

Optimal Multi-Product Inventory Problem Algorithm with Target In-Stock Ratio Constraints

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목표 재고보유매장비율 달성을 위한 다중품목 재고수준 최적화 알고리즘

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This paper studied the problem of determining the optimal inventory level to meet the customer service target level in a situation where the customer demand for each branch of a nationwide retailer is uncertain. To this end, ISR (In-Stock Ratio) was defined as a key management indicator (KPI) that can be used from the perspective of a nationwide retailer such as Samsung, LG, or Apple that sells goods at branches nationwide. An optimization model was established to allow the retailer to minimize the total amount of inventory held at each branch while meeting the customer service target level defined as the average ISR. This paper proves that there is always an optimal solution in the model and expresses the optimal solution in a generalized form using the Karush-Kuhn-Tucker condition regardless of the shape of the probability distribution of customer demand. In addition, this paper studied the case where customer demand follows a specific probability distribution such as a normal distribution, and an expression representing the optimal inventory level for this case was derived.

Keywords : Key performance Indicator, In-Stock Ratio, Karush-Kuhn-Tucker Condition, Optimal Inventory Level

1. Introduction

The retail industry is currently facing a vastly different environment compared to before, primarily due to the contraction in consumption and economic recession caused by the COVID-19 pandemic. KPMG, one of the world's largest accounting firms, has predicted that polarization in the distribution industry will intensify due to the shrinking consumption. They have also stated that companies that have preemptively established an online shopping environment will

enjoy relatively special benefits.

Apparel specialty stores and department stores are experiencing a decline in sales due to the decrease in offline customers. However, the increase in online purchases due to COVID-19 will accelerate the digital transformation of the retail industry. Consumers who previously did not purchase items online are now shopping online due to the pandemic.

To survive in this new retail environment, distribution companies that have already entered the online market should further strengthen their online capabilities. Meanwhile, off-line-based distribution companies should quickly establish an online platform and last-mile delivery system to secure online consumers [6].

In addition to the challenges mentioned above, retailers must also address the fundamental problem of matching customer demand in an agile and cost-efficient manner. To achieve efficient operations and gain a competitive edge against other retailers, there are numerous activities that retailers can undertake. These include inventory optimization, price discounting, and enhanced collaboration with manufacturers through various collaboration schemes such as Collaborative Planning Forecasting and Replenishment (CPFR), Vendor Managed Inventory (VMI), and Customer Management Inventory (CMI), among others.



<Figure 1> The Standard CPFR Model

<Figure 1> displays the core dynamics of the CPFR (Collaborative Planning, Forecasting, and Replenishment) collaboration process between a manufacturer and a retailer [4]. In this process, the manufacturer possesses accurate supply information, while the retailer has accurate market sales trend information. The channel inventory that accumulates in their supply chain can only be minimized through the sharing of their supply and market sales information via CPFR.

The fundamental philosophy for collaboration among supply chain partners, including retailers, suppliers, and distributors, is to assume that the entire supply chain can generate more profits when it is operated by a single centralized party. One of the most significant business hurdles in achieving maximum sales or market share is the frequent occurrence of product shortages during customers' shopping experiences. In this paper, we will provide solutions on how to minimize product shortages and maximize product availability in meeting the dynamic customer demand in both online and offline shopping environments.

In today's era of digital transformation, customers have become increasingly impatient. With abundant other options just a click away, they seldom tolerate product shortages or backlogs. In fact, their expectations for shopping experiences, including brand recognition, impeccable product quality, attractive pricing, delivery speed, and more, are persistently growing. Moreover, customers are placing more emphasis on accompanying after-sales services, even after their purchases. If a company wants to establish a good reputation, it must avoid frequent product shortages or low product availability for their customers.

The existing body of research closely aligned with our study can be classified into three main groups. In the first group, researchers approach the problem by framing it as an inventory management issue, focusing on determining the optimal inventory levels for retailers faced with stochastic demand, stochastic supply, or both sources of uncertainty. These studies aim to find the optimal solution that maximizes (or minimizes) the retailer's overall profit (or total cost) by striking an optimal balance between shortages and surpluses. One prominent model within this category is the newsvendor model, which has gained significant recognition. For a detailed analysis of the newsvendor model, interested readers can refer to [5, 7].

The second category of studies focuses on the optimization of shelf space, aiming to determine the optimal positioning of each product to maximize total sales. Moreover, marketing research has revealed that customers make their final purchasing decisions at the point of purchase, simultaneously (for example, refer [8]). Additionally, Bae et al. [1] found that, with the exception of relatively short time periods, buyers of a particular brand tend to purchase other brands more frequently than their preferred brand. This suggests that customers' product choices may be influenced by in-store factors, including the allocation of shelf space. According to Yang and Chen [8], retailers can attract a greater number of customers, reduce instances of stock-outs, and enhance the financial performance of their stores through well-designed shelf space allocation.

The third category of research endeavors involves the development of Key Performance Indicators (KPIs) aimed at quantifying product availability within a defined time frame, either for a specific product or a group of products. However, it is important to acknowledge that in practice, the precise definition of such KPIs may prove challenging due to the need for suitable assumptions that facilitate the simplification of the intricate dynamics inherent to the market. For a compre-

hensive exploration of metrics within the domain of supply chain management, it is recommended to consult [3], wherein the authors introduce three fundamental dimensions—service, assets, and speed—as essential components of supply chain metrics. Furthermore, the authors emphasize the importance of incorporating at least one performance measure for each of these three key dimensions within every supply chain.

In our prior publication [2], we introduced a novel Key Performance Indicator (KPI) for product availability known as the In-Stock Ratio (ISR), within the context of a simplified single product setting. The ISR is computed as the percentage of stores that possess more than one sellable product in stock. As depicted in <Figure 2> of Han and Kim [2], the ISR emerges as a suitable inventory metric for retailers operating in multiple countries, or those with a wide network of offline stores or online distribution channels within a particular nation

This paper aims to expand upon our prior findings by investigating a multi-product environment, in contrast to the simplistic single product setting employed in previous research. The current study introduces an optimization model designed to minimize the cumulative inventory across all individual stores of the retailer, while concurrently ensuring that each store maintains a product availability level (referred to as ISR) surpassing a predetermined target threshold denoted as T.

The subsequent sections of this paper are structured as follows: Section 2 presents a comprehensive description of the mathematical model employed to maximize the product quantity constraint, accompanied by an exposition of the optimality conditions. In Section 3, a concise yet illustrative example is provided to demonstrate the practical application of the proposed model. Finally, Section 4 encapsulates the key contributions of this study and deliberates on potential avenues for future research.

2. Mathematical Model

This study focuses on the context of nationwide retailers, such as Samsung, LG, and Apple, which operate retail stores on a global or national scale. It is assumed that these retailers receive products exclusively through their central warehouse on a weekly basis. Furthermore, each individual store faces its own unique random weekly demand, characterized by a cumulative distribution function and probability density function. To enhance the tractability of the problem, the following assumptions are made: the inventory management cycle

for the retailer is conducted on a weekly basis, and the In-Stock Ratio (ISR) is adopted as the performance measure for the retailer's inventory operations. The ISR represents the percentage of stores that possess available products on their shelves at the conclusion of each week.

Notation:

- M: Number of product types (j = 1, ..., M)
- N: Total number of stores (i = 1, ..., N)
- X_{ij}: Initial inventory level of product j at store i at the beginning of the operational period
- D_{ij}: Demand for product j at store i during the operational period (we assume D_{ij}=D_j~iid N(μ_j, σ_j) for all j)
- F_{ij}: Probability distribution of the demand for product j at store i during the operational period
- α_j: Target ISR level for product j at store i during the operational period

Let I_{ij}(x_{ij}) be the indicate variable taking value ‘1’ only when there exists any product j on the shelf in store i at the end of each week where x_{ij} represents the initial inventory level, then

$$E[I_{ij}(x_{ij})] = \int_0^{x_{ij}} f_{ij}(t) dt = F_{ij}(x_{ij})$$

This means that E(I_{ij}(x_{ij})) equals to the probability for the initial inventory x_{ij} to cover the store i’s weekly demand for product j, that is demand fill-rate of the product j in the store I.

Product	Store	S ₁	S ₂	S _i	S _N	
P ₁		X ₁₁	X ₂₁	X _{i1}	X _{N1}	ISR(Product 1)
P ₂		X ₁₂	X ₂₂	X _{i2}	X _{N2}	ISR(Product 2)
⋮						
P _j		X _{1j}	---	X _{ij}	X _{Nj}	ISR(Product j)
⋮						
P _M		X _{1M}	X _{2M}	X _{iM}	X _{NM}	ISR(Product M)

<Figure 2> Product-Store Matrix

To describe the entire supply chain, we need total of MN initial inventory values as shown in <Figure 2>, then we can define ISR for each product j as $ISR(j) = \sum_{i=1}^N \frac{I_{ij}(x_{ij})}{N}$, and, the expected value of ISR is calculated as follows:

$$E[ISR(j)] = \sum_{i=1}^N \frac{F_{ij}(x_{ij})}{N} \tag{1}$$

Now the problem is to minimize each retailer's initial inventory level while meeting the desired expected ISR level for each product kind. One of the most important mission of the central distribution centers for an enterprise is to effectively distribute products to the stores to meet weekly demands. Here, the optimal level of initial inventory for each product at each store will be the necessary input to accomplish the mission. In order words, if the optimal inventory level is $X^* = (X_{11}^* X_{21}^*, \dots, X_{NM}^*)$ and the current inventory level is, $X_{cur} = (X_{11}, \dots, X_{NM})$ then the central distribution centers need to deliver $(X_{ij}^* - X_{ij})$ units to the retail store i and hold $\sum (X_{ij}^* - X_{ij})$ units to meet the desired expected target ISR level.

This problem can be represented as the following model, minimizing the sum of the initial inventory level for the entire retail stores while the expected value of ISR is kept above the pre-determined target level.

$$\begin{aligned} \text{Model 1: } \min \quad & \sum \sum X_{ij} \\ \text{s.t. } \quad & E[\text{ISR}(j)] \geq \alpha_j \quad \text{for } j=1, \dots, M \\ & X_{ij} \geq E[D_{ij}] \quad \text{for } i=1, \dots, N \end{aligned} \quad (2)$$

The inclusion of the second constraint in this model is motivated by the common practice among retailers of maintaining an initial inventory level that meets or exceeds the expected weekly demands. This formulation exhibits similarities with the well-known knapsack problem, which is renowned for its computational complexity. Additionally, solving the problem explicitly poses challenges due to the expected value of ISR generally being a nonlinear function of X . Consequently, a proposed method to address the problem within a relaxed environment will be presented in the subsequent section.

3. Calculation of Optimal Inventory Level

As discussed in Section 2, Model 1 presents a challenging integer nonlinear programming problem that is inherently difficult to solve directly. To render the problem more manageable, we make the assumption that the functions $F_{ij}(x)$ and $f_{ij}(x)$ are continuous with respect to both x and x_{ij} , allowing them to take real nonnegative values. The significance of the integer constraint becomes more pronounced when the product demand at each retailer is scarce. However, in cases where the product demand at each retailer is relatively substantial (e.g., exceeding 100), the integer constraint can be reasonably disregarded.

Given that Model 1 is formulated as a nonlinear programming problem, the optimality conditions can be derived by employing the Karush-Kuhn-Tucker (KKT) conditions. To solve the optimization problem using the KKT conditions, let's introduce Lagrange multipliers for each constraint. Denote the Lagrange multipliers as $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_M$, for the constraints $(1/N)E[\text{ISR}(j)] \geq \alpha_j$, for $j=1, \dots, M$ and as $\beta_{11}, \beta_{21}, \dots, \beta_{NM}$ for the remaining constraints with the form of $X_{ij} \geq E[D_{ij}]$ in Model 1.

The Lagrangian function for the optimization problem is given by:

$$\begin{aligned} L = \quad & \sum \sum X_{ij} + \lambda_1 [(1/N)E[\text{ISR}(1)] - \alpha_1] + \lambda_2 [(1/N)E[\text{ISR}(2)] \\ & - \alpha_2] + \dots + \lambda_M [(1/N)E[\text{ISR}(M)] - \alpha_M] \\ & + \beta_{11}(X_{11} - E[D_{11}]) + \dots + \beta_{NM}(X_{NM} - E[D_{NM}]) \end{aligned}$$

To apply the KKT conditions, we need to differentiate the Lagrangian with respect to each decision variable ($X_{11}, X_{21}, \dots, X_{NM}$) and set the derivatives equal to zero. Additionally, we need to consider the complementary slackness conditions for the inequality constraints.

$$\begin{aligned} \text{Differentiating the Lagrangian function with respect to } X_{ij}: \\ \partial L / \partial X_{ij} = 1 + \lambda_j * (1/N) * f_{ij}(X_{ij}) * f'_{ij}(X_{ij}) + \beta_{ij} \\ = 0, \text{ for all, } i, j. \end{aligned}$$

Now, let's consider the complementary slackness conditions for the inequality constraints:

$$\begin{aligned} \lambda_1 * [(1/N) * \sum f_{i1}(X_{i1}) - \alpha_1] &= 0 \\ \lambda_2 * [(1/N) * \sum f_{i2}(X_{i2}) - \alpha_2] &= 0 \end{aligned}$$

$$\lambda_M * [(1/N) * \sum f_{iM}(X_{iM}) - \alpha_M] = 0$$

$$\beta_{11} * (X_{11} - E[D_{11}]) = 0$$

$$\beta_{21} * (X_{21} - E[D_{21}]) = 0$$

$$\beta_{NM} * (X_{NM} - E[D_{NM}]) = 0$$

Finally, we have the constraints:

$$(1/N) * E[\text{ISR}(j)] \geq \alpha_j, \text{ for } j=1, \dots, M$$

$$(1/2) * f_1(X_{12}) + (1/2) * f_2(X_{22}) \geq 0.95$$

$$X_{ij} \geq E[D_{ij}] \text{ for all } i, j.$$

Solving these equations and inequalities simultaneously will yield the optimal values for $X_{11}, X_{21}, X_{12}, \dots, X_{NM}$.

Summarizing all above equations leads to as follows:

$$\lambda_j(\alpha_j - E[ISR(j)]) = 0 \quad \text{for } \forall j \quad (3)$$

$$(1 - \frac{\lambda_j}{N} f_{ij}(x_{ij}))(E(D_{ij}) - x_{ij}) = 0 \quad \text{for } \forall i, j \quad (4)$$

$$\alpha_j - E[ISR(j)] \leq 0 \quad \text{for } \forall j \quad (5)$$

$$E(D_{ij}) - x_{ij} \leq 0 \quad \text{for } \forall i, j \quad (6)$$

The Lagrangian multipliers corresponds to the sensitivity of the product amount with respect to changes in the expected value of $ISR(j)$. Additionally, since the objective function is linear and the feasible region, determined by the two constraints, forms a convex set due to $F_{ij}(x)$ being a non-decreasing function, it can be ensured that any solution satisfying the optimality conditions is a global optimal solution to our problem. Utilizing the optimality condition presented in equation (3), we deduce that λ_j can be either 0 or $E[ISR(j)] = \alpha_j$.

If $\lambda_j=0$, it implies that $X_{ij} = E(D_{ij})$ for all i, j according to equation (4). Therefore, if $E[ISR(j)]$ is greater than α_j , the optimal value is $X_{ij}^*=E(D_{ij})$ for all i and j . Conversely, if λ_j cannot be zero, indicating that $F_{ij}(E(D_{ij})) < \alpha_j$. In this case, the optimal value is $X_{ij}^* > E(D_{ij})$.

When $\lambda_j \neq 0$, we have $E[ISR(j)] \geq \alpha_j$. As $E[ISR(j)]$ is equivalent to $F(x)$, we can determine X^* by solving the equation $X^* = F^{-1}(\alpha_j)$. Moreover, the corresponding value of λ_j^* can be calculated using equation (4) if $X_{ij} \neq E(D)$. Specifically, $\lambda_j = N / f_{ij}(x^*)$.

In this paper, we focus exclusively on the scenario where the weekly demand follows a normal distribution with a mean of μ_j and a standard deviation of σ_j . Any normal distribution function is a decreasing function in $[\mu, \infty]$, and this property makes sure that the feasible region of our problem is convex. Since the objective function is a linear function, we can guarantee that the solution driven by optimality conditions is the global optimum.

Though it is not easy to calculate X_{ij}^* and $\lambda_j^*, \beta_{ij}^*$ from the set of equations (2) through (6) explicitly, To solve the problem numerically, we can use the SciPy optimization library in Python as illustrated in the following example.

Example 3.1: We consider an illustrative case with two retail stores with two kinds of product in <Table 1>. The demand for product j of each retail store i is normally distributed with mean μ_{ij} and standard deviation σ_{ij} . In addition, μ_{ij} and σ_{ij} are assumed as shown in <Table 1>.

<Table 1> Sample Data

Store product	1	2
1	$D_{11} \sim N(\mu_{11}, \sigma_{11})$	$D_{12} \sim N(\mu_{12}, \sigma_{12})$
2	$D_{21} \sim N(\mu_{21}, \sigma_{21})$	$D_{22} \sim N(\mu_{22}, \sigma_{22})$
	$\alpha_1 = 0.95$	$\alpha_2 = 0.95$
where $\mu_{11}=200, \mu_{21}=150, \mu_{12}=100, \mu_{22}=30$ $\sigma_{11}=20, \sigma_{21}=15, \sigma_{12}=10, \sigma_{22}=5$		

To solve the optimization problem using the KKT conditions, let's introduce Lagrange multipliers for each constraint. Denote the Lagrange multipliers as λ_1, λ_2 for the constraints $(1/2)(F_1(X_{11}) + F_2(X_{21})) \geq 0.95, (1/2)(F_1(X_{12}) + F_2(X_{22})) \geq 0.95$. And denote as $\beta_{11}, \beta_{21}, \beta_{12}$, and β_{22} for the remaining constraints $X_{11} \geq 100, X_{21} \geq 50, X_{12} \geq 100,$ and $X_{22} \geq 50$, respectively.

The Lagrangian function for the optimization problem is given by:

$$L = X_{11} + X_{21} + X_{12} + X_{22} + \lambda_1[(1/2)(F_1(X_{11}) + F_2(X_{21})) - 0.95] + \lambda_2[(1/2)(F_1(X_{12}) + F_2(X_{22})) - 0.95] + \beta_{11}(X_{11} - 100) + \beta_{21}(X_{21} - 50) + \beta_{12}(X_{12} - 100) + \beta_{22}(X_{22} - 50)$$

To apply the KKT conditions, we need to differentiate the Lagrangian with respect to each decision variable ($X_{11}, X_{21}, X_{12}, X_{22}$) and set the derivatives equal to zero. Additionally, we need to consider the complementary slackness conditions for the inequality constraints.

Differentiating the Lagrangian with respect to X_{11} :

$$\partial L / \partial X_{11} = 1 + \lambda_1 * (1/2) * f_1(X_{11}) * f_1'(X_{11}) + \beta_{11} = 0$$

Differentiating the Lagrangian with respect to X_{21} :

$$\partial L / \partial X_{21} = 1 + \lambda_1 * (1/2) * f_2(X_{21}) * f_2'(X_{21}) + \beta_{21} = 0$$

Differentiating the Lagrangian with respect to X_{12} :

$$\partial L / \partial X_{12} = 1 + \lambda_2 * (1/2) * f_1(X_{12}) * f_1'(X_{12}) + \beta_{12} = 0$$

Differentiating the Lagrangian with respect to X_{22} :

$$\partial L / \partial X_{22} = 1 + \lambda_2 * (1/2) * f_2(X_{22}) * f_2'(X_{22}) + \beta_{22} = 0$$

Now, let's consider the complementary slackness conditions for the inequality constraints:

$$\begin{aligned}\lambda_1 * [(1/2) * f_1(X_{11}) + (1/2) * f_2(X_{21}) - 0.95] &= 0 \\ \lambda_2 * [(1/2) * f_1(X_{12}) + (1/2) * f_2(X_{22}) - 0.95] &= 0 \\ \beta_{11} * (X_{11} - 100) &= 0 \\ \beta_{21} * (X_{21} - 50) &= 0 \\ \beta_{12} * (X_{12} - 100) &= 0 \\ \beta_{22} * (X_{22} - 50) &= 0\end{aligned}$$

Finally, we have the constraints:

$$\begin{aligned}(1/2) * f_1(X_{11}) + (1/2) * f_2(X_{21}) &\geq 0.95 \\ (1/2) * f_1(X_{12}) + (1/2) * f_2(X_{22}) &\geq 0.95 \\ X_{11} &\geq 100 \\ X_{21} &\geq 50 \\ X_{12} &\geq 100 \\ X_{22} &\geq 50\end{aligned}$$

We used the SciPy optimization library in Python. In the Python code we defined the objective function as the sum of the decision variables X_{11} , X_{21} , X_{12} , and X_{22} . The constraints are defined using the cumulative distribution functions (CDFs) of the normal distributions, which are calculated using `norm.cdf` from the `scipy.stats` module.

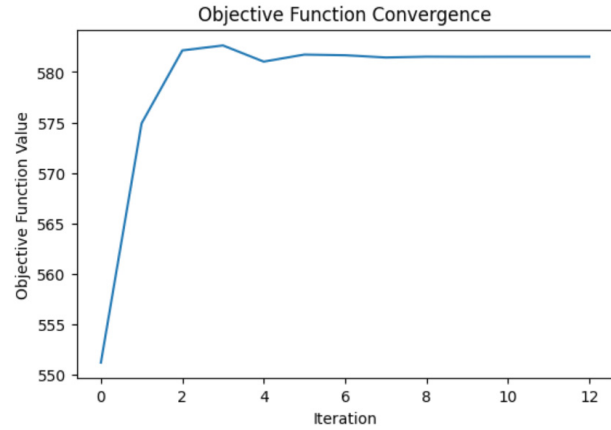
The initial guess for the decision variables is set to $[0, 0, 0, 0]$, and the bounds for the decision variables are defined to satisfy the constraints. The constraints are defined as inequality constraints. Solving these equations and inequalities simultaneously yielded the optimal values for X_{11} , X_{21} , X_{12} , and X_{22} , $X^* = [X_{11}^*, X_{21}^*, X_{12}^*, X_{22}^*] = [231.3, 176.1, 114.7, 59.4]$. The objective function value at the optimal solution is 581.5 and this optimal solution was reached pretty fast after 4 iterations as shown in <Figure 3>.

4. Conclusion

In this paper, we extended our previous research on determining the optimal inventory level for a single product in a retail store to the multi-product setting. Our goal was to address the challenges faced by retailers in managing inventory levels for multiple products while ensuring a desired level of customer service.

To tackle this problem, we introduced the concept of the Inventory Stock Ratio (ISR) as a metric to measure product availability in a multi-product context. We developed an optimization model that aimed to minimize the total inventory level across all product types in each store, while ensuring

that the expected ISR for each product met or exceeded a specified threshold.



<Figure 3> Iterations vs. Objective Function Value

Through our analysis, we demonstrated the existence of an optimal solution for this problem and derived a generic expression for the optimal inventory level that is applicable to any specific customer demand distribution. This expression allows retailers to determine the optimal inventory levels for multiple products in their stores without relying on the exact form of the demand distribution.

Moreover, we extended our investigation to consider cases where the customer demand for each product at the retailer-owned stores follows a known probability distribution, such as the normal distribution. In these scenarios, we derived a general expression for the optimal inventory level, incorporating the distribution parameters of the demand distribution.

While our study focused on the normal distribution as a representative example, future research could explore alternative probability distributions to capture a wider range of demand patterns in retail settings. Investigating different demand distributions would provide valuable insights into the robustness and adaptability of the proposed inventory management approach.

In conclusion, our research contributes to the understanding of optimal inventory management in a multi-product setting, considering stochastic demand and customer service levels. The developed optimization model, along with the concept of the Inventory Stock Ratio, offers a valuable framework for retailers to make informed decisions about inventory allocation and achieve efficient inventory management across multiple product types.

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