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FUZZY TRANSPORTATION PROBLEM IS SOLVED UTILIZING SIMPLE ARITHMETIC OPERATIONS, ADVANCED CONCEPT, AND RANKING TECHNIQUES

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Abstract. In this article, a new penalty and different ranking algorithms are used to find the lowest transportation costs for the fuzzy transportation problem. This approach utilises different ranking techniques when dealing with triangular fuzzy numbers. Also, we find that the fuzzy transportation solution of the proposed method is the same as the Fuzzy Modified Distribution Method (FMODI) solution. Finally, examples are used to show how a problem is solved.

AMS Mathematics Subject Classification : 65D30, 65D32.
Key words : Fuzzy transportation problem, initial basic feasible solution (IBFS), fuzzy modified distribution method (FMODI), different ranking techniques, triangular fuzzy numbers.

1. Introduction

Fuzzy transportation problems have transportation costs, supply, and demand that are fuzzy. Several mathematical models require ranking fuzzy numbers. Zadeh initially developed the concept of fuzzy sets. In recent years, a new ranking technique based on the region between the fuzzy number's centroid and the original point was proposed by Ta-Chung Chu et al. [1]. The fuzzy number centroid equations for trapezoidal and triangular fuzzy numbers were provided by Chin, K. S., et al. [2] They also justified the formulas using analytical geometry. Normal and non-normal, invertible and non-invertible triangular and trapezoidal numbers can be efficiently ranked by Shuo-Yan Chou et al. [3]. As one of the most widely used methods in the application of ranking fuzzy numbers, Nazirah Ramli et al. [4] evaluated and analysed the ranking approaches based on the centroid idea. S. Rezvani [5] proposed a new method for ranking two generalised trapezoidal fuzzy numbers. Fuzzy Russell's method was combined with Yager's

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ranking approach by S. Narayanamoorthy et al. [6]. To solve the fuzzy transportation problem using fuzzy number ranking, Hadi Basirzadeh [7] proposed a simple but effective parametric approach. For the fuzzy optimal solution to the given fuzzy transportation problem, K. Ganesan et al. [8] proposed a new strategy using triangular fuzzy numbers without converting the problem into a classic transportation problem. Darunee Hunwisai [9] offered a solution to the fuzzy transportation problem (FTP) using Robust's ranking methodology to estimate the representative value of the fuzzy number. A. Edward Samuel et al. [10] came up with the Modified Vogel's Approximation Method (MVAM) to help solve fuzzy transportation problems. Our goal is to find the least amount of transportation. Sections 2–5 contain basic definitions. Section 6 will introduce a novel technique, and Section 7 will apply it to the transportation issue. Results, discussion, and conclusion are in sections 8 and 9

2. MATHEMATICAL REPRESENTATION OF THE FUZZY TRANSPORTATION PROBLEM

Assume that the fuzzy transportation has r origins and e destinations with fuzzy numbers [11]. where $\widetilde{K_a}(\widetilde{K_a} \geqq 0)$ denotes source availability at a, and $\widetilde{m_j}(\widetilde{m_j} \geqq 0)$ denotes destination requirement at j. We'll call this a unit of fuzzy transportation cost $\widetilde{c_{aj}}$ and $\widetilde{x_{aj}}$denote the total number of fuzzy units suggested for transportation from source to destination [12-13]. This issue requires minimum cost transportation and a feasible path. The fuzzy transportation mathematical model is shown below. The following is the mathematical model for the fuzzy transportation problem.

$$Minimize \quad z = \sum_{a=1}^{a=r} \sum_{j=1}^{j=e} \widetilde{c_{aj}}\widetilde{x_{aj}} \tag{1}$$

$$Subject\ to \sum_{j=1}^{e} \widetilde{x_{aj}} = \widetilde{K_a}; a = 1, 2, ...r \tag{2}$$

$$\sum_{a=1}^{r} \widetilde{x_{aj}} = \widetilde{m_j}; j = 1, 2, ...e \tag{3}$$

$$\widetilde{X}_{aj} \ = 0 for a = 1, 2, ...r, j = 1, 2...e \tag{4}$$

3. Fuzzy Set

A fuzzy set $\tilde{D}$ in x (a set of real numbers) is a set of order pairs $\tilde{D} = (x, \mu\tilde{D}(x))/x\epsilon X$, where $\mu\tilde{D}$ (x) is called the membership function of x in $\tilde{D}$ which maps x to [0,1] .

4. TRIANGULAR FUZZY NUMBER

A fuzzy number $\tilde{D}$ = (t, m, s) is said to be a triangular fuzzy number as shown in figure 1 if its membership function is described by the equation below, which mappings x to [0,1].

$$\mu\tilde{D}(x) = \begin{Bmatrix} \frac{x-t}{m-t}, t \leqq x \leqq m \\ 1, x = m \\ \frac{s-x}{s-m}, m \leqq x \leqq s \\ 0, otherwise \end{Bmatrix}$$

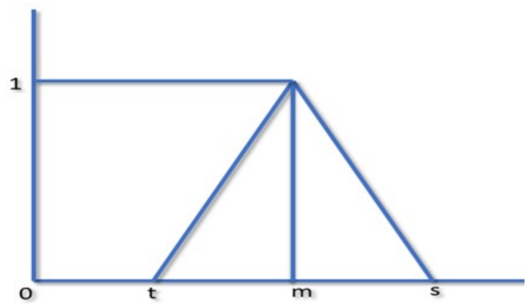

FIGURE 1

4.1. OPERATIONS OF TRIANGULAR FUZZY NUMBERS. It is considered that $\tilde{D}1$ = (t1, m1, s1) and $\tilde{D}2$ = (t2, m2, s2) are two triangular fuzzy numbers[14]. The basic arithmetic of a fuzzy triangular number is defined by the following expressions:

Addition:

$$\tilde{D}_1 + \tilde{D}_2 = (t1, m1, s1) + (t2, m2, s2) = (t1 + t2, m1 + m2, s1 + s2)$$

Subtraction:

$$\tilde{D}_1 - \tilde{D}_2 = (t1, m1, s1) - (t2, m2, s2) = (t1 - s2, m1 - m2, s1 - t2)$$

Multiplication:

$$\tilde{D}_1 * \tilde{D}_2 = (t1, m1, s1)(t2, m2, s2) = (t1R(\tilde{D}_2)), m1R(\tilde{D}_2)), s1R(\tilde{D}_2)) if R(\tilde{D}_2) \geqq 0$$

$$\tilde{D}_1 * \tilde{D}_2 = (t1, m1, s1)(t2, m2, s2) = (s1R(\tilde{D}_2)), m1R(\tilde{D}_2)), t1R(\tilde{D}_2)) if R(\tilde{D}_2) < 0$$

5. DIFFERENT RANKING OF TRIANGULAR FUZZY NUMBERS

1. AVERAGE RANKING TECHNIQUE OF TRIANGULAR FUZZY NUMBERS The triangular fuzzy number's ranking function $\tilde{D}$ = (t, m, s). The mapping from the set of all fuzzy numbers to the set of real numbers is defined as

$$R(\widetilde{D}) = \frac{t+m+s}{3} \tag{5}$$

2. THE RANKING FUNCTION OF THE CENTROIED OF GENERALIZED TRIANGULAR FUZZY NUMBERS The centroid of the triangle fuzzy number $\tilde{D}$ = (t, m, s) as $G_{\tilde{D}} = \left(\frac{t+m+s}{3}, \frac{w}{3}\right)$. The generalized triangular fuzzy number's ranking function $\tilde{D}$ = (t, m, s;w). The mapping from the set of all fuzzy numbers to the set of real numbers is defined as

$$R(\widetilde{D}) = \left(\frac{t+m+s}{3}, \frac{w}{3}\right) \tag{6}$$

$$R(\widetilde{D}) = \left(\frac{t+m+s}{3}, \frac{1}{3}\right) where\ w = 1 \tag{7}$$

3. GRADED MEAN INTEGRATION REPRESENTATION OF CHEN AND HSIEH RANKING OF TRIANGULAR FUZZY NUMBERS The triangular fuzzy number's ranking function $\tilde{D}$ = (t, m, s). The mapping from the set of all fuzzy numbers to the set of real numbers is defined as

$$R(\widetilde{D}) = \left(\frac{t+4m+s}{6}\right) \tag{8}$$

3. ADAMO AND CAMPOS et al. RANKING OF TRIANGULAR FUZZY NUMBERS The triangular fuzzy number's ranking function $\tilde{D}$ = (t, m, s) . The mapping from the set of all fuzzy numbers to the set of real numbers is defined as

$$R(\widetilde{D}) = am + (1-\alpha)s, \alpha\epsilon\ [0,1] \tag{9}$$

$$R(\widetilde{D}) = 0.5m + (1-0.5)\ s\ with\ \alpha = 0. \tag{10}$$

5. YAGER'S CENTER OF GRAVITY RANKING OF TRIANGULAR FUZZY NUMBERS The triangular fuzzy number's ranking function $\tilde{D}$ = (t, m, s) .The mapping from the set of all fuzzy numbers to the set of real numbers is defined as

$$R(\widetilde{D}) = \left(\frac{t+m+s}{3}\right) \tag{11}$$

6. KAUFMANN et al AND CHEN RANKING OF TRIANGULAR FUZZY NUMBERS The triangular fuzzy number's ranking function $\tilde{D}$ = (t, m, s) .The

mapping from the set of all fuzzy numbers to the set of real numbers is defined as

$$R(\widetilde{D}) = \left(\frac{t + 2m + s}{4}\right) \tag{12}$$

7. LIOU et al RANKING OF TRIANGULAR FUZZY NUMBERS with $\alpha = 0.3$
The triangular fuzzy number's ranking function $\tilde{D}$ = (t, m, s) . The mapping from the set of all fuzzy numbers to the set of real numbers is defined as

$$R(\widetilde{D}) = \left(\frac{\alpha s + m + (1 - \alpha)t}{2}\right) where\ \alpha = 0.3 \tag{13}$$

8. LIOU et al RANKING OF TRIANGULAR FUZZY NUMBERS with α=0.5
The triangular fuzzy number's ranking function$\tilde{D}$ = (t, m, s) . The mapping from the set of all fuzzy numbers to the set of real numbers is defined as

$$R(\widetilde{D}) = \left(\frac{\alpha s + m + (1 - \alpha)t}{2}\right) where\ \alpha = 0.5 \tag{14}$$

9. LIOU et al RANKING OF TRIANGULAR FUZZY NUMBERS with α=0.5
The triangular fuzzy number's ranking function$\tilde{D}$ = (t, m, s) . The mapping from the set of all fuzzy numbers to the set of real numbers is defined as

$$R(\widetilde{D}) = \left(\frac{\alpha s + m + (1 - \alpha)t}{2}\right) where\ \alpha = 0.9 \tag{15}$$

10. ROBUST'S RANKING TECHNIQUES: The ranking approach meets the requirements of compensation, linearity, and addictiveness in a more accurate way[15-16]. The results of the proposed approach have been shown to be consistent with human perception. If $\tilde{D}$ is a fuzzy number, the ranking is defined as follows:

$$R(\widetilde{D}) = \int_0^1 (0.5)(a_a^L, a_a^U)da, (a_a^L, a_a^U) = \{(m-t)\alpha+t, s-(s-m)\alpha\}, \alpha\epsilon[0,1] \tag{16}$$

where (a_a^L, a_a^U) is the α -cut of the triangular fuzzy number $\tilde{D}$.

6. PROPOSED METHOD

Step-1: Using the transportation problem, prepare the transportation table. Following that, the cost of transportation is included in the allocation table. Step-2: Check whether the given problem is balanced or not. Proceed to the next level if it is balanced. If not, convert it to balance with the help of by adding dummy row or dummy column. Step-3: Add supply, the first minimum cost, and the second minimum cost to each row. This added value is divided by the number of columns in each row and written on the side as the row penalty. Step-4: Add demand, the first minimum cost, and the second minimum cost to each column. The column penalty is calculated by dividing the added value by the number of rows in each column and is written at the bottom. Step-5: Pick the biggest value from these (row penalty and column penalty). We must assign the smallest supply/demand to the smallest element of the specified row/column. Delete the columns or rows that relate to the supply or demand. Step-6: If the condition achieved in step 5 is incongruent, that is, if the highest value is tied, choose the value with the smallest element. If there is a tie for the least element, allocate the least element with the lowest supply/demand. Step-7: The same steps from steps 3 to 6 are repeated until all supply and demand are met. Step-8: Next, different techniques are used to turn the fuzzy transportation problem into a crisp value problem. Finally, the minimal transportation cost is determined as the product of the cost and the supply/demand values assigned to it. Algorithm key points For example, if the row or column values are 3,4,6 and 5,5,7, respectively, the First Minimum =3, the Second Minimum =4, and the First Minimum =5, the Second Minimum =5.

7. NUMERICAL EXAMPLE

A production is produced by three factories F1, F2, and F3. Their unit production costs are Rs (2, 2.25, 2.50), Rs (3, 3.10, 3.20), and Rs (1, 1.15, 1.30) respectively. The production capacities of the factories are (10,20,30), (15,30,45) and (5,10,15) units respectively. The product is supplied to three stores with S1, S2 and S3 requirements, which are (7,12,17), (10,25,40) and (13,23,33) respectively. Unit cost of transportation are given below.

	Stores				Capacity
Factory		S1	S2	S3	
	F1	(1,2,3)	(2,4,6)	(4,7,10)	(10,20,30)
	F2	(7,11,15)	(11,16,21)	(16,22,28)	(15,30,45)
	F3	(22,29,36)	(29,37,45)	(37,46,55)	(5,10,15)
	Requirement	(7,12,17)	(10,25,40)	(13,23,33)	(30,60,90)

Find the transportation plan such that the total production and transportation cost is minimum. **Solution:**
The above fuzzy transportation table is solved using the proposed method, step 1. The following table consists of both production costs and transportation costs (Fuzzy production costs + Fuzzy transportation costs).

	Stores				
Factory		S1	S2	S3	Capacity
	F1	(3, 4.25, 5.50)	(4, 6.25, 8.50)	(6, 9.25, 12.50)	(10,20,30)
	F2	(10,14.10,18.20)	(14,19.10,24.20)	(19,25.10,31.20)	(15,30,45)
	F3	(23,30.15,37.30)	(30,38.15,46.30)	(38,47.15,56.30)	(5,10,15)
	Requirement	(7,12,17)	(10,25,40)	(13,23,33)	(30,60,90)

Table 1

The above fuzzy transportation table is solved using the proposed method, step 2.

	Stores				
Factory		S1	S2	S3	Capacity
	F1	(1,2,3) +(2, 2.25, 2.50)	(2,4,6) + (2, 2.25, 2.50)	(4,7,10) +(2, 2.25, 2.50)	(10,20,30)
	F2	(7,11,15) +(3,3.10,3.20)	(11,16,21) + (3,3.10,3.20)	(16,22,28) +(3,3.10,3.20)	(15,30,45)
	F3	(22,29,36) +(1,1.15,1.30)	(29,37,45) + (1,1.15,1.30)	(37,46,55) +(1,1.15,1.30)	(5,10,15)
	Requirement	(7,12,17)	(10,25,40)	(13,23,33)	(30,60,90)

Table 2

It is a balanced TP because Capacity = Requirement. The above fuzzy transportation table is solved using the proposed method, step 3 to step 8. Finally, the Initial feasible solution is

	Stores				
Factory		S1	S2	S3	Capacity
	F1	(3, 4.25, 5.50)	(4, 6.25, 8.50)	(6, 9.25, 12.50) (10,20,30)	(10,20,30)
	F2	(10,14.10,18.20) (-8,2,12)	(14,19.10, 24.20) (10,25,40)	(19,25.10,31.20) (-17, 3, 23)	(15,30,45)
	F3	(23,30.15,37.30) (5,10,15)	(30,38.15,46.30)	(38,47.15,56.30)	(5,10,15)
	Requirement	(7,12,17)	(10,25,40)	(13,23,33)	

Table 3

Here, the number of allocated cells = 5 is equal to m + n - 1 = 3 + 3 - 1 = 5. Therefore, this solution is non-degenerate.The minimum total fuzzy transportation cost = (6, 9.25, 12.50) (10,20,30) + (10,14.10,18.20) (-8,2,12) + (14,19.10, 24.20) (10,25,40) + (19,25.10,31.20) (-17, 3, 23) + (23,30.15,37.30) (5,10,15). The minimum total fuzzy transportation cost = (256.4,352.29,448.14) Use the FMODI approach and table-3 to find an optimal solution

	Stores				
		S1	S2	S3	Capacity
Factory	F1	(3, 4.25, 5.50)	(4, 6.25, 8.50)	(6, 9.25, 12.50) (10,20,30)	(10,20,30)
	F2	(10,14.10,18.20) (-8,2,12)	(14,19.10, 24.20) (10,25,40)	(19,25.10,31.20) (-17, 3, 23)	(15,30,45)
	F3	(23,30.15,37.30) (5,10,15)	(30,38.15,46.30)	(38,47.15,56.30)	(5,10,15)
	Requirement	(7,12,17)	(10,25,40)	(13,23,33)	

Table 4

Here,$dij > 0$ So, the final optimal solution is arrived. The minimum total fuzzy transportation cost = (256.4,352.29,448.14). Above fuzzy transportation cost is converted to crisp valued transportation cost using different ranking techniques.

8. Results and Discussion

RANKING TECHNIQUES	D=(t, m, s).	TRANS. COST
Average ranking technique (5)	R(D)=t+m+s/3	352.28
The centroid ranking technique(7)	R(D) =(t+m+s/3) (w/3) , where w=1	**116.25**
Chen and Hsieh ranking (8)	R(D) =(t+4m+s)/6	352.28
Adamo and Campos et al. ranking (10)	R(D)=αk+(1-α)u,where α=0.5	400.22
Yager's center of gravity ranking (11)	R(D)=t+m+s/3	352.28
Kaufmann et al and Chen ranking (12)	R(D) =t+2m+s/4	352.28
Liou et al ranking with $\alpha = 0.3$ (13)	R(D)=αs+m+(1-α)t / 2 , α=0.3	333.1
Liou et al ranking with $\alpha = 0.5$ (14)	R(D)=αs+m+(1-α)t / 2 , α=0.5	352.28
Liou et al ranking with $\alpha = 0.9$ (15)	R(D)=αs+m+(1-α)t / 2 , α=0.9	390.63
Robust's ranking techniques (16)	R(D)=$\int_0^1 (0.5)(a_a^L, a_a^U)da$, $(a_a^L, a_a^U) = \{(m-t)\alpha + t, s-(s-m)\alpha\}, \alpha\epsilon[0,1]$	352.28

Minimum Transportation cost =116.25

Fuzzy numbers can be ranked in a variety of ways, as seen in the above findings. The centroid ranking approach gave minimum fuzzy transportation cost 116.25 in the above results.

9. Conclustion

A variety of ranking techniques, including a brand-new penalty technique, are used to solve fuzzy transportation problems. As a result, we were able to develop a fuzzy transportation solution that was completely compatible with the Fuzzy Modified Distribution Method (FMODI) technique. Furthermore, fuzzy transportation problems are calculated using various ranking approaches. Finally, the centroid ranking approach gave a lower transportation cost than other techniques

Conflicts of interest : The authors declare no conflict of interest

Data availability :Not applicable

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References

1. T.C. Chu, & C.T. Tsao, *Ranking fuzzy numbers with an area between the centroid point and original point*, Computers & Mathematics with Applications **43** (2002), 111-117.
2. Y.M. Wang, J.B. Yang, D.L. Xu, & K.S. Chin, *On the centroids of fuzzy numbers*, Fuzzy sets and systems **157** (2006), 919-926.
3. N. Ramli, & D. Mohamad, *A comparative analysis of centroid methods in ranking fuzzy numbers*, European Journal of Scientific Research **28** (2009), 492-501
4. H. Basirzadeh, *An approach for solving fuzzy transportation problem*, Applied Mathematical Sciences **5** (2011), 1549-1566.
5. A.E. Samuel, & M. Venkatachalapathy, *Modified Vogel's approximation method for fuzzy transportation problems*, Applied Mathematical Sciences **5** (2011), 1367-1372.
6. S. Rezvani, *A new method for ranking in perimeters of two generalized trapezoidal fuzzy numbers*, International Journal of Applied Operational Research **2** (2012), 85-92.
7. L.Q. Mdat, V.F. Yu, & S.Y. Chou, *An improved ranking method for fuzzy numbers based on the centroid-index*, International Journal of Fuzzy Systems **14** (2012), 413-419.
8. S. Narayanamoorthy, S. Saranya, & S. Maheswari, *A method for solving fuzzy transportation problem (ftp) using fuzzy russell's method*, International Journal of Intelligent Systems and Applications **5** (2013), 71.
9. D. Hunwisai, & P. Kumam, *A method for solving a fuzzy transportation problem via Robust ranking technique and ATM*, Cogent mathematics **4** (2017), 1283730.
10. S.H. Chen, & C.H. Hsieh, *Representation, ranking, distance, and similarity of LR type fuzzy number and application*, Australian Journal of Intelligent Processing Systems **6** (2000), 217-229.
11. A. Singh, *Importance of Fuzzy Logic in Traffic and Transportation Engineering*, In: Vasant, P., Zelinka, I., Weber, GW. (eds) Intelligent Computing & Optimization. ICO 2021. Lecture Notes in Networks and Systems, vol 371, 2022, Springer, Cham. $https : //doi.org/10.1007/978-3-030-93247-3_10$
12. F. Abbasi, M. Zarei, A.M. Rahmani, FWDP: a fuzzy logic-based vehicle weighting model for data prioritization in vehicular ad hoc networks. Veh. Commun. 100413 (2021).

13. A. Jovanovic, K. Kukik, A. Stevanovic, *A fuzzy logic simulation model for controlling an oversaturated diverge diamond interchange and ramp metering system*, Math. Comput. Simul. **182** (2021), 165–181.
14. T.D. Tohan, Y.D. Wong, *Fuzzy logic-based methodology for quantification of traffic congestion*, Physica A. **570** (2021), 125784.

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