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How Consignment Should be Implemented to be an Effective Supply Chain Collaboration Program?

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Abstract

Purpose: This study observes how the consignment performs over time and proposes the proper way to operate the consignment to bring its best benefit to the supply chain system. **Research design, data, and methodology:** The supply chain system is represented as the mathematical model where a manufacturer and a retailer trade one type of products to maximize their own profits. In the numerical examples of the proposed models, the consignment is compared with the fully coordinated system as well as the traditional system, and both advantages and limitations of the consignment are detected. **Results:** The consignment makes greater profit than the traditional system after a certain time period. The consignment is still outperformed by the fully coordinated system. The numerical examples also show that the performance of the consignment is sensitive to the consignment ratio. **Conclusions:** This study finds out that a certain time period must be spent until the consignment brings the benefit to the supply chain system. The numerical outcomes also imply that the consignment requires additional features to be a completely effective supply chain collaboration program. To obtain the best result from the consignment, the supply chain members are recommended to decide the appropriate consignment ratio by consent.

Keywords: Supply Chain Collaboration, Consignment, Profit Maximization, Optimal Control

JEL Classifications: M11, M19, M21, M29

1. Introduction

The supply chain collaboration is the management scheme that improves the supply chain performance by coordinating individual members' operations (Hariga & Al-Ahmari, 2013). As one of supply chain collaboration programs, the consignment, which has already been used as a common business practice, has attracted heavy attentions from both business practitioners and academic researchers (Hung et al., 1995; Zahran et al., 2015). Instead of the conventional trade style, the supply chain members would choose the consignment contract and collaborate their individual operations to achieve the improved supply chain performance (Gümüş et al., 2008).

The main research goal of this study is to evaluate the performance of the consignment and propose the effective way to operate this supply chain collaboration program. While most past studies focus on merely the final outcome,

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this study monitors how the consignment performs over time and intends to identify its detailed nature that implies the effective managerial guidelines. The optimization models are formulated to represent three supply chain systems, which are the traditional system, consignment, and fully coordinated system. The proposed mathematical model describes two stage supply chain system where a manufacturer supplies one type of products to a retailer. In the numerical examples of the proposed mathematical models, three supply chain systems are compared in terms of the system profit to figure out the strengths and weaknesses of the consignment as the supply chain collaboration program.

The numerical analysis reveals that the consignment eventually improves the overall supply chain performance, even though it takes a certain amount of time to notice the improvement. While the consignment achieves its original goal of supply chain collaboration and increases the whole system profit, the manufacturer experiences severe profit loss due to increased production volume and extra cost burden for holding retailer's inventory. Since the benefit from the consignment unequally distributed to the supply chain members, they need to consider the incentive program to sustain this collaboration program in practice.

Meanwhile, the supply chain system cannot achieve the complete collaboration by using the consignment alone, and the system requires the additional collaborative features combined with the consignment. This study recognizes the consignment ratio as a key element of the consignment contract, which should be carefully determined to obtain the best outcome from the consignment.

2. Research Subject and Related Literature

The consignment has been applied to the broad areas of businesses in its extensive history (Fenton & Sanborn, 1987). In present industries, the consignment is even perceived to be one of the specialized programs, which are designed to coordinate the partners in the supply chain system and improve its overall performance (Chen & Liu, 2007). In general, the consignment represents the special contract under which the vendor owns the products stored at the retailer's warehouse. Since the ownership of those inventories is held by the vendor, the retailer does not pay to purchase the products from the vendor until they are sold at the retail market (Yang et al., 2020; Zhang et al., 2022).

Once its effective role of coordinating the supply chain operations and accomplishing the business success is noticed, the consignment have been applied to various industries including the recent virtual markets as well as the conventional retailing of appliances, medical supplies, clothing, books, and musical instruments (Tao et al., 2019; Yang et al., 2020; Zahran et al., 2015).

As the supply chain collaboration program, the consignment has attracted great interest from researchers. A group of researchers investigate how the consignment influences the supply chain performance by comparing different supply chain systems. Chen and Liu (2007) develop Stackelberg gaming model representing the relationship between one manufacturer and one retailer. They evaluate the performance of the optimal consignment policy by comparing with the traditional system without the consignment. Their numerical analysis reveals that the consignment increases the total supply chain profit as well as the manufacturer's.

In two stage supply chain system with a vendor and a customer, Gümüş et al. (2008) compare three different supply chain systems including the traditional inventory sourcing, consignment inventory, and VMI combined with consignment. The numerical examples of their proposed supply chain models indicate that the consignment can save both vendor's and customer's costs depending on who pays the transportation cost. Their study still shows that the combination of VMI and consignment outperforms the consignment alone.

Under the situation that the dynamic adverting is conducted to boost the demand, Lu et al. (2019) examine two different contracts made between a manufacturer and a retailer. They find out that the consignment contract results in greater system profit than the wholesale price contract. According to their model analysis, the manufacturer makes higher profit with the wholesale price contract than with the consignment contract when the retailer's advertising does not perform effectively or chain members is not patient.

Focusing on the manufacturer's flexibility due to VMI, Savasaneril and Erkip (2010) examines the impact of the consignment stock by comparing the consignment stock system with the no-consignment stock system. Their study identifies the specific conditions that the manufacturer makes more profit under VMI with consignment than VMI without consignment.

The consignment frequently appears as a supply chain collaboration program combined with Vendor-Managed Inventory (VMI) in a large group of the past studies (Batarfi et al., 2019; Taleizadeh & Moshtagh, 2019). Most of them examine how VMI with the consignment performs by comparing with the case without it (Ferretti et al., 2017; Hemmati et al., 2017; Lee & Cho, 2014; Marchi et al., 2019).

Different from the past studies that evaluate mere the final outcome from the consignment, this study monitors how this supply chain collaboration program performs over time. By identifying when the consignment becomes more profitable than the traditional system, this study provides sophisticated managerial guidelines about how to effectively implement the consignment.

In this study, the question whether the consignment is the effective collaboration program is one of the research issues and the consignment is compared with the traditional no-consignment system as shown in many past studies. Furthermore, this study considers the fully coordinated system to be an ideal form of supply chain collaboration and figures out the weaknesses of the consignment. The comparison with the fully coordinated system gives the useful practical implications about how to improve the consignment to be an advanced supply chain collaboration program.

3. Research Model

This study examines how the consignment performs over time by using the supply chain model. The proposed mathematical model represents two-stage supply chain system where one manufacturer and one retailer exist. The manufacturer makes one type of the products and supplies them to the retailer. The retailer purchases the products from the manufacturer and then sell them at the retailer market.

On the purpose of understanding the impact of the consignment on the supply chain performance, this study compares three different types of the supply chain systems, which are the traditional system, consignment, and fully coordinated system. Table 1 describes the notations used in the proposed mathematical models.

Table 1: Notation of Mathematical Models

Manufacturer		Retailer	
π_M	Profit	π_R	Profit
p_t	Price	r_t	Price
y_t	Production rate	q_0	Lot size
o_M	Setup cost per order	o_R	Ordering cost
h_M	Unit inventory holding cost	h_R	Unit inventory holding cost
υ	Unit production cost	h_R^F	Financing cost for inventory
δ	Transportation cost per price	h_R^S	Stocking cost for inventory
x_t^M	Estimated market demand	x_t^R	Market demand
$x_t^{M\prime}$	Rate of demand change	$x_t^{R\prime}$	Rate of demand change
N_M	Potential market size	N_R	Potential market size
d_M	Price elasticity of demand	d_R	Price elasticity of demand
B	Initial market demand	γ	Discount rate
T	Time to leave the market		

3.1. Traditional System

This study evaluates the performance of the consignment compared with the traditional system to identify the superiority of the consignment as the supply chain collaboration program. The traditional system indicates the plain form of supply chain systems, which has no particular feature to enhance the collaboration between supply chain members. The following optimization model represents the traditional system where the manufacturer and the retailer determine the operations to maximize their own profits.

$$\max_{p_t, y_t} \ \pi_M = \int_0^T e^{-\gamma\cdot t}\left(p_t\cdot x_t^M - \frac{o_M\cdot x_t^M}{q_0} - q_0\left(1-\frac{t}{T}\right)h_M\cdot \frac{x_t^M}{y_t} - \upsilon\cdot y_t - \delta\cdot x_t^M\right)dt \qquad (1)$$

$$\text{subject to} \ \ x_t^{M\prime} = N_M\cdot e^{-d_M\cdot p_t} \qquad (2)$$

$$x_0^M = B \qquad (3)$$

$$\max_{r_t, q_0} \ \pi_R = \int_0^T e^{-\gamma\cdot t}\left(r_t\cdot x_t^R - p_t\cdot x_t^R - \frac{o_R\cdot x_t^R}{q_0} - q_0\left(1-\frac{t}{T}\right)h_R\right)dt \qquad (4)$$

$$\text{subject to} \ \ x_t^{R\prime} = N_R\cdot e^{-d_R\cdot r_t} \qquad (5)$$

$$x_0^R = B \qquad (6)$$

Equation (1) describes that the manufacturer determines transfer price (p_t) and production rate (y_t) for maximizing his profit (π_M). The manufacturer's profit contains the revenue from the sales to the retailer and the costs for setup, inventory holding, manufacturing, and transportation. As the inventory control policy, the extended Economic Order Quantity model is applied to the mathematical models (Tungalag, Erdenebat, & Enkhbat, 2017).

The retail market demand estimated by the manufacturer is described in Equation (2). The market demand is assumed to depend on the price according to the naive model of innovation diffusion (Kalish, 1985). Equation (3) specifies the initial demand at the retail market.

Equations (4), (5) and (6) represent the retailer's problem in the traditional supply chain system. In Equation (4), the retailer decides the retail price (r_t) and lot size (q_0) to maximize his profit (π_R). The retailer's profit is composed of his revenue from the sales at the retail market, purchasing cost, ordering cost, and inventory holding cost.

Equations (5) and (6) describe the demand change in the retail market, which is recognized by the retailer, and the initial market demand. The retailer's unit inventory holding cost (h_R) is assumed to be proportional to the transfer price (p_t) as Equation (7) shows.

$$h_R = \beta \cdot p_t \quad (7)$$

3.2. Consignment

In contrast with the traditional system, the consignment represents the collaborative supply chain system where the manufacturer consigns his products to the retailer. Equations (8), (9), and (10) describes the manufacturer's problem in the consignment system.

$$\max_{p_t, y_t} \ \pi_M = \int_0^T e^{-\gamma \cdot t} \left(p_t \cdot x_t^M - \frac{o_M \cdot x_t^M}{q_0} - q_0 \left(1 - \frac{t}{T}\right) h_M \cdot \frac{x_t^M}{y_t} - \upsilon \cdot y_t - \delta \cdot x_t^M - q_0 \left(1 - \frac{t}{T}\right) h_R^F \right) dt \quad (8)$$

$$\text{subject to} \ \ x_t^{M\prime} = N_R \cdot e^{-d_M \cdot p_t} \quad (9)$$

$$x_0^M = B \quad (10)$$

$$\max_{r_t, q_0} \ \pi_R = \int_0^T e^{-\gamma \cdot t} \left(r_t \cdot x_t^R - p_t \cdot x_t^R - \frac{o_R \cdot x_t^R}{q_0} - q_0 \left(1 - \frac{t}{T}\right) h_R^S \right) dt \quad (11)$$

$$\text{subject to} \ \ x_t^{R\prime} = N_R \cdot e^{-d_R \cdot r_t} \quad (12)$$

$$x_0^R = B \quad (13)$$

In Equation (8), the manufacturer still determines the transfer price and production rate to maximize his profit including the sales revenue, setup cost, inventory holding cost, production cost, and transportation cost. In the consignment, meanwhile, the retailer's inventory holding cost is separated into financing cost (h_R^F) and stocking cost (h_R^S), and the manufacturer pays even the financing cost of holding the inventory at the retailer's warehouse (Valentini & Zavanella, 2003). Equations (9) and (10) indicate the demand change and initial demand size estimated by the manufacturer.

The retailer's decision is to set the retail price and order quantity for the purpose of obtaining the maximum profit (Equation (11)). According to the consignment contract, the retailer is responsible for only the stocking cost of holding inventory at his warehouse. Equations (12) and (13) describe the demand size at the beginning and its change over time, which the retailer observes.

Based on the assumption that the retailer's inventory holding cost is in proportion to the value of inventory (Equation (7)), both financing and stocking costs of retailer's inventory are dependent on the transfer price as shown in Equation (14) and (15).

$$h_R^S = \beta_S \cdot p_t \quad (14)$$

$$h_R^F = \beta_F \cdot p_t \quad (15)$$

3.3. Fully Coordinated System

This study compares the consignment with the fully coordinated system as well as the traditional system and it intends to identify the deficiency of the original consignment to realize the full collaboration in the supply chain system. The fully coordinated system is defined to be the supply chain system where every operational decision is made in a way to maximize the whole system profit instead of the individual member's profit.

$$\max_{p_t, y_t, q_0} \ \pi_M = \int_0^T e^{-\gamma \cdot t} \left(r_t \cdot x_t^R - \frac{o_R \cdot x_t^R}{q_0} - q_0 \left(1 - \frac{t}{T}\right) h_R - \frac{o_M \cdot x_t^R}{q_0} - q_0 \left(1 - \frac{t}{T}\right) h_M \cdot \frac{x_t^R}{y_t} - \upsilon \cdot y_t - \delta \cdot x_t^R \right) dt \quad (16)$$

$$\text{subject to} \ \ x_t^{R\prime} = N_R \cdot e^{-d_R \cdot r_t} \quad (17)$$

$$x_0^R = B \quad (18)$$

Equation (16) represents that the manufacturer and retailer coordinate their operational decisions completely and agree to pursue the maximum of the whole system profit. Equations (17) and (18) provide the detailed information about the demand at the retail market under the fully coordinated system. The retail price is assumed to depend on the transfer price with the fixed profit margin.

The optimal control theory is used to obtain the optimal solutions of three supply chain models (Sethi & Thompson, 1981). Appendix illustrates the detailed procedure of solving the optimization problem of the consignment. The identical method is applied to the cases of the traditional system and fully coordinated system.

4. Numerical Analysis

This study compares three different supply chain systems, which are the traditional system, consignment, and fully coordinated system by analyzing the numerical examples of the proposed mathematical models. Table 2 illustrates the parameters used in the bases case of the numerical examples, and they have arbitrarily determined values.

Table 2: Parameters of Base Case

N_M = 190	N_R = 200	d_M= 0.0025	d_R = 0.0023
o_M = 15,000	o_R = 10,000	υ = 10	δ = 5
h_M = 3	β = 0.30	β_F = 0.15	β_S = 0.15
B = 5,000	α = 1.20	γ = 0.01	T = 30

The numerical examples equip with five parameters varied in five different levels (ordering cost, setup cost, retailer's inventory holding cost, manufacturer's inventory holding cost, and potential market size). The numerical examples contain 3,125 cases in total ($5^5 = 3{,}125$) and the period of each case is 30.

4.1. Performance of Consignment

On the purpose of identifying the detailed nature of the consignment as the supply chain collaboration program, this study evaluates how the consignment performs over time and compare it with two different supply chain systems. Figure 1 shows the differences in the system profit between the distinct supply chain systems.

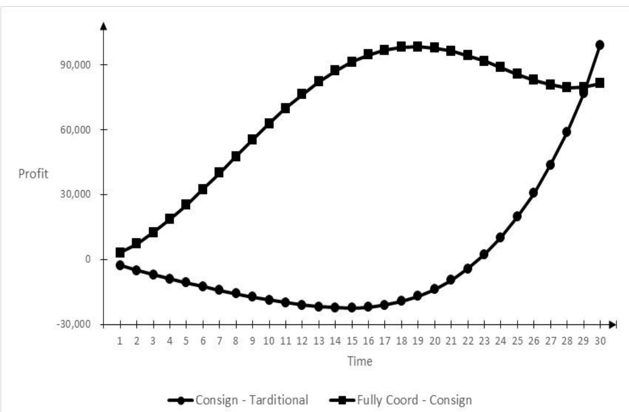

Figure 1: Profit Differences between Supply Chain Systems

The supply chain profit under the consignment is lower than the one in the traditional system in the early time, and then it becomes higher later. This results indicates that the consignment achieves the poor performance in the early time and it needs to spend a certain time period until it makes greater supply chain profit than the non-collaborative system.

Compared with the fully coordinated system, the consignment maintains lower supply chain profit over the whole time period. According to Figure 1, the fully coordinated system always outperforms the consignment in terms of the supply chain profit. This outcome implies that the classical consignment is not still the completely effective collaboration program even though it improves the supply chain performance.

The consignment has different impacts on the individual supply chain members. Figure 2 shows that the retailer obtains higher profit under the consignment than in the traditional system. Meanwhile, the consignment results in lower manufacturer's profit compared with the traditional system.

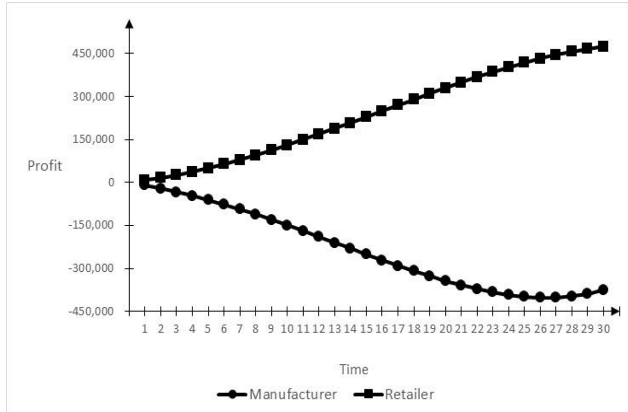

Figure 2: Differences in members' profits between Consignment and Traditional System

Under the consignment, the manufacturer obtains lower profit due to greater production rate compared with the traditional system according to Figure 3. Even though manufacturer is able to increase his sales by reducing the transfer price significantly under the consignment, he need to make relatively greater amount of products to meet the increased demands. The burden of the cost for holding the retailer's inventory according to the consignment contract also results in the manufacturer's profit loss.

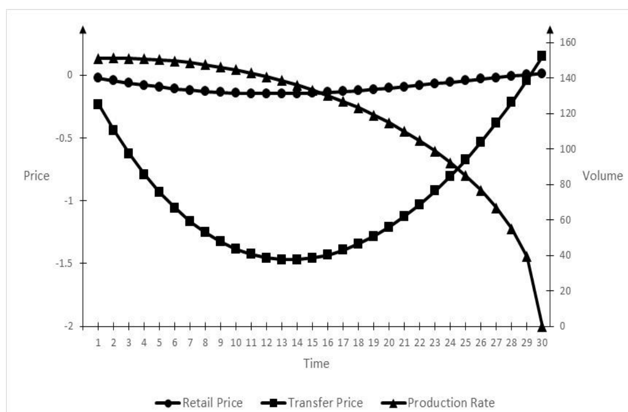

Figure 3: Difference in Decision Variables between Consignment and Traditional System

Figure 3 shows that the decreased retail price enables the retailer to achieve greater profit under the consignment than the traditional system. After all, the increased demands due to the properly controlled price is the main reason that the consignment increases the overall supply chain profit.

Figure 4 indicates that the fully coordinated system has lower retail and transfer prices than the consignment, in particular during the later period of time. Meanwhile, the production rate is increased by a relatively small amount. The fully coordinated system makes greater supply chain profit than the consignment because the lower retail and transfer prices lead more demands and the controlled production volume saves the cost.

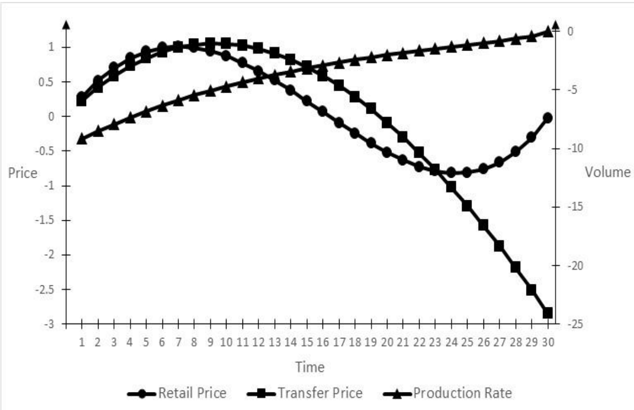

Figure 4: Difference in Decision Variables between Fully Coordinated System and Consignment

4.2. Impacts of Consignment Ratio on Performance of Consignment

This study investigates how the consignment contract content affects the performance of the consignment by conducting an extensive experiment on the consignment ratio. The consignment ratio represents the proportion of the financing cost to the entire cost of holding the retailer's inventory $\left(\frac{\beta_F}{\beta_F+\beta_S}\right)$. Figure 5 shows the profit differences between distinct supply chain systems over different consignment ratios.

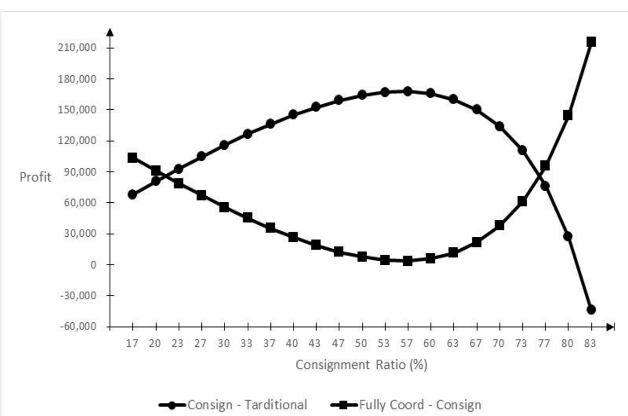

Figure 5: Impact of Consignment Ratio on Profit Difference between Supply Chain Systems

The profit difference between the consignment and traditional system increases as the consignment ratio becomes higher until a specific consignment ratio. After that, the profit difference decreases. Meanwhile, the profit difference between the fully coordinated system and consignment decreases with higher consignment ratio and then it increases after a certain value of consignment ratio. In the experiment case of this study, when the consignment ratio is 57%, the consignment makes the greatest supply chain profit, which is most different from one of the traditional system and closest to one of the fully coordinated system.

5. Discussion

This study evaluates the performance of the consignment and finds out the proper way to implement this supply chain collaboration program for achieving the best outcome. Three supply chain models including the consignment, traditional system, and fully coordinated system are tested in the numerical examples. The significant outcomes are identified in the numerical analysis and they provide valuable managerial implications for business practitioners.

First, the consignment eventually achieves greater supply chain profit, while it has lower profit than the traditional system in the early period of time. This result indicates that the consignment is the effective supply chain collaboration program that would bring the certain benefit to the supply chain system as the past studies describe (Chen & Liu, 2007; Lu et al., 2019). Meanwhile, the numerical analysis of this study implies that the immediate achievement is not expected right after the implementation of the consignment, and instead it requires a certain period of time until making improvement.

Second, the consignment has a different impact on the individual supply chain member's profit. Under the consignment, the retailer increases his profit because lower retailer price leads to greater demands compared with the traditional system. This result is consistent with the past studies that consider the consignment to be beneficial to the buyer because he can mitigate the influence of demand uncertainty and save the inventory holding cost (Gümüş et al., 2008; Valentini & Zavanella, 2003; Zhang et al., 2022).

Compared with the traditional system, the manufacturer in the consignment, however, has to increase the product rate to meet the increased demand and he pays higher production cost and financing cost for holding retailer's inventory. After all, the manufacturer's profit is less in the consignment than the traditional system. A group of researchers already point out that the manufacturer may experience the financial loss due to the consignment (Chen & Liu, 2007; Gümüş et al., 2008). This outcome implies that the consignment requires the additional supporting scheme that compensates the manufacturer's loss to be the practical supply chain program that can be acceptable to every member.

Third, the consignment results in less supply chain profit than the fully coordinated system. The numerical examples reveal that the consignment outperforms the traditional system. Compared with the fully coordinated system, which is the completely collaborated system, however, the consignment achieves smaller supply chain profit. By implication, to fully collaborate the supply chain operations and obtain the best performance from the complete collaboration, the consignment needs to equip with the additional collaborative features such as Vendor-Managed Inventory and Revenue Sharing Contract (Savasaneril & Erkip, 2010; Yang et al., 2020; Zahran et al., 2015; Zhao et al., 2020).

Finally, the consignment makes the greatest supply chain profit when the consignment ratio is set at the specific level. According the numerical examples, the consignment obtains the highest system profit when it has the certain level of consignment ratio. This result provides the implication that the supply chain members need to agree on the proper consignment ratio in their contract to achieve the best outcome from the consignment.

In summary, once the consignment is implemented for collaborating the supply chain operations, the supply chain system should wait for a sufficient amount of time until it obtains the real benefit from this collaboration program. Since the consignment is not beneficial to every supply chain member, this study suggests that the consignment is operated along with the certain compensation scheme to induce all members to participate in this supply chain collaboration program. Under the consignment contract, the supply chain members need to determine the appropriate level of consignment ratio to obtain the best outcome from the consignment. Meanwhile, the consignment alone does not fully collaborate the supply chain operations, and this study recommends the combination with other collaborative functions such as Vendor-Managed Inventory and Revenue Sharing Contract.

6. Conclusion

The consignment has been used to build the close coordination between supply chain members in various industries. This study focuses on the consignment as a supply chain collaboration program. By investigating how the consignment performs over time, this study seeks to understand how to operate this supply chain collaboration program to achieve the best outcome.

The mathematical models are formulated to represent three different supply chain systems. In the numerical examples, this study compares the consignment with the traditional system and fully coordinated system to figure out both superiority and inferiority that the consignment possesses.

The numerical analysis on the proposed supply chain models reveals the critical characters of the consignment and these findings provide the valuable managerial guidelines about how to effectively operate this collaboration program. Once the consignment is implemented, the supply chain system needs to spend a sufficient amount of time until it realizes a certain benefit from this collaboration program. Since most financial gains from the consignment goes to only the retailer, the consignment requires the compensation program that makes up for the manufacturer's loss to be the collaboration program that is welcomed by every supply chain member.

The consignment definitely makes less supply chain profit than the fully coordinated system, while it outperforms the non-consignment system. Since the consignment alone fails to bring the complete collaboration, this study recommends that the consignment would be combined with additional collaborative functions such as Vendor-Managed Inventory and Revenue Sharing Contract.

This study also discovers that the consignment makes the greatest supply chain profit with the particular value of the consignment ratio. This result implies that the supply chain members should carefully decide the consignment ratio in their consignment contract to achieve the best outcome.

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Appendix

In the consignment case, Equation (A1) shows the Hamiltonian equation of the manufacturer's model with the transfer price (p_t) and production rate (y_t) as the decision variables, the market demand (x_t^M) as the state variable, and λ_t^M as the adjoint variable.

$$H_M = e^{-\gamma\cdot t}\left(p_t\cdot x_t^M - \frac{o_M\cdot x_t^M}{q_0} - q_0\left(1-\frac{t}{T}\right)h_M\cdot\frac{x_t^M}{y_t} - \upsilon\cdot y_t - \delta\cdot x_t^M - q_0\left(1-\frac{t}{T}\right)\beta_F\cdot p_t\right) + \lambda_t^M\cdot N_R\cdot e^{-d_R\cdot p_t} \quad \text{(A1)}$$

Based on the optimal control theory, the necessary conditions for optimality appear as the following equations show.

$$\frac{\partial H_M}{\partial p_t} = 0 \quad \text{(A2)}$$

$$\frac{\partial H_M}{\partial y_t} = 0 \quad \text{(A3)}$$

$$\frac{\partial H_M}{\partial x_t^M} = -\lambda_t^{M\prime} \quad \text{(A4)}$$

$$\frac{\partial H_M}{\partial \lambda_t^M} = x_t^{M\prime} \quad \text{(A5)}$$

The Hamiltonian equation of the retailer's model (A6) contains the retail price (r_t) as the decision variable, the market demand (x_t^R), as the state variable, and λ_t^R as the adjoint variable. Equations (A7) through (A9) indicate the optimal conditions for optimality.

$$H_R = e^{-\gamma\cdot t}\left(r_t\cdot x_t^R - p_t\cdot x_t^R - \frac{o_R\cdot x_t^R}{q_0} - q_0\left(1-\frac{t}{T}\right)\beta_S\cdot p_t\right) + \lambda_t^R\cdot N_R\cdot e^{-d_R\cdot r_t} \quad \text{(A6)}$$

$$\frac{\partial H_R}{\partial r_t} = 0 \quad \text{(A7)}$$

$$\frac{\partial H_R}{\partial x_t^R} = -\lambda_t^{R\prime} \quad \text{(A8)}$$

$$\frac{\partial H_R}{\partial \lambda_t^R} = x_t^{R\prime} \quad \text{(A9)}$$

Equations (A10) through (A17) represent the optimal solutions of all given variables,

$$q_0 = \sqrt{\frac{2\cdot o_R\cdot x_T^r}{\beta_S\cdot p_t}} \quad \text{(A10)}$$

$$p_t = \frac{\gamma\cdot t}{d_M}\cdot ln\left(\frac{x_t^M - q_0\left(1-\frac{t}{T}\right)\beta_F}{d_M\cdot\lambda_t^M\cdot N_M}\right) \quad \text{(A11)}$$

$$y_t = \sqrt{\frac{q_0\left(1-\frac{t}{T}\right)\cdot h_M\cdot x_t^M}{\upsilon}} \quad \text{(A12)}$$

$$\lambda_t^M = \lambda_T^M + \int_t^T e^{-\gamma\cdot t}\left(p_t - \frac{o_M}{q_0} - q_0\left(1-\frac{t}{T}\right)\frac{h_M}{y_t} - \delta\right)dt \quad \text{(A13)}$$

$x_t^M = B + \int_0^T N_M \cdot e^{-d_M \cdot p_t}\, dt$ (A14)

$r_t = \frac{\gamma \cdot t}{d_R} ln\left(\frac{x_t^R}{d_R \cdot \lambda_t^R \cdot N_R}\right)$ (A15)

$\lambda_t^R = \lambda_T^R + \int_t^T e^{-\gamma \cdot t}\left(r_t - p_t - \frac{o_R}{q_0}\right) dt$ (A16)

$x_t^R = B + \int_0^T N_R \cdot e^{-d_R \cdot r_t}\, dt$ (A17)

The following procedure shows the shooting method that this study uses to obtain the numerical optimal solutions.

Step 1. Set arbitrary values for old_sum_x1, old_sum_x2, old_sum_x3, and old_sum_x4. The following discrete summation forms estimate the integral portions of Equations (A13), (A14), (A16), and (A17).

$old\_sum\_x1 = \sum_{t=\tau+1}^{T} e^{-\gamma \cdot t}\left(p_t - \frac{o_M}{q_0} - q_0\left(1 - \frac{t}{T}\right)\frac{h_M}{y_t} - \delta\right)$ (A18)

$old\_sum\_x2 = \sum_{t=1}^{T} N_M \cdot e^{-d_M \cdot p_t}$ (A19)

$old\_sum\_x3 = \sum_{t=\tau+1}^{T} e^{-\gamma \cdot t}\left(r_t - p_t - \frac{o_R}{q_0}\right)$ (A20)

$old\_sum\_x4 = \sum_{t=1}^{t} N_R \cdot e^{-d_R \cdot r_t}$ (A21)

Step 2. Compute the optimal solutions of λ_t^M, x_t^M, λ_t^R, x_t^R according to Equations (A13), (A14), (A16), and (A17) with the values of old_sum_x1, old_sum_x2, old_sum_x3, and old_sum_x4 from Step 1. Obtain the values of q_0, p_t, y_t, and r_t by using Equations (A10), (A11), (A12), and (A15). Set old_q_0=q_0

Step 3. Compute new values of old_sum_x1, old_sum_x2, old_sum_x3, and old_sum_x4 (new_sum_x1, new_sum_x2, new_sum_x3, and new_sum_x4) with the values of λ_t^M, x_t^M, λ_t^R, x_t^R, q_0, p_t, y_t, and r_t from Step 2. Set new_$q_0 = q_0$.

Step 4. If |new_sum_x1 - old_sum_x1| > tol_x1, |new_sum_x2 - old_sum_x2| > tol_x2, |new_sum_x3 - old_sum_x3| > tol_x3, |new_sum_x4 - old_sum_x4| > tol_x4, with the small tolerances for tol_x1, tol_x2, tol_x3, and tol_x4,

set old_sum_x1 = new_sum_x1×ch+old_sum_x1 × (1-ch)
old_sum_x2 = new_sum_x2×ch+old_sum_x2 × (1-ch)
old_sum_x3 = new_sum_x3×ch+old_sum_x3 × (1-ch)
old_sum_x4 = new_sum_x4×ch+old_sum_x4 × (1-ch)
where 0 < ch < 1
and then go to Step 2
else continue.

Step 5. If |new_q_0 - old_q_0| > tol_q_0, with the small tolerance for tol_q_0,

set old_q_0=new_q_0×ch+old_q_0×(1-ch)
where 0 < ch < 1
and then go to Step 2
else exit.

In the numeircal examples of the traditioal system and fully coordinated system, the same shooting procedure is applied as it apears above, except the different optimal solution eqautions of the variables are used.