [0.4, 1]$ and
$h_5(a)(0, 0.2)$,$h_5(b) = (0, 0.3)$,$h_5(c) = (0, 0.4)$.
Hence (X, τ) is hesitant fuzzy topological space. Since $MinInt_H(h_5) = h_3$ (resp. $MaxCl_H(h_5) = h_3$) may not be hesitant fuzzy minimal open (resp. hesitant fuzzy maximal) set.

Theorem 4.4. Let $\xi \in X$ be a hesitant fuzzy subset. Then $h^1 - MinInt_H(\xi) = MaxCl_H(h^1 - \xi)$.

Proof. Consider $MinInt_H(\xi) = h^0$. This would mean that ξ containing no hesitant fuzzy minimal open set. If possible, suppose that $h^1 - \xi$ contains a hesitant fuzzy maximal set β. Then $h^1 - \beta$ is a hesitant fuzzy minimal open set contained in ξ, a contradiction. Now we could have $h^1 - MinInt_H(\xi) = MaxCl_H(h^1 - \xi)$. Assume that ξ contains a hesitant fuzzy minimal open set υ. Then $h^1 - \xi < h^1 - \upsilon$. In addition, $h^1 - \upsilon$ is a hesitant fuzzy maximal set. Hence we get $MaxCl_H(h^1 - \xi) \neq h^1$ iff $MinInt_H(\xi) \neq h^0$. It could be easy to verify that if $\{\upsilon\}$ is a family of all hesitant fuzzy minimal open sets contained in ξ, then $\left\{h^1 - \upsilon\right\}$ is the family of all hesitant fuzzy maximal sets containing $h^1 - \xi$ and vice-versa. Therefore

$$\begin{aligned} & h^1 - MinInt_H(\xi) = h^1 - \bigvee \{\upsilon \mid \upsilon \text{ is hesitant fuzzy minimal open contained in } \xi\} \\ & = \bigwedge \left\{h^1 - \upsilon \mid h^1 - \upsilon \text{ is hesitant fuzzy maximal containing } h^1 - \xi\right\} \\ & = MaxCl_H(h^1 - \xi). \end{aligned}$$

□

Theorem 4.5. For a hesitant fuzzy subset ξ of X, $MinInt_H(\xi)$ is hesitant fuzzy minimal open iff ξ contains one and only one hesitant fuzzy minimal open set.

Proof. The sufficiency follows easily. Let $MinInt_H(\xi)$ be hesitant fuzzy minimal open set and ξ contains two hesitant fuzzy minimal open sets υ, σ. Then, $MinInt_H(\xi) = \upsilon \vee \sigma < \xi$. Since $\upsilon, \sigma < \upsilon \vee \sigma$ and $MinInt_H(\xi)$ is hesitant fuzzy minimal open , we have $\upsilon = \upsilon \vee \sigma$ and $\sigma = \upsilon \vee \sigma$. $\upsilon = \upsilon \vee \sigma$ implies that $\sigma < \upsilon$ and $\sigma = \upsilon \vee \sigma$ implies that $\upsilon < \sigma$. Hence we have $\upsilon = \sigma$.

"Theorem 4.6 is a dual of Theorem 4.5. The proof of the theorem is omitted as the proof is similar to that of Theorem 4.5." □

Theorem 4.6. For a hesitant fuzzy subset ξ of X, $MaxCl_H(\xi)$ is hesitant fuzzy maximal iff ξ contained in one and only one hesitant fuzzy maximal set.

Theorem 4.7. A hesitant fuzzy topological space X is fuzzy strongly regular iff for each hesitant fuzzy open set σ and each $x_\upsilon \in \sigma$, there exists a hesitant fuzzy minimal open set δ and a hesitant fuzzy maximal set β such that $x_\upsilon \in \delta < MaxCl_H(\delta) < \beta < \sigma$.

Proof. Let X be a hesitant fuzzy strongly regular space and σ be a hesitant fuzzy open set. For $x_\upsilon \in \sigma$, we obtain by hesitant fuzzy strong regularity of X two distinct hesitant fuzzy minimal open sets δ, γ such that $x_\upsilon \in \delta$, $h^1 - \sigma < \gamma$. δ, γ being distinct hesitant fuzzy minimal open sets, we have $\delta \wedge \gamma = h^0$. Now $\delta \wedge \gamma = h^0 \Rightarrow \delta < h^1 - \gamma$. Since γ is hesitant fuzzy minimal open, $h^1 - \gamma$ is hesitant fuzzy maximal and so $MaxCl_H(h^1 - \gamma) = h^1 - \gamma$. Thus $MaxCl_H(\delta) < MaxCl_H(h^1 - \gamma) = h^1 - \gamma < \sigma$. Putting $\beta = h^1 - \gamma$, we see that β is a hesitant fuzzy maximal and $x_\upsilon \in \delta < MaxCl_H(\delta) < \beta < \sigma$.

Conversely, let β be a hesitant fuzzy cover and $x_\upsilon \in X$ such that $x_\upsilon \notin \beta$. As $h^1 - \beta$ is a hesitant fuzzy open set with $x_\upsilon \in h^1 - \beta$, there exist a hesitant fuzzy minimal open set δ and a hesitant fuzzy maximal set ϱ such that $x_\upsilon \in \delta < MaxCl_H(\delta) < \varrho < h^1 - \beta$. We put $\gamma = h^1 - \varrho$. Then γ is a hesitant fuzzy minimal open set with $\beta < \gamma$ and $\delta \wedge \gamma = h^0$. □

Theorem 4.8. A hesitant fuzzy topological space X is hesitant fuzzy strongly normal iff for a hesitant fuzzy cover set β and for a hesitant fuzzy open set σ with $\beta < \sigma$, there exist a hesitant fuzzy minimal open set δ and a hesitant fuzzy maximal set ϱ such that $\beta < \delta < MaxCl_H(\delta) < \varrho < \sigma$.

Proof. Analogous to proof of Theorem 4.7. □

Conflicts of interest : The authors declare no conflict of interest.

Acknowledgments : The authors are thankful to the referees for their suggestions and commands to develop this manuscript.

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