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RESEARCH ARTICLE

Understanding Whether and How Prospective Teachers Support Elementary Students to Compare Multiple Strategies in Their Enacted Number Talks

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Abstract

Number talks as a brief instructional routine benefits students and teachers. In general, the routines consist of four steps- introducing, posing questions, collecting answers, sharing ideas. This paper focuses on the *sharing ideas* step in which multiple strategies are shared by students because teachers sometimes do not know what to do with these multiple ideas. One way is to support students to engage in comparison given that teachers are expected to support students to compare strategies in number talks. This paper explores whether and how 15 prospective teachers supported students in their practicum classroom to compare different strategies in their enacted number talk. In this paper, 15 videos of number talks enacted by the prospective teachers were collected. Analyzing the videos produced multiple episodes in relation to comparing strategies, including 1) where prospective teachers created pre-conditions for comparison, 2) where they invited students for comparison, 3) where they pressed students to compare, and 4) where they offered their own way to compare. There were two patterns that might limit the potential of having multiple strategies as conditions for comparison. Additionally, this paper found that even though the prospective teachers missed opportunities to support students to compare different strategies, there were two ways for teachers to support students to engage in comparison. These findings can be used for mathematics teacher educators to support prospective teachers.

Keywords number talks, sharing strategies, comparing strategies, prospective teachers

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I. INTRODUCTION

Number talks are brief instructional routines (5-15 minutes) in which students use mental mathematics to solve computational problems (Parrish, 2011, 2014). In number talks, teachers pose a computational problem on the board and ask students to share their strategies to solve the problem, followed by asking them to explain their strategies and justify their ideas in public. Participating in number talks, students have opportunities to explain and justify their strategies, which supports them to make sense of multiple strategies shared by their peers and reasoning behind the strategies. The routine offers students opportunities to develop number sense as well as number relationships (Humphreys & Parker, 2015; Parker & Humphreys, 2018; Parrish, 2011, 2014; Sun et al., 2018). In the long-term, number talks can support teachers and students to co-construct a math-talk learning community (Hufferd-Ackles et al., 2004, 2015) that puts students' reasoning, problem-solving, and reflection at the center, which reflects what teachers and students do in inquiry-based instruction (Murata et al., 2017; Wood, 2020). Simply implementing number talks in their classroom, however, does not automatically result in benefits for students as evidenced in many reform efforts that were unsuccessfully realized in classrooms (Cuban, 1990). Sherin (2002) argues that a challenging part of inquiry-based instruction is what to do with students' multiple ideas. Number talks might share this challenge because, like inquiry-based instruction, number talks put students' reasoning at the center of instruction. The challenge comes from what to do with such multiple ideas or how to build discussion on different ideas in ways to make sense to students and promote conceptual understanding.

Following number talks routines is likely to produce multiple strategies because the routines explicitly expect teachers to encourage students to share their ideas. In this paper, strategy refers to solutions generated by students to solve computational problems when the word strategy is used. Given Sherin (2002), what to do with the multiple strategies in number talks also will be challenging for teachers, especially for prospective teachers (PSTs), who just began to learn to implement number talks during teacher preparation programs. Kazemi and Hintz (2014) suggest comparing and connecting as a way to deal with such a challenge. Comparing and connecting among strategies can be a way for PSTs to facilitate mathematical discussion in number talks. Comparing multiple strategies has been identified as a policy recommendation for improving mathematical reasoning and problem solving (Woodward et al., 2012). The field, however, knows little of how PSTs support students in comparing different strategies in their enacted number talks. Matney et al. (2020) called for more empirical studies on variation of number talks enacted by teachers and how the variation influences students' learning. This paper addresses such a call for more empirical evidence by looking into PSTs' enacted number talks in terms of whether and how they engage students in comparing strategies.

The purpose of this paper is to explore whether and how PSTs support students in comparing different strategies as implementing number talks. In doing so, this paper presents related literature on number talks and comparing strategies and how this paper draws on the ideas and perspectives in the related literature. This paper explains research

context and participants, how the data were collected and analyzed. This paper illustrates findings from the data analysis, followed by discussion and implications. This paper can help teacher educators as well as researchers with an interest in number talks enacted by PSTs know more about challenges faced by PSTs in relation to comparing strategies. This paper can contribute to understanding ways to support PSTs in developing a knowledge and skill related to comparing strategies in number talks.

II. RELATED LITERATURE

Number Talks Routines

Number talks benefit teachers and students. The instructional routines are known as promoting students' sense-making of mathematical concepts (Humphreys & Parker, 2015; Parker & Humphreys, 2018; Parrish, 2011, 2014; Sun et al., 2018) because students can have more chances to connect different ideas by thinking and talking about how different ideas work together to get the same answer. Sense-making allows students to deepen and solidify their understanding related to number sense and number relationships (Humphreys & Parker, 2015; Parker & Humphreys, 2018). Number talks also support teachers to create a math-talk learning community where students have more ownership of their learning (Murata, 2017; Woods, 2022).

Such benefits may result from enacting number talk routines, which consist of multiple steps- *introducing*, *posing computational problems*, *collecting answers*, *sharing ideas* (Parrish, 2010; Parker & Humphreys, 2018). In more detail, the number talks begin with the *introducing* step in which teachers briefly state their expectations and procedures, including use of hand signals and multiple strategies. In *posing computational problems*, teachers pose a computational problem on the board and give students ample time to think about answers and strategies. The problem can be whole numbers or fractions as well as algebraic equations. In the *collecting answers* step, without evaluating, teachers collect answers by recording them on the board after all students show that they are ready to share their answers by putting their thumbs on their chest. Answers could be single or multiple depending on how students think. As recording answers, teachers are expected to be no evaluative. Once answers are recorded on the board, the *sharing ideas* step follows. Teachers ask students to share multiple ways of how they get the answer. It is important to have multiple ways to solve a problem in number talk because multiple strategies can allow teachers and students to engage in each other's ideas and reasoning. By asking how and why questions for justifications, teachers try to make sense of students' ideas as they record each strategy. As such, the representations of the strategy on the board tend to be specific and detailed. In this *sharing ideas* step, teachers also invite students to engage in each other's strategies to help them make sense of their peers' ideas. Students' sense-making of their peers' strategies help them to compare and connect different strategies recorded on the board. Teachers sometimes offer their ways to solve the computational problem. If there is another computational problem, the cycle of number talk routines (*introducing*, *posing computational problems*, *collecting answers*, *sharing ideas*) is iterated. Matney et al. (2020)

and Woods (2022) show that what teachers do in each step of the number talk routines is supported by literature on teaching practices when they explain number talk routines.

To increase the benefits from the number talks on the part of students, teachers need to carefully implement number talks especially in the *sharing ideas* step (Parrish, 2011, 2014; Parker & Humphreys, 2018). Given what Wood et al. (2006) argue for developing an inquiry/argument classroom culture in mathematical instruction, as implementing number talks, teachers need to go beyond strategy reporting, which means that students share their strategies without having a chance to compare and connect strategies. They have to support students to express mathematical thinking by justifying and comparing and connecting in the *sharing ideas* step, which is more likely to produce a greater quality of mathematical thinking. As a mathematics teacher educator who has taught PSTs about number talks over the last three years, I observed that in the *sharing ideas* step, asking students to share their answers and strategies is hardly followed by comparing and connecting different strategies. Comparing strategies is a crucial way for students to make connections between strategies (Kazemi & Stipek, 2001) and engage in each other's ideas (Franke et al., 2015). Comparing different strategies is also rarely seen when teachers learn to implement number talks in their classrooms (Woods, 2022). As such, in this paper, I pay attention to how PSTs who learn to implement number talks support students in comparing and connecting multiple strategies in their enacted number talks.

Comparing Multiple Strategies

As mentioned above, simply sharing strategies does not guarantee deeper conceptual understanding on the part of students (Wood et al., 2006). Students more likely develop a much deeper understanding when the teachers ask them "to examine the mathematical similarities and differences among multiple strategies" (Kazemi & Stipek, 2001, p. 130). Goldstone et al. (2010) suggest that "the process of comparison plays a critical role in problem solving, judgment, decision making, categorization, and cognition, broadly construed" (p. 103). In classrooms with high-press teacher-students interaction patterns where students are pressed to extend their mathematical thinking, the teacher tended to invite students to "compare the strategies that had been presented thus far", which helped them see how "different solutions differed mathematically" (Kazemi & Stipek, 2001, p. 131). This kind of comparing different strategies to the same problem is known as increasing students' procedural flexibility and procedural and conceptual knowledge (Rittle-Johnson, Star, & Durkin, 2009; Rittle-Johnson & Star, 2011).

There may be multiple conditions for students to effectively compare multiple strategies. First, there need to be multiple strategies to compare with. Even though having multiple strategies to compare is obvious, it is a fundamental condition for comparison. Teachers need to pose problems purposefully that can encourage students to come up with specific strategies (National Council of Teachers of Mathematics [NCTM], 2014 Boaler, 2015; Stein & Smith, 1998). Kazemi and Hintz (2014) suggest beginning to compare two different strategies that are mathematically connected. Second, teachers need to invite students to compare different strategies (Kazemi & Stipek, 2001). The invitation could be a simple talk move ("What's similar and what's different about these two students'

strategies?") or probing sequence of specific questions (Franke et al., 2009). Third, teachers need to press students to think about and discuss similarities and differences across strategies (Franke et al., 2015; Kazemi & Hintz, 2014; Kazemi & Stipek, 2001). Overall, teachers need to facilitate meaningful mathematical discourse (NCTM, 2014)

The multiple conditions mentioned above suggest what teachers need to think about to facilitate mathematical discussion when students compare different strategies in number talks. In number talks, students are more likely to have multiple strategies, especially when teachers pose purposeful problems and ask students to share different strategies after giving enough time to students (Parrish, 2014). Students also need to be supported by teachers to make sense of each strategy shared by their peers. The support can come from teachers' detailed recordings that represent well what students explain about their strategy and teachers' facilitation of discussion on each strategy. Teachers also need to invite students to compare different strategies based on their sense-making of each strategy, followed by pressing them to identify similarities and differences across strategies.

Comparing strategies in the *sharing ideas* step of the number talk routines seems to be challenging for especially PSTs. Research shows that some teachers rarely provide students with opportunities to compare different strategies when they begin to implement number talks (Woods, 2022). Knowing more of how PSTs support students to compare different strategies in their enacted number talks can help mathematics teacher educators with an interest in number talks support PSTs to facilitate more meaningful discussion in terms of comparing different strategies. How PSTs support students to compare different strategies in their enacted number talks, however, is unclear. As such, this paper explores ways PSTs support students to compare different strategies in their enacted number talks. Building on the related literature, I pose a research question that drives this study: whether and how do PSTs support students to compare multiple strategies in their enacted number talks?

III. METHODS

Research Context and Participants

In this paper, I analyzed the data from a research project. The research project got approved from the Institutional Review Board (IRB) at a southwestern university in the U.S in Spring 2021. The purpose of the research project was to examine how PSTs in mathematics methods courses learned to effectively implement facilitating meaningful mathematical discourse, one of the effective teaching practices in mathematics education (NCTM, 2014). The course was designed to support the PSTs to learn to implement the teaching practice of facilitating meaningful mathematical discourse through enacting number talks and mathematics lessons in their practicum classroom (Grade 3 to 5). The author was the instructor of the course. There were 29 PSTs enrolled in the mathematics methods course in Spring 2021. The PSTs were in their second semester of the four-semester-long elementary education program when the research project was conducted. They took two mathematics content coures as pre-requisites before entering the program.

All of them were whites and one of them was male. The school sites where their practicum took place were in a school district with about 20 elementary schools. In the course, the PSTs were expected to complete multiple course projects, including Number Talks and Math Lesson. In the two course projects, the PSTs engaged in a learning cycle of designing a plan, revising the plan based on the instructor's comments, rehearsing the revised plan in the class, enacting the revised plan in their practicum classroom as video-recording, and reflecting on the teaching practices on the video. The PSTs submitted plans, revised plans, videos, and reflection papers to a virtual learning management system.

In the research project, two types of data were collected: 1) electronically submitted course assignments and 2) 30-minute semi-structured interviews. Twenty one of 29 PSTs consented to data collection related to electronic course submissions. Files electronically submitted by the participants, including plans, revised plans, videos, and reflection papers were collected. Eleven of 21 PSTs agreed to participate in the interviews. The interviews were conducted to understand more specific information on their enacted number talks and their perceptions of the relationships between number talks and facilitating meaningful mathematics discourse. The 11 interviews were audio-recorded and transcribed for analysis. These interview data were not analyzed for this paper because they were not designed for the purpose of this paper.

Number Talks Project

As for the Number Talks project, the PSTs were expected to design a number talk plan, revise the plan based on the instructor's comments, rehearse the revised plan in the class, enact the revised plan in their practicum classroom as video-recording, and reflect on the teaching practices. In particular, when designing their number talk plan, they needed to use a number talk plan template created by the instructor, which included their plan on what to do in each component of the number talk routines- introducing, posing questions, collecting answers, sharing ideas (Parrish, 2011, 2014; Parker & Humphreys, 2018). The PSTs were asked to plan how to ask students to share strategies, how to have students compare different strategies, and how to ensure students' wide participation. To complete the project, they submitted number talk plans, revised plans, videos, and reflection papers to a virtual learning management system. Even though the learning cycle in the methods course may not guarantee that PSTs apply what they learned to the enactment of number talks in their field placement classroom, PSTs' engagement in such a learning cycle, especially rehearsal of the revised plan or feedback on their rehearsal, as the approximation of number talks may increase the possibility for PSTs to enact their number talks as planned.

Data Collection

For this specific paper, only number talk videos were used as data sources because videos of number talks could provide me with instances of whether and how the PSTs asked students to compare the strategies right after having students share their strategies. Initially, 21 number talk videos were initially collected, but I decided not to use six number talk videos with a low quality of audio, which did not clearly provide what the teacher and students talked about for the analysis. As such, only the 15 number talk videos were used

as data sources.

The 15 number talks were different from one to another in three ways (Table 1). First, the time length of number talks ranged from about five minutes to 15 minutes, which met the recommendation of the experts on the number talks (Parrish, 2011, 2014; Parker & Humphreys, 2018). Second, the grade levels were different (Grade 3 to 5). The differences occurred because each PST was placed in different classrooms and grade levels Third, they were different with respect to the number of students they worked with for the number talks. Most PSTs did number talks in the whole class. A few PSTs chose to do number talks with a small number of students. Even though the instructor expected them to work with the whole class, deciding the number of students to work with was left to each PST's discretion to meet different needs (e.g., social distancing) of their practicum classroom under the COVID-19 pandemic.

Table 1. Contextual information of number talks collected as data sources

ID	Grades	Lengths	Numbers of problem (operation type)	Working with
PST01	3	11:43	4 (addition)	5 students
PST02	4	9:04	2 (addition/ subtraction)	3 students
PST03	5	10:44	4 (multiplication)	whole class
PST04	4	11:30	3 (multiplication)	whole class
PST05	3	13:49	2 (addition)	whole class
PST06	5	11:06	3 (multiplication/ addition)	5 students
PST07	3	11:08	1 (addition)	whole class
PST08	5	10:44	4 (multiplication)	whole class
PST09	4	12:41	2 (division)	whole class
PST10	3	4:34	1 (addition)	whole class
PST11	4	15:15	1 (measurement story problem)	whole class
PST12	3	13:54	2 (multiplication)	whole class
PST13	5	8:45	1 (addition)	whole class
PST14	5	9:52	3 (multiplication/ addition)	5 students
PST15	4	12:12	3 (addition)	whole class

Data Analysis

A thematic analysis (Saldana, 2015) was used to analyze the video data collected from the number talks project. As mentioned earlier in the research context, the PSTs were not required to have students to compare different strategies in their enacted number talks. Instead, the project expected them to support students in comparing strategies if they could because they were learning to effectively implement number talks for the first time. As such, the videos of number talks might be the ones that could show whether and how the PSTs support students in comparing strategies. There were three steps to analyze the video data.

The first step was to begin with coding individual number talk videos using four codes (introducing, posing questions, collecting answers, sharing ideas), which

corresponded to each step of the number talk routine (Parrish, 2010; Parker & Humphreys, 2018). As a result of this step, individual number talk videos were divided into multiple episodes, each of which corresponded to one of the four codes.

The second step was to examine the episodes with the sharing ideas code and identified and developed sub-codes (Table 2). The sub-codes were initially built on the three conditions for comparison (See the Related Literature section). As analyzing the videos, additional sub-code emerged. That is, some PSTs offered their own ways to compare different strategies. As a result, the sub-codes included 1) *creating conditions for comparison*, 2) *inviting students to compare*, 3) *pressing students to compare*, and 4) *teachers offering ways to compare*. These four sub-codes were used to code episodes, which were coded as sharing ideas in the first step. This second step helped me identify episodes of the four sub-codes and investigate whether and how PSTs supported students to compare different strategies in the episodes.

The third step was to look for patterns regarding the four sub-codes of the sharing ideas code within and across episodes. The focus was given to whether and how each sub-code played out in individual episodes by looking at how individual strategies were treated by the PSTs. For example, in number talks with two computational problems, students might come up with different numbers of strategies as working on each computational problem. Even within an episode with several strategies, the time that PSTs spend digging each strategy might not be the same. Building on the patterns within episodes, I also identified certain patterns emerging across episodes in terms of the four sub-codes. The patterns will be presented in the next section.

Table 2. Descriptions of the four sub-codes of the sharing idea code

Sub-codes	Descriptions	Examples
Creating conditions for comparison	PSTs ask students to share different students in relation to each computational problem. As a result, there are multiple problems that might be used for comparison. (Kazemi & Hintz, 2014; Stein & Smith, 1998)	Does anyone have another strategy? Is there a different idea to get the answer?
Inviting students to compare	PSTs provide students with an opportunity to think about and talk about similarities and differences (Franke et al., 2009; Kazemi & Stipek, 2001). This code is usually related to a single talk move.	Did you guys see we got different strategies but we got the same answers? Does everyone see how Jane and John did differently?
Pressing students to compare	PSTs take up the opportunity for students to compare different strategies. This code may appear after PSTs invited students to compare strategies (Franke et al., 2015; Kazemi & Hintz, 2014; Kazemi & Stipek, 2001).	What's similar and what's different about these two students' strategies?
Teachers offering ways to compare	PSTs explicitly provide students with ways to compare different strategies.	Class, see these two strategies. They are similar to each other in terms that they are using place value to make the numbers easy to add.

IV. FINDINGS

First, this section presents an overview of the flow of number talks enacted by the 15 PSTs. Second, it explains whether and how the PSTs created multiple strategies as pre-conditions for comparing different strategies. Third, it illustrates two different ways for the PSTs to support opportunities for students to compare different strategies.

An Overview of the Flow of Number Talks Enacted by the PSTs

As shown in Figure 1, all number talks but one followed the number talk routines, which means 14 number talks consisted of introducing, posing questions, collecting answers, and sharing ideas (Parrish, 2011, 2014; Parker & Humphreys, 2018). The number talks enacted by PST14 did not have collecting answers. The PST14 did not collect answers after posing a problem and waiting. Instead, the PST asked students to share their answers and strategies. Overall, all number talks had the *sharing ideas* step.

In relation to the sharing idea, all number talks were able to create pre-conditions for comparison by putting multiple strategies on the board. Even though the numbers of the strategies provided by students varied from one to eight, each number talk had an episode(s) with more than two strategies (See PST01 in Figure 1 as an example). Three of 15 number talks had episodes where PSTs invited students to compare different strategies. There were only three episodes where a PST pressed students to compare different strategies (PST13) and where a PST offered students her way to compare (PST11 and PST12), respectively.

Multiple Strategies as Pre-conditions for Comparison

Overall, the enacted number talks showed that the PSTs were able to create pre-conditions for students to compare different strategies. Out of 38 episodes where students shared multiple strategies, 36 episodes had multiple strategies shared by different students. The two episodes had only one strategy shared by one student, which would not be a pre-condition for students to compare.

Two patterns were found that might limit the potential of having multiple strategies as pre-conditions for comparison. The two patterns were related to 1) the time length for students to spend on each strategy and 2) engagement of students in other strategies across number talks. These patterns are presented in this section because each pattern shapes whether and how effectively comparing strategies takes place.

The first pattern was that the actual time students spent sharing their strategies was typically short. The whole number of the strategies in the 36 episodes were 119. Eight-night of 119 strategies were short between 7 and 60 seconds. Sixty-three of 89 strategies were shared in length between 7 seconds and 30 seconds. As for strategies in this time length, students merely reported what they did step by step and were not asked for justification. Twenty-six strategies had more than 30 seconds for sharing. Of the 21 strategies, seven strategies were shared for more than two minutes. They took longer because some students in the episodes had a hard time clearly explaining their ideas or there were interruptions by other teachers.

The second pattern was that as the PSTs asked students to share their strategies,

they did not offer chances for students to engage deeply in the strategies on the board. The PSTs tended to limit conversation to the students who were sharing strategies and sought another strategy without having other students engage in the strategies. This pattern is tied to the first pattern. The less students engaged in each other's strategies, the shorter time on strategies was spent. Without engaging more fully in each other's ideas, students do not have more chance to make sense of how strategies work.

Opportunities to Support Students to Compare

In relation to opportunities to support students to compare different strategies, the first finding was that the 12 number talks missed opportunities to support students in comparing different strategies. The second finding was that the three number talks provided a space for students to engage in comparing strategies. These two findings will be presented by providing excerpts from some PSTs' enacted number talks.

First, the 12 number talks did not have episodes where the students were pressed to compare strategies shared by their peers. The students did not have any opportunities to engage in comparing strategies. Eight of the 12 number talks did not have any episodes with teachers supporting students in comparing strategies. The interaction between students and PSTs in these episodes were similar to a teacher-students interaction pattern in a strategy reporting classroom culture, in which students reported several strategies in a serial manner without being pressed to compare and connections among the strategies (Wood et al., 2006).

One example comes from PST03's number talks working with the whole class in her 5th grade practicum classroom. The number talks lasted for about 11 minutes and had four multiplication problems (2×18, 2×24, 16×3, 28×2). The following number talk occurred when they worked on the last problem (28×2). In the number talk, the PST03 asked students to share their strategies, which produced four strategies from different students. Only three strategies will be presented here in this example because the last student's explanation was inaudible and the teacher's re-presentation of the student's strategy on the board was not clear.

The PST03 posed the problem and collected different answers but there was only one answer (56), which was agreed by the class. The interaction excerpt began when the PST asked students to share their strategy.

As shown in the figure, the PST03 asked students to share their strategies in a serial manner. The total amount of time spent on the three strategies was about one minute. Scott spent about 7 seconds explaining his strategy to the teacher. The PST only interacted with the student who was sharing a strategy. From watching the video, there was no opportunity for other students to engage in the strategies on the board.

Only three of the 12 number talks included five episodes followed by another episode where teachers invite students to compare different strategies. Even when the teacher invited students to compare different strategies, the students were not pressed enough for the students to compare different strategies as a follow-up move.

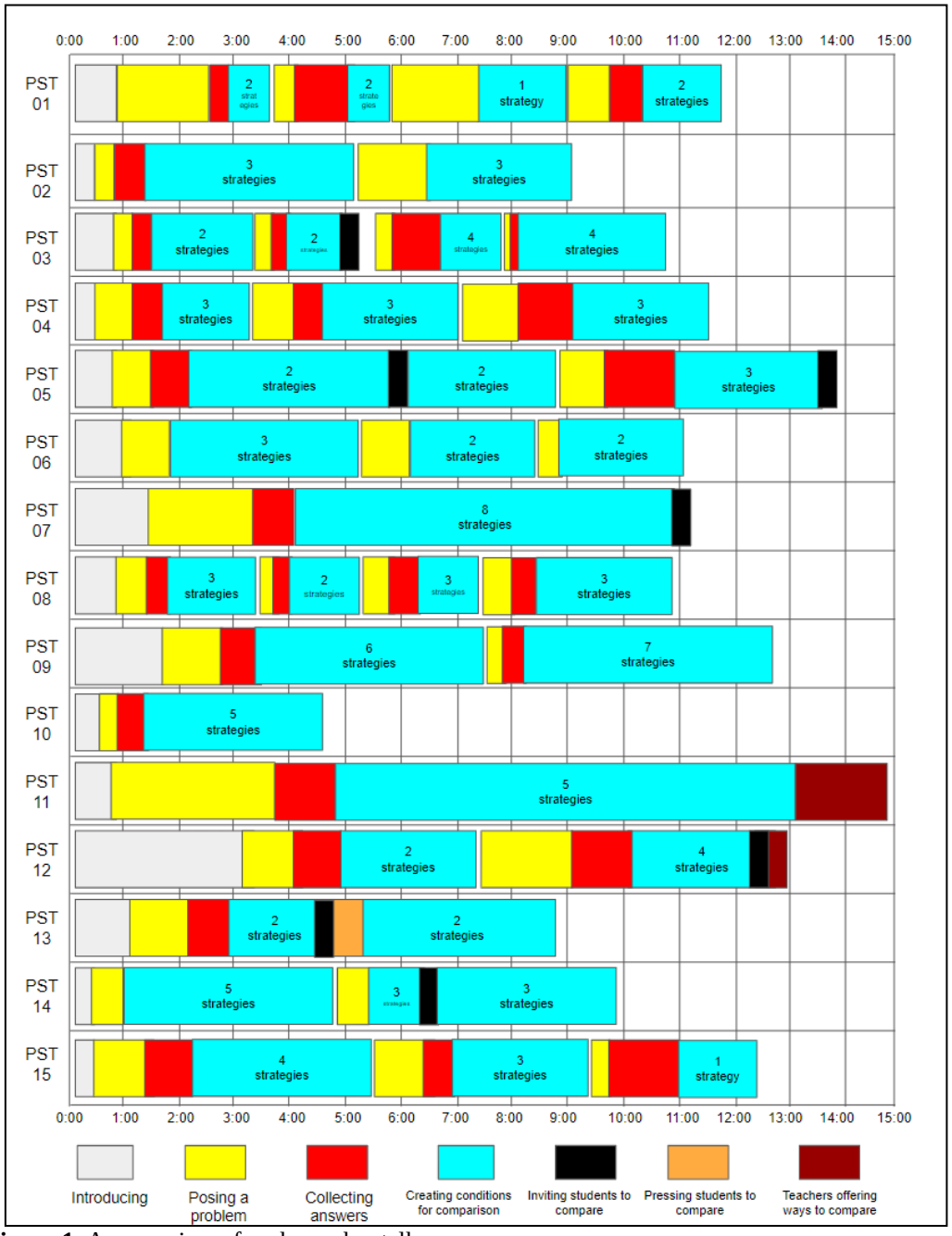
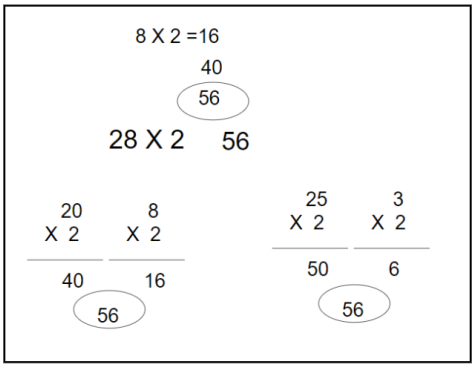


Figure 1. An overview of each number talk

One example comes from the PST03's number talks mentioned above. The following number talk occurred when they worked on the second problem (2 × 24). In the number talk, the PST03 asked students to share their strategies, which produced three

strategies from different students. One of the four problems was not recorded because the student (Riley [all students' names are pseudonyms]) said she just knew it. As such, Figure 3 shows only two strategies. The PST03 posed the problem and collected different answers but there was only one answer (56), which was agreed by the class. The interaction excerpt began when the PST asked students to share their strategy, following the students agreeing on 48 as an answer.



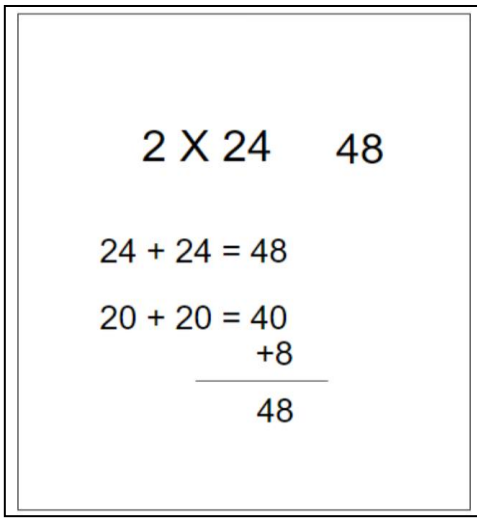
[8:19] **PST03**: Does anyone want to share it? [Looking around]
[8:23] **PST03**: Kay, how did you solve it? [Looking at the student]
[8:26] **Kay**: I just did 20 plus 20 to get 40. And 8 plus 8 equals 16. Add 40 plus 16 to get 56 [The PST is recording Kay's strategy in a vertical form on the board]
[8:43] **PST03**: Scott, how'd you do it? [Looking at the student]
[8:44] **Scott**: I just did 25 times 2, which equals 50. [The PST is recording what Scott said in a vertical form]. And I added 6.
[8:51] **PST03**: nice. [Still recording]
[9:04] **PST03**: Cole, how did you solve it? [Looking at the student]
[9:06] **Cole**: I did 8 times 2 to get 16. And 20 times 2 equals 40. Add 16 plus 40 equals 56.
[9:26] **PST03**: So you did the standard method. okay.

Figure 2. Strategies and interaction excerpts of the PST03's number talk

As shown in Figure 3, the PST03 asked students to share their strategies in a serial manner. The total amount of time spent on the three strategies was about one minute. Scott spent about 7 seconds explaining his strategy to the teacher. The PST only interacted with the student who was sharing a strategy. This episode was followed by teachers inviting students to compare strategies when she said, "Did you guys see how we did all the different thinking but we got the same answers? That's why we are doing this. I wanna show you guys there are different ways so we can think about, not just get the answer" (4:50). In this invitation, the PST03 initiated a chance to compare strategies but did not go further to have students compare different strategies. The PST03 went back to ask for another strategy, instead of pressing students to compare the two strategies on the board. Overall, regardless of whether the PSTs invited students to compare different strategies, the PSTs in the 11 number talks missed opportunities for students to compare different strategies.

Second, three number talks offered students with opportunities to engage in comparing multiple strategies in two different ways: 1) The students were pressed to compare strategies (PST13) and 2) the PSTs offered a way to compare different strategies (PST11, PST12). As for pressing students to compare strategies, the number talks enacted by PST13 included one episode with pressing students to compare strategies. The episode comes from the PST13's number talks working with the whole class in her 5th grade practicum classroom. The number talks lasted for about nine minutes and had one addition problem (15 + 16). In the number talk, the PST13 asked students to share their answers and there was only one answer shared (31). The PST13 asked students to share their strategies, which produced four strategies from different students. After recording the two strategies on the board, the PST13 decided to press students to compare the two strategies. The other

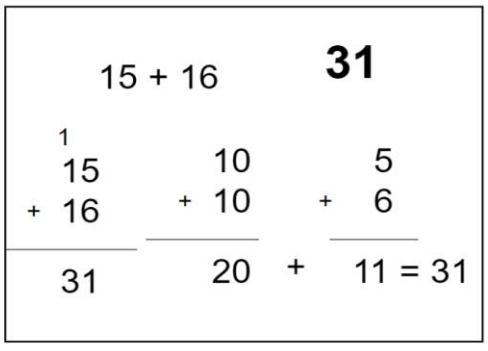
two strategies were shared after students had a discussion. As such, Figure 4 shows only two strategies. The excerpt below also shows when the PST13 began to have students compare the two strategies.



[3:50] **PST03**: What did you do to solve it? [Looking around]
[3:53] **PST03**: Riley. [Looking at the student raising hands]
[3:54] **Riley**: I just looked at it and got it.
[3:57] **PST03**: You just looked at it and knew?
[3:59] **PST03**: Bart, you wanna share it?
[4:02] **Bart**: I added 24 plus 24 to get 48 (The PST is recording his strategy horizontally on the board).
[4:11] **PST03**: Does anyone solve it in a different way?
[4:14] **Sara**: I solved like 12 times 4 because [inaudible]...
[4:24] **PST03**: You double this number? [Pointing to the number 2 on the board.] Perfect!
[4:37] **PST03**: Lily, how did you do it to solve it?
[4:39] **Lily**: I did 20 plus 20 to get 40 and added 4 and 4 to get 8. Added 40 to 8 and got 48. (The PST is recording his strategy horizontally on the board).
[4:47] **PST03**: Awesome.
[4:50] **PST03**: Did you guys see how we did all the different thinking but we got the same answers? That's why we are doing this.I wanna show you guys there are different ways so we can think about, not just get the answer"
[5:00] **PST03**: Does anyone solve it another way? [Students silent]

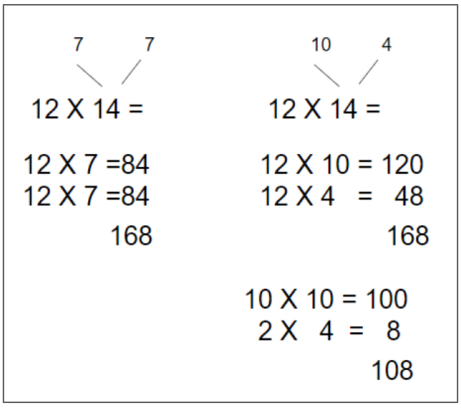
Figure 3. Strategies and interaction excerpts of the PST03's number talk

As shown in Figure 4, the PST13 asked students to press students to compare the two strategies on the board, followed by the PST inviting students to compare the two strategies ("Let's first compare these two strategies. What's similar and what's different about these two students' different strategies?" (4:28)). The PST tried to engage students in comparing strategies. Instead of moving to another strategy, the PST waited for students to raise their hands after having them think. By pressing students to compare the two strategies, two students volunteered their ways to compare the two strategies ("One of them has multiple steps and another just has…" (4:28)).



[4:20] **PST13**: Great! Alright! Does anybody have a different strategy on that? I can see [inaudible]. I am gonna extend this. Do you remember what this looks like?
[4:27] **Students**: Yes
[4:28] **PST13**: Let's first compare these two strategies. What's similar and what's different about these two students' different strategies?
(Students are raising their hands to share their ideas.)
[4:41] **PST13**: Katie.
[4:43] **Jason**: They both got the same answers.
[4:45] **PST13**: They both got the same answer. That's a similarity. Do you know the difference between these two? (Looking at Jason) Do you see the difference between these two strategies? (Looking around and calling on another student). Cecelia.
[4:51] **Cecelia**: One of them has multiple steps and another just has…
[4:57] **PST13**: That's right, right. Jim broke the problem into two different addition problems while Audrey kept it in one. (Looking at all students). Great! Those are great observations.
[5:05] **PST13**: Raise your hands if you have another strategy.

Figure 4. Strategies and interaction excerpts of the PST13's number talk



[12:09] **PST12**: Anybody has a different way? [Looking around] (Some students saying no.)
(The PST is waiting)
[12:14] **PST12**: Okay. Would anybody want to restate thinking of somebody else? (Looking around but no raising hands).
[12:18] **PST12**: Let's compare these answers. (Pointing out the top-left strategy and top-right strategy.)
[12:21] **PST12**: So these two people just decomposed numbers the same way, right? (Looking at the students). They just broke this number apart in a different way, right?
[12:29] **PST12**: (Pointing the bottom-left strategy) She also broke it up into tens… (Pausing and thinking of how to explain) And she broke it up… ten times ten… two times four, right? (Looking at the students)
[12:43] **PST12**: Okay, alright. And… I am gonna show you another way I think about it. (The PST explaining her way)

Figure 5. Strategies and interaction excerpts of the PST12's number talk

As for PSTs offering a way to compare different strategies, the number talks enacted by PST11 and PST12 included two episodes offering a way to compare different strategies. One example comes from the PST12's number talks working with the whole class in her 3rd grade practicum classroom. The number talks lasted for about 14 minutes and had two multiplication problems (5 × 14, 12 × 14). The following number talk occurred when they worked on the second problem (12 × 14). In the number talk, the PST12 asked students to share their strategies, which produced three strategies from different students. The PST12 posed the problem and collected different answers but there were two answers (168 and 108). The interaction excerpt began when the PST offered a way to compare the three strategies on the board.

As shown in the figure, the PST12 invited students to compare the two strategies ("Would anybody want to restate thinking of somebody else?" (12:14)) and waited for a short time (about five seconds). The PST12 went on to offer her own way to compare the three strategies on the board. She began with the two strategies on top of the board ("They just broke this number apart in a different way, right?" (12:21)). The PST12 also tried to compare the strategy on the bottom, which had an incorrect answer (108). The PST12 focused on how numbers were decomposed but had a hard time explaining the strategy ("She also broke it up into tens… And she broke it up… ten times ten… two times four, right?" (12:29)). The PST tried to have students engage in comparing the strategies but ended up with providing her own way to compare the three strategies on the board.

V. DISCUSSIONS AND IMPLICATIONS

In summary, this study found that the PSTs were able to successfully create the pre-conditions for students to compare different strategies in number talks, but they did not go further to engage students in comparing strategies. It found that many number talks missed opportunities to support students to compare strategies. It also found two ways that

teachers supported students to compare different strategies in number talks; 1) pressing students to compare strategies and 2) offering their own way to compare strategies.

There are two points to discuss in relation to the findings of this paper. First, comparing strategies is challenging for the PSTs as they implement number talks. Second, there are some conditions for PSTs to support their students to compare different strategies. The first discussion point is that this study is similar to Kazemi and Stipek (2001). Kazemi and Stipek (2001) included examples that experienced mathematics teachers who valued reform-oriented practices (e.g., valuing students' thinking) did not always go beyond sharing multiple solutions to compare strategies in the whole class discussion. This study differs from their finding in that it paid attention to PSTs' enactment of number talks in their field placement classroom. The findings reported in this paper are important in terms that they showed multiple episodes regarding comparison. Number talks may be a powerful tool for teachers to support students in developing sense-making and conceptual understanding (Humphreys & Parker, 2015; Murata, 2017; Parker & Humphreys, 2018; Parrish, 2011, 2014; Sun et al., 2018; Woods, 2022); However, just following the number talks routines may not guarantee such benefits for students without having them engage in each other's ideas, if possible, by comparison. This paper shows comparing strategies may be rarely seen in the PSTs' enacted number talks (See Figure 1), which suggests a challenging aspect of implementing number talks. Only two episodes were followed by pressing students to compare or offering their own way to compare. This paper argues that given the findings of this paper, it is worth investigating what PSTs bring to the mathematics content/methods courses in relation to content knowledge of and perspectives on comparing strategies. It is also worth exploring ways to support PSTs in teacher preparation programs in engaging students in comparing strategies in number talks. One way is to design an instructional activity for PSTs to compare multiple strategies students might generate before designing a number talk plan and implementing the plan. Another way is to press PSTs to plan ahead to pose a computational problem and anticipate strategies that are likely to be generated by students in their field placement classroom.

As for the second discussion point, the field has to more fully support PSTs to develop how to prepare for and facilitate comparing strategies. Like what Sherin (2001) emphasized in her paper, PSTs also need to develop ways to build on multiple strategies in number talks. What the paper illustrated suggests that comparing strategies is one way to make number talks benefit students and there might be multiple ways. PSTs need to go beyond asking students to share their ideas in a serial manner like shown in the PST03's number talks (Figure 2) or simply inviting students to compare strategies (Figure 3). Instead, like in the number talks enacted by PST12 (Figure 5) and PST13 (Figure 4), PSTs may need to give students ample time to think about how they would compare strategies and furthermore press students to compare strategies. Students need to make fuller sense of each strategy and process what makes sense to them before comparing. Students cannot know what to compare during discussion without making a fuller sense of how each strategy works to get the answer to the problem. This paper provides clear instances of what it looks like to engage students in comparing strategies, which is a contribution of this paper to the field of mathematics teacher education. This paper suggests exploring ways to

support PSTs to develop a sense of how to compare different strategies as they choose problems.

This paper has several limitations. First, this paper cannot show how the 15 PSTs understood multiple strategies because the study was not designed to measure such knowledge. Second, this finding may not tell how PSTs in other teacher preparation programs enact number talks and support students in comparing different strategies. Third, the number of the videos was limited. This paper examined 15 videos of number talks dominantly with whole numbers. It might not be enough to say what PST could do with comparing strategies. The future study may need more number talks building on the findings of this paper. Fourth, this paper did not use other types of data rather than videos of number talks. Analyzing videos may not be enough to fully understand the contextual factors that limit the PSTs to sharing strategies. The future study needs to collect and analyze interviews with a focus on their intention on (non-)verbal actions.

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