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RESEARCH ARTICLE

A Cross-case Analysis of the Use of Qualitative Research Methods in Mathematics Education Focusing on Series E Journal: Exploring to Current Practices and Future Possibilities

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Abstract

In the context of Korean educational research, the number of qualitative research studies has gradually increased since 2000. It has become one of the most important research methods today. The field of math education is no exception to this trend, and qualitative approaches are now becoming one of the main research methods. This increase in qualitative research has contributed to the provision of detailed information about educational practice, but at the same time, the overall level of credibility in the results of qualitative research seems to be lower than that of quantitative research. This study started with the problem consciousness that the number of qualitative studies is increasing in the field of mathematical education, but there is a lack of discussion on the methodology of applying qualitative research methods. In this study, among the papers published in the journal related to mathematical education, papers using a qualitative approach are analyzed focusing on cross-case analysis. Based on the analysis results, the tendency to use qualitative approaches is diagnosed, ways of improving the validity and trustworthiness of qualitative research results in the field of mathematical education are examined, and implications and suggestions are presented.

Keywords mathematics education, qualitative research, cross-case analysis, validity in qualitative research

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I. INTRODUCTION

Over the past two decades, the number of studies using qualitative methods has been gradually increasing. As a result, qualitative methods have become one of the primary research methods in mathematics education in Korea. However, along with this trend, an important issue has been continuously raised. This is a controversy over research methods and findings using qualitative approaches, and consequently qualitative approaches have become less recognized than their quantitative counterparts (Hammersley, 1992; Guba & Lincoln 1989; Koch & Harrington, 1998; Morse et al 2002; Patton, 1990, Phillips, 1987; Sandelowski, 1986). In other words, it is mainly related to the validity and quality of the individual studies using qualitative approaches, raising concerns about the credibility of qualitative research.

With this in mind, this study aims to examine how qualitative methods are being applied in mathematics education in Korea. Specifically, among all the papers published in a mathematics education journal between 2019 and 2020, 6 papers that mainly used qualitative approaches are selected and analyzed. This study seeks to provide implications for improving the validity and quality of qualitative research practice based on a detailed qualitative analysis of individual papers followed by cross case analysis. Based on a detailed qualitative analysis of individual papers and a cross-case analysis, this study provides implications for the validity and quality of qualitative research practices.[1] Here are research questions that this study attempted to answer.

1. How are individual papers using qualitative research methods in math education?
2. What are the results of a cross-case analysis based on the results of a case-by-case analysis of individual papers applying qualitative research methods??
3. Based on the results of the cross-case analysis, what discussions and recommendations can be made to improve the quality of qualitative research?

II. RELATED LITERATURE

Analysis of Qualitative Research: Within Case Analysis and Cross Case Analysis

The purpose of analyzing an individual qualitative study is to seek a detailed description, understanding, and explanation of what happened and how in an individual case or research site (Milles & Huberman, 1994; Patton, 2002). Within-case analysis and cross-case analysis are also practiced in individual studies. The case refers to the unit of

[1] The results of the individual case analysis were published in Series E, 35(2), 137-152, and this paper focuses on the results of the cross-case analysis.

analysis, i.e., if the purpose of the study is to look at the different responses of students, the unit of analysis is the students; if the purpose of the study is to look at the responses of different schools, the unit of analysis is the schools.

To illustrate the process of analyzing an individual qualitative study, this researcher uses an example of analyzing interview data. Suppose a researcher interviews 10 students using a structured interview questionnaire (10 questions presented in a sequence). The researcher would first write down each individual student's responses to the 10 questions and then read through them repeatedly, looking for important themes, patterns, etc. This is the equivalent of a within case analysis. Then, analyzing the 10 students' responses in aggregate for common or differentiated patterns of response to the overarching themes found is a cross-case analysis. In general, within-case analysis is done first in individual studies, and then cross-case analysis is done based on that. It is not advisable to do both at the same time, as this can lead to confounding. However, if the focus of the study was on a specific mathematics education program for students, the analysis could start with a description of variations in their answers to common questions. for example, what were patterns of the program experiences, what did they like or dislike, how did they think they had improved, and so forth (Patton, 2002)..

If a researcher utilized a structured questionnaire, as in the example above, the cross-case analysis is relatively easy because the students' answers are presented in a set order. On the other hand, if the researcher used an interview guideline (i.e., a set of topics and flexible order), the responses of different students would be categorized into themes generated by the interview guideline. However, the relevant data is not found in the same place in each student interview.

Within (individual) case analysis: Definition, Process, Example

Within-case analysis is a method used in qualitative research to analyze data within a single case. It involves a detailed examination of all available data sources pertaining to a specific case to generate a comprehensive understanding of the phenomenon being studied (Milles & Huberman, 1994; Patton, 2002; Stake, 1995; Yin, 2002).

The process of within-case analysis typically involves the following steps:

1. Familiarization with the case: Researchers become familiar with the case through repeated readings and immersion in the data.
2. Coding: Researchers code the data by identifying and labeling key themes, concepts, and patterns.
3. Categorization: Researchers categorize the data into broader themes and categories.
4. Interpretation: Researchers interpret the data to generate a comprehensive understanding of the phenomenon being studied.
5. Verification: Researchers verify their interpretations by checking them against the data and seeking feedback from others.

One example of within-case analysis in mathematics education could involve a study of a single classroom, where researchers examine the teaching practices and student learning experiences to gain an understanding of how students learn mathematics. In this study, researchers might collect data through classroom observations, student work samples, and interviews with the teacher and students. They would then analyze the data within the context of the single case (classroom) to identify key themes and patterns related to teaching and learning mathematics. For example, researchers might analyze how the teacher structures mathematical tasks, how students engage with the tasks, and how the teacher provides feedback to students. They might also examine how students collaborate and communicate with each other during mathematical activities. Through within-case analysis, researchers could generate a comprehensive understanding of the teaching and learning practices within the single classroom, which could inform future research and instructional practices in mathematics education.

Cross-case Analysis: Definition, Process, Example

Cross-case analysis is a method used in qualitative research to compare and contrast findings across multiple cases, or participants, in order to identify common themes, patterns, or variations. It involves analyzing data from each individual case separately, and then synthesizing the results to develop a more comprehensive understanding of the research topic (Milles & Huberman, 1994; Patton, 2002; Yin, 2002).

The process of cross-case analysis typically involves the following steps:

1. Identifying the research questions or hypotheses that will guide the analysis.
2. Selecting a sample of cases that represent diverse perspectives, contexts, or experiences related to the research topic.
3. Collecting and organizing data from each case using various methods, such as interviews, observations, documents, or artifacts.
4. Conducting a within-case analysis of each case to identify themes, patterns, or variations within the data.
5. Conducting a cross-case analysis by comparing and contrasting the findings across cases to identify similarities, differences, or relationships.
6. Synthesizing the results of the cross-case analysis to develop a conceptual framework, theory, or model that explains the research phenomenon.

One example of cross-case analysis in mathematics education could involve a study of multiple classrooms, where researchers compare and contrast the teaching practices and student learning experiences across different contexts to gain a broader understanding of how students learn mathematics. In this case, researchers might collect data from several classrooms, using similar methods such as classroom observations, student work samples, and interviews with teachers and students. They would then analyze the data across the cases to identify similarities and differences in teaching and learning practices. For example, researchers might compare how different teachers structure

mathematical tasks, how students in different classrooms engage with the tasks, and how teachers in different cases provide feedback to students. They might also compare how students in different classrooms collaborate and communicate with each other during mathematical activities. Through cross-case analysis, researchers could generate a comprehensive understanding of the range of teaching and learning practices in mathematics education, which could inform future research and instructional practices in the field.

In sum, this study aims to explore the application of qualitative research methods in individual qualitative studies and suggest implications for improving the quality of qualitative research. Through a cross-case analysis of the results of individual case analyses of individual studies, common and differentiated themes and aspects are found and analyzed. Based on the results of these analyses, we will also discuss ways to improve the quality of qualitative research in the field of mathematics education

III. METHOD

Selection of Potential Samples (papers to analyze)

In this study, the journals of the Korean Society of Mathematical Education were considered as the target of analysis. The reasons for selecting the journals of the Korean Society of Mathematical Education are as follows. First of all, the Korea Society of Mathematical Education Association is the largest academic society related to mathematical education in Korea, and they publish five academic journals (series A to E). In particular, anyone interested can directly download all series of published articles from their website.

Another reason is that all four journals, with the exception of one, are journals registered with the National Research Foundation of Korea. this researcher first considered selecting papers published in five academic journals, including A, B, C, D and E, based on the year of publication. However, this researcher confirmed that a total of 96 papers (A24, B19, C13, D13 and E27) were published in 2020. The number of 96 papers is not a problem in the case of quantitative analysis (similar to general quantitative research, showing the frequency of comparison items such as research topics, research subjects and research methods, and adding brief explanations based on quantitative information). On the other hand, ninety-six articles are beyond the capacity of one researcher to conduct a qualitative analysis that focuses on in-depth analysis of individual cases and subsequent cross-case analysis. For these reasons, this researcher decided that it would be appropriate to select a single journal and to analyze the articles that were published over 2 or 3 years.

In 2020, E journals have the highest number of articles (27), followed by A journals (24), and the remaining B, C and D journals have relatively few articles. Given the number of published articles, it seems appropriate to select journal E as the target of the analysis and to consider journal A as a secondary choice. When searching for the number of articles published in Series A and Series E from 2018 to 2020, the total number of articles in Series A was 80, while Series E was 81. This means that Series E had one more article. In this case, this researcher chose Series E <Communications of Mathematical Education> for the

analysis because it was considered to have the highest number of articles published in the last three years (although there is no significant difference in the logic of choosing either journal).

A total of 81 articles from 2018 to 2020 were retrieved from the website of the Korean Mathematics Education Association. This researcher carefully examined the abstracts, research objectives, research questions, research methods, and results of the 81 articles, and categorized the research methods used in each study. The 81 articles were categorized by research method: 21 quantitative studies, 15 qualitative studies, 3 mixed studies, 35 literature studies, and 6 other (literature + quantitative, literature + qualitative). Table 1 shows the breakdown by year and methodology.

Table 1. Categorized by research methods for Series E papers published in 2018-2020

	number of papers by research method				
	quantitative	**qualitative**	**mixed**	Literature analysis	**others**
2020 (27)	9	**6**	**3**	8	**1**
2019 (25)	4	**4**	0	17	0
2018 (29)	8	**5**	1	12	3
total 81 papers	21	15	4	37	4

As shown in Table 1, out of the 81 articles, 20 articles were identified that applied at least a partial qualitative approach (15 qualitative, 4 mixed, and 1 other). For the 20 selected articles, this researcher conducted a preliminary analysis to identify the research objectives, research questions, research methods, data collection, research participants/samples, analysis focus/methods/ framework, main findings. The preliminary analysis process required more time than expected. While it was relatively easy to find the research objectives (research questions) and main findings, it took a lot of effort to identify the parts related to the research methods. This was particularly true when it came to understanding the overall context of the studies and finding evidence of how the qualitative data collection and analysis was conducted by going through the whole content of the papers (rather than having a detailed and clear description of the methodology in the individual studies). In addition, at the end of the preliminary analysis of the 13 papers analyzed in 2020 and 2019, this researcher found that the discussions on the use of qualitative research were unique to each study, but at the same time, there were many similarities. In other words, the preliminary analysis of 13 papers and detailed discussion of individual papers could provide some valuable information about the use of qualitative approaches in mathematics education, ways to improve validity and reliability, and

implications for the use of qualitative research in mathematics education. Ideally, all 20 articles from all three years would be analyzed. However, given the limited volume of the journal, the amount of work required to analyze and discuss individual articles, and the number of articles that a single researcher could cover in an in-depth analysis, the final analysis was limited to a total of 13 articles published in 2019 and 2020. In other words, this decision was a trade-off for more in-depth analysis and focused discussion.

IV. RESERCH FINDINGS

Preliminary Analysis for Sample of 13 Papers (individual case analysis)[2]

The 13 articles analyzed were categorized into three groups according to the degree of use of the qualitative approach: Six active qualitative research (③, ⑥, ⑦, ⑩, ⑪, ⑫), four semi-qualitative research (①, ④, ⑨, ⑬), three mixed research (②, ⑤, ⑧). Based on the classification results, active **qualitative** studies are **bolded,** semi-qualitative studies are underlined, and *mixed research is italicized.* In the case of semi-qualitative classified papers, the title says "case study," but the characteristics of a qualitative case study are not clear, and it looks more like a literature analysis. Another reason for categorizing these papers as semi-qualitative is that they treat student responses collectively. For example, in the cited student worksheets, there is no information or code about the individual students. This is inconsistent with the nature of qualitative research, which focuses on the overall pattern of responses but also seeks to respond to the diverse responses of individual students. The new educational initiatives and programs introduced in the semi-qualitative papers seem to have achieved significant results, and the intention to share the results and information is understandable. However, the descriptions of the research methods are very brief or absent, and do not fully reveal the characteristics of qualitative research. For this reason, they are not appropriate for the purpose of this study, which is to analyze the application of qualitative research methods. In addition, the three articles categorized as mixed methods (as shown in the table above) are either surveys plus open-ended questionnaires, or surveys plus interviews. Only one of the three articles presents direct quotes from the interviews. The other two present the results of the interviews in summary form without direct quotes.

The seven papers categorized as semi-quantitative or mixed methods in the preliminary analysis were found to have either a very low reliance on qualitative approaches or a very poor record of their use. In these cases, the discussion of improving the quality of qualitative research remains superficial. This means that it is not possible to develop an in-depth methodological discussion with specific reference to the qualitative research methods utilized in the papers. In light of these issues, six active qualitative research (③, ⑥, ⑦, ⑩, ⑪, ⑫) is selected for final analysis the following section focuses on the results of a cross-case analysis of six articles categorized as active qualitative

[2] For more information on the individual case analysis, see 000 (2021), Analysis for the potential sample of 13 papers, Communications of Mathematical Education, 35(2), 137-152.

research. The results of the Preliminary analysis (research objectives, research questions, research methods, data collection, research participants/samples, analysis focus/methods/ framework, main findings) for the potential sample of 13 papers are shown in the following Table 2.

Table 2. Results of the preliminary analysis for the potential sample of 13 papers

	purpose/ RQ	method/ data collection	participant/ sampling	focus of analysis/ framework	key findings
① V34 (4)1 lee et al.	Create fair and effective assessments in online college math classes	Quali. (class case study: activity sheets, PBL reports) 7-week class	39 students in the Challenge (summer) semester at S Univ. in Seoul	Content analysis of PBL report: imprvs. subj. knowl., PBL classr. & prcss. -ori. PBL (direct quote) *. brief description on research method	More engaging than traditional courses. Feedback that online courses are effective & fairly evaluated.
② V34 (4)3 heo & 00	Assessing the impact of a math field trip program on student attitudes toward math learning.	Mixed method: survey & interv. conducted during pre-test & post-test (19.12.14~27)	36 students at a M.S. in Jeju City (19 in 1st grade, 17 in 2nd grade / 9 high, 16 middle, 11 low / M14, F21)	Survey results are detailed in tables, but interv. results are presented in summary form (without direct quotes).	Math tour programs have a positive impact on students' interest, confidence, and desire to learn math
③ V34 (4)8 kim et al.	Developing math-coding materials for Python class (6 sessions on prime factorization)	**Quali**.: interview & observation, Pre & post interview (6 sessions).	2 first graders at K M.S in Gwacheon	Anal. of activity sheets & intervs. (math & prog. interest, achv. std. reached). Stdt. intervs., direct quotes & disc. from activity sheets	Develop materials that meet cognitive standards, achieve learning goals, and boost interest in math & coding
④ V34 (3)1 lee et al.	Sharing various cases to ensure the quality of non-face-to-face univ. math education	Although the title says "case study," it appears to be a mixture of literature anal. & quali.	Students in a virtual class at S Univ. in Seoul (exact number not available)	*.NA *. brief description on research method	Info. on recording lectures, preparing materials, assessing & communicating. Covers a variety of

	after COVID-19	research			issues & suggestions for running a virtual classroom.
⑤ *V34 (3)2 shin*	Pre-service math tchrs' perceptions of the need for AI in future math educa. & role of tchrs. in utilizing AI	Mixed method: survey (open-ended questionnaire for awareness, optional 4-point scale for need) (data collection period unknown)	46 students (M25, F21) in the 1st & 2nd year of mathematics educa. at OO univ. in the capital region	Analyzed by the constant comparison of open coding, categorization, and category checking" open-ended survey anal.: thematic summaries & direct quotes, response types presented as frequencies (%)	Positive view of AI in teaching, learning, & assessment Tchrs: teach, interact emotionally, assess informally, counsel. AI: individualized/ rote learning,, struct., assess,, admin., tks.
⑥ **V34 (3)3 lee & 00**	Analyzing preservice math tchrs.' experiences in a design-based classroom and practicum program	**Quali.**: indiv.& grp interview, classroom activity products. (4 days, 2h per team for grp interv., 1h for indiv. interv.)	4th-year students in 'Algebra I' at Jeonnam OO Univ.'s Math Ed Dept.“ 8(M2, F6)	Analyzing indiv. & grp. interv.: presenting themes in direct quotation format [No examples of comparing the results of indiv. & grp interv. available].	Explore pre-serv. tchrs' percep. shifts (from confusion to active role, prototype's applicability) Emph. univ.-sch. site connection to promote design thinking-based classes
⑦ **V34 (2)1 kim et al.**	Diff. btw. lesson plnng. & impl. perceived by pre-serv. math tchrs. completing discursive competency dev. program	**Quali.**: Presentation & final report (including activity sheets) (18.3.7.~6.15)	15 stds. in "Theories of mathematics educ." at K Univ. (4th year, prospective math tchrs.)	PCK analysis framework (content, teaching, student understanding, assessment, classroom situation. Discourse competency anal. framework	Pre-serv. tchrs recognize the difference btw. lesson planning & implementation, Knowledge to consider in class practice, & value of discursive competence.

⑧ *V34 (2)5 yoo*	Effect of one-to-many tutoring math cooperative learning on academic achievement, affective domains, & math competency	Mixed method: survey(+open ended, 2 times) & interv. (13 indivs.) - 17.5.30~7.14	Sophomore at a H.S.l in Gyeonggi. 19 experimental (M12, F7). 38 controls (two groups, 19 each)	Anal. open-ended surveys & indivi. intervs: counting the frequency of key phrases (no direct quotes)	Only experimental group had notable pre/post-test differences." Imprvd. stdt-tchr self-efficacy, confidence in general stdts., math competency.
⑨ V34 (1)1 lee et al.	Sharing examples of running a "Basic Math for AI" course to understand math concepts needed for AI.	Quali.: (class case study, activity sheets, Q &A, pbl report) - fall of 2019	Stdts. taking Basic Math for AI at S univ. in Seoul (exact number unknown)	Content analysis: PBL report, monthly reaction paper (direct quote) *. brief description on research method	Stdts. understand core concept of AI. Increased knowl., PSS, & confidence by collaboration
⑩ **V33 (3)2 park et al.**	How do stdts' motivation & feelings of learning occur in a peer-discussion-oriented math class, and what is the relationship btw. the two?	**Quali.**: case study, observation & interview (criterion sampling) (data collection period unknown)	31 stdts. in Complex Variable Function Theory, OO Univ., Seoul, Korea. 7 studts. interv.(Learning Motivation Behavior Indicator), using 'Interv. & Stimulus Image'	Develop an analysis framework (Deci & Ryan, 1985; Schunk et al, 2008). Direct quote & multiple researchers	Math learning is motivated by need satisfaction. Positive learning experience (V.V.) when fulfilled. Positive learning feelings (V.V.) when satisfied.
⑪ **V33 (3) 11 choi & 00**	Analyze math & discursive competencies of teachers to identify how teachers' discourse skills	**Quali**.: class observation for 1 sem. (44 sessions) (data collection period unknown)	A female teacher at K Middle School in Gyeonggi Province	Analysis framework: math & discursive Competencies(MOE,2015). Reviewed by three experts	Propose ways to enhance students' real-world math competencies by refining discourse competencies & analyzing key utterance

	enhance students' math competencies.				strategies.
⑫ **V33 (2)3 Kang**	Analyze tasks developed through activities that emphasize mathematical connections and provide implications for teacher development	**Quali**.: grp. & indiv. task (undergrad. 8 teams, grad. 7 teams). 1st semester 2016 (11 weeks, 44h)	Stdts. taking essay class at a univ. in C city C, 56 undergrads. 28 grad. stdts. (including 1 teacher).	Analysis framework: CCK logic, SCK expression, KCS level, KCT strategy: Ball et al (2008) & Chapman (2013). Refer to portfolio, presentation, Self & Peer Assessment	Internal & external connections in pre-serv. tchr. development tasks. CCK logic, SCK expression (positive), KCT strategy (lack of diversity), KCS level (some tasks were out of level).
⑬ V33 (1)1 lee & lee	Sharing cases of discrete math class aimed at active participation, improvement, and success of students	Quali. (class case study, activity sheets, Q &A, pbl report) 2019 spring & fall sem.	Students taking Discrete math at S univ. (exact number unknown)	Lecture note, activity sheets, PBL report, monthly reaction paper(direct quote) *. brief description on research method	Instructor can make discrete math instruction personal, leveled, customized, and creative.

*. The number after the V is the "volume", the number in parentheses is the issue, and the number next to the parentheses is the table of contents number [Example: V34(4)1 means the first paper in volume 34, issue 4].

*. Explanations of word abbreviations:

A Cross-case Analysis of Six Active Qualitative Research Papers

This section focuses on the results of the cross-case analysis of common themes that emerged across the papers. Common themes include triangulation, citation of participants and interview data, combination of individual and group interviews, purposeful sampling, qualitative research questions, analytical framework, organization of research results, and others (researcher effect, grounded theory) (Milles & Huberman, 1994; Morse et al. 2002; Patton, 2015).

In other words, by focusing on the results of cross-case analysis, this researcher hopes to promote a better understanding of how to improve the validity of qualitative research. The results of cross-case analysis are organized into 8 themes and provide with discussions to improve the quality of qualitative research.

Data Sources Triangulation. There are three types of data in qualitative research: documents, interviews, and observations. The choice of which of these three types of data to collect depends on the nature of the research question. If all three types of data are needed to address the research question, the researcher in the study should consider whether it is practical to obtain all three types of data. If all three types of data are needed to address the research question but observation is not possible, such as historical events in the past, the researcher's decision to exclude observation can be justified. However, in situations where observations and interviews are clearly possible, it is not advisable to exclude one or the other for the sake of research convenience. Similarly, there is no justification for collecting and analyzing documentary data alone when observations or interviews are available.

Triangulation of sources including documents, interviews, and observations is important to check validity of each data source. Some papers only analyzed documents (class activity sheets/assignments/reports) and did not conduct interviews, even though it appears that interviews were available. In addition, in the methodology section, it was stated that observations were conducted, but it is impossible to see the analysis of the observation data in the findings section. There are obvious limitations to exploring only document data, especially if a research question is related to the effectiveness of a program, quality management/improvement, and perceptions of research participants. In fact, data source triangulation is a matter of prioritizing whether the research question requires a combination of documents, interviews, and observations, or whether it is feasible to obtain all three types of data (Milles & Huberman, 1994; Patton, 1990).

As in study #7's case, if the research question was to explore perceptions, interviews would be more informative because they allow the researcher to get close to the participants' inner world by asking probing questions that follow up on their answers (Patton, 1990). If the participant was a student in the researcher's course, the interview would not have been impossible. This lack of triangulation (Milles & Huberman, 1994; Patton, 1990) by comparing the documents with the interviews with some of the pre-service teachers who had unusual responses is somewhat disappointing.

Even if the study is designed from the beginning to focus on documents such as student journals, reflections, and final reports, it is a natural part of qualitative research that new questions and interests will arise as the analysis is nearing completion. Therefore, depending on the accessibility of the data, it is necessary to include at least a triangulation of documents, interviews, and observations (this does not necessarily mean having three types of data, but having multiple sources of data to compare with each other).

Of the six papers analyzed, only study #3 properly applied triangulation. Considering that triangulation does not necessarily mean obtaining three types of sources, this researcher can say that study #10, which used a combination of interviews and observations, also applied triangulation. However, paper #6 excluded observations, papers #7 and #11 excluded interviews, and paper #12 excluded interviews and observations and analyzed classroom activity sheets/tasks. The point of data source triangulation is to compare different sources to reach a clearer conclusion.

Regarding Participant Labeling and Citation of Interview Data. The use of pseudonyms or codes for participants was generally well-represented. While it is important

to provide demographic information about participants, qualitative research should also include narrative descriptions of their personal characteristics. These descriptions make qualitative research more qualitative. The following example illustrates the importance of assigning a code/pseudonym to research participants and the significance of providing a narrative description of their personal characteristics.

<Comparison Examples>

Specific description of the research participant: "Student A usually finds math more interesting and easier than other subjects and enjoys math class the most in school.(middle) He had high expectations and interest in the convergence class of math and coding (Study 3: 568)."

The research participants are given only the code such as A, B. C. D. E without any descriptive explanation (the commentary on the interview quotes provides a glimpse into the personal characteristics of the research participants): "In the case of the above pre-mathematics teacher H, as revealed in the course of the in-depth interview, he was a student who had always had good grades in his major and was regarded as an excellent student in his department (Study 6: 243)."

Participants are not given a code/pseudonym and a narrative description of their personal characteristics is not provided: In this case, it becomes impossible to distinguish which research participant is quoted directly in the paper, i.e., it is impossible to know whose statement is being quoted, and it is impossible to compare responses between research participants (Study 10).

In qualitative research, quotations from research participant interviews should be "verbatim" It is not advisable to rephrase participants' words into more formal language. Quoting participants verbatim reveals their linguistic and cultural characteristics, attitudes, and feelings(Patton, 2002). If there are words that make it difficult for the reader to understand what the participant is saying, you can quote them directly and add an explanation in parentheses afterwards. The following examples illustrate the use of verbatim quotations of the participant's language and the use of paraphrases of the participant's statements.

<Comparison Examples>

Verbatim quote from a participant: "Here, we just made a basic plan... I just made a basic plan, so I didn't know what grade we were going to do, so I just made it right away, so it (took) a bit of time there... It was a bit difficult because the kids came with their textbooks already solved... I had a glance at it in class, and when I asked them to do a little bit of work, they didn't respond, but when I showed them the PPT of the mini-book, they tried to do it, and it was a little bit good (Study 6: 243, Pre-service Teacher E)."

- It is similar to everyday language usage and seems to be quoting the language of the research participants, the university students. It also suggests that the researcher's relationship with the participants is very intimate. In addition, as shown in the quote from pre-service teacher E, appropriate explanations in parentheses facilitate the reader's understanding of the interview situation.

It seemed to be paraphrasing the words of a research participant: "It was interesting and difficult because I made it myself, and it was a pity that the commands were not in

Korean (written by the students themselves, but it is not known who wrote them)" and "Student B: I felt that this class was more like a coding class to study commands than a math class, but it made math easier and more interesting because I was able to examine the mathematical principles carefully using coding, which I had only memorized for no reason (Study 3: 574, interview on math interest)."

- In the interviews, the students' responses were more refined than in the written activity sheets. In qualitative research, the language of the participants should be quoted verbatim. This is because it allows you to capture the unique culture, speech patterns, and personal characteristics of the research participants.

Combining Individual and Group Interviews. In qualitative research, it is worth considering combining individual and group interviews whenever possible (although this is only done in some papers). One advantage of combining group and individual interviews is the ability to compare comments made in group interviews with comments made in individual interviews (Morgan, 1996; Patton, 2002). When group interviews are conducted with a group of people with homogeneous attributes, it is possible to obtain a large amount of information in a relatively short period of time due to the group dynamic (Morgan, 1996). However, in some cases, the overall reaction of the participants to the topics covered in the group interview may be in one direction, and there may be some participants who disagree with the overall opinion of the group but are reluctant to express themselves. This is why it is necessary to compare and confirm the results of group interviews with individual interviews. If the responses in the group interviews and the individual interviews on the same topic are consistent, this will increase the validity of the interview data.

Purposeful Sampling. Qualitative research involves a much smaller number of participants than quantitative research. One way to turn this weakness of qualitative research into a strength is purposive sampling, i.e., intentionally selecting informed participants to gain a richer understanding of the phenomenon (Patton 1990). Most of the studies were found to utilize convenience sampling. Instead of convenience sampling, which is easier to apply but less likely to be logically generalizable, it is necessary to utilize a more logical purposive sampling technique. The following is a discussion of purposive sampling strategies that can be utilized when a relatively small number of students can only be interviewed.

What considerations need to be made when a researcher can only interview two students due to various circumstances at the research site. For example, if the focus of the study is to determine whether the developed teaching and learning materials are comprehensible to average students, a "typical case sampling" can be applied, where it is more important to know what is average than what is out of average. Although not explicitly stated in Study 3, the selection of two students to be interviewed in Study 3 is similar to 'intense case sampling' (Onwuegbuzie & Leech, 2007). Intense case sampling involves selecting cases that are not extreme, but are experiencing the phenomenon with some intensity and are rich in information, i.e., not the best and the worst (extreme case sampling, where it is meaningful to know both extremes or only one extreme), but the top quarter and the bottom quarter of students. If both the top and bottom students had no difficulty with the developed teaching and learning materials, it is a logical generalization

that the students in between the top and bottom students would not have any difficulty.

As in Study #10, the selection of the final interviewees based on the "behavioral indicators of motivation to learn mathematics" that emerged from the observations of the 31 students is similar to criterion-based sampling (Onwuegbuzie & Leech, 2007). Criterion-based sampling seeks to focus on cases that meet certain criteria. This technique is often used for program quality control and evaluation (e.g., conducting in-depth interviews with people who relapse to alcoholism within two months of completing an alcohol rehabilitation center program is very important for program quality control and improvement).

Research Question: Alignment with Quantitative/Qualitative Paradigms. The compatibility of the research problem with qualitative research should be fully considered. This process requires a review of whether the qualitative approach can provide sufficient data to solve the research problem and the compatibility between the research problem and the qualitative paradigm (Patton, 2002). In some cases, the research question is appropriate for quantitative research.

The two research questions in Study 10 are "How do math learning motivation and math learning emotions occur?" and "How do math learning motivation and math learning emotions relate?". The first research question is qualitative in that it seeks a detailed description of a phenomenon or situation. However, the second research question seems to be very similar to the type of question commonly found in quantitative research: what is the relationship between A and B? In contrast, qualitative research, which seeks a deeper, more contextual, and holistic understanding of a phenomenon, generates research questions that ask about experiences, understandings, perceptions, and life stories (Patton, 2015). If the word relationship appears in a qualitative research question, it is likely to be something like, "How do childhood experiences of domestic abuse affect interpersonal relationships in adulthood?" In other words, simply looking at the relationship between two variables is a quantitative paradigm, while looking at how a particular experience affects current interpersonal relationships is a qualitative paradigm.

Analytical Framework. It is important to note that the analysis of qualitative research requires a continuous iteration of induction and deduction (Patton, 2015). Even if there are assumptions or analytical frameworks that the researcher brings to the research site, the researcher must be diligent in the process of discovering new lists and themes through an open approach to the data (induction), testing them against the theory or analytical framework (deduction), analyzing the data again based on the revised analytical framework (deduction), and discovering new themes and patterns from the data again (induction).

In qualitative research, the analytical framework is sometimes created before data analysis. For example, if you want to analyze the effectiveness of a program by applying an input-output model, the components of inputs (human, material, time, etc.) and outputs (completion rate, employment rate, salary increase, etc.) are usually clearly distinguished, and it will be easy to classify the collected data according to the pre-set analysis framework. However, given the nature of qualitative research, which requires a close response to the dynamics of the research site, participants, and data, it is generally recommended that the

analytical framework be finalized after several revisions during the data analysis process to categorize and include various cases.

A closer look at the analytical framework of Study 11 reveals that for the mathematical content competencies, it provides definitions for each competency (problem solving, reasoning, creativity and synthesis, and communication). On the other hand, it does not provide definitions for each of the sub-strategies (exploring mathematical relationships, clarifying thinking, generating discussion, and focusing on important elements) of the core talk strategy. If teachers' discourse (discursive competence) is categorized based on the analytical framework and each competence and key talk strategy is attempted to be captured, adding definitions of the four key talk strategies would help to improve the validity of the results. In addition, the four key talk strategies in Study 11 were modified from Boaler & Brodie's (2004) nine strategies. If you cite Boaler & Brodie's (2004) nine strategies but reorganize them, you need to provide a rationale and explanation for why and how you modified them.

Reorganization of the Research Findings. Researchers should seek to reorganize their findings in a way that best reveals the meaning of the study (Patton, 2015). The presentation of the results of Study 12 seems to be based on the analytical framework in Table 1, i.e., the 15 team tasks are arranged according to each of the analytical categories (intra-mathematical, extra-mathematical; CCK, SCK, KCS, KCT) and some tasks that meet the analytical criteria are shown in direct quotation format. Of course, it is possible to organize the findings in the same way. However, if you are a qualitative researcher who has already identified overall trends and what is important through a qualitative approach, you may want to consider a better way to present the meaning of your findings. For example, a researcher might consider presenting a description and summary table of the overall trends in the results first, and then directly cite examples (or good and bad practices) that are rich in diversity, quality, and discussion points when analyzed across all metrics. Rather than presenting each team's work in segments, a researcher might explore the interconnectedness of the different metrics in more detail around selected individual assignments (e.g., noting that one assignment demonstrates both internal and external connections to math, or that one assignment contains two or more complex connections within internal and external connections). For example, a researcher might consider presenting a description and summary table of the overall trends in the results first, and then directly cite examples (or good and bad practices) that are rich in diversity, quality, and discussion points when analyzed across all metrics. Rather than presenting each team's work in segments, a researcher might explore the interconnectedness of the different metrics in more detail around selected individual assignments (e.g., noting that one assignment demonstrates both internal and external connections to math, or that one assignment contains two or more complex connections within internal and external connections).

Other: Researcher Effects, Use of Specific Methodologies. Interestingly, based on the results of this study, it appears that the most common research participant in papers using qualitative research methods in mathematics education is students taking the researcher's class (13 papers in total, not just the 6 papers in the final analysis).

A qualitative researcher needs to be aware of the researcher(investigator) effects (Patton, 1990). Although not directly addressed in this research, most of the studies used students as research participants. In this case, in order to minimize the researcher effect that can be derived from the researcher effect, the relationship of knowing each other, and the relationship between instructor and student, a humble attitude, reflection on the status and behavior of the researcher, etc. are more required.

There are cases where a particular methodology has been used, but the nature of the methodology is not well characterized. For example, grounded theory emphasizes steps and procedures to link inductive and deductive methods in the analysis process by utilizing continuous comparative methods, comparison of research sites, theoretically based sampling techniques, and additional fieldwork (data collection) to validate concepts generated in the initial analysis (Strauss & Corbin, 1998). However, in Study 11, it is difficult to identify continuous comparative analysis, microanalysis, theoretical tabulation, data/theoretical saturation, range of diversity, open coding, axial coding, and relational statements (Strauss & Corbin, 1998). It is worth noting that the analysis cannot be called grounded theory simply because it is base

V. CONCLUSIONS AND RECOMENDATIONS

This study was motivated by the realization that despite the ever-increasing number of qualitative studies, there is a lack of in-depth discussion of research methods. Despite minor qualitative methodological errors, the six papers analyzed provide findings that cannot be found in quantitative studies. The issues discussed in this paper are common to many qualitative studies, but at the same time, they are general enough that even researchers with limited experience in qualitative research can apply them with a little care.

The results of this study are discussed in terms of how to improve the quality of qualitative research, focusing on common themes that were found by conducting a cross-case analysis based on case-by-case analysis. Specifically, the common themes classified into eight groups are 1) data source triangulation, 2) research participant labeling and citation of interview data, 3) combining individual and group interview, 4) purposeful sampling, 5) research question (alignment with qualitative paradigm), 6) analytical framework, 7) reorganization of the research finding, and 8) others (researcher effects, use of specific methodologies), and the problems and improvement measures are explained by directly quoting from the articles. Researchers interested in qualitative research and wanting to see qualitative research become a more widely recognized research method will want to take note of the findings of this study and hopefully lead to useful follow-up discussions.

It is also necessary to discuss the seven papers that were excluded from the in-depth qualitative analysis of this study. The clear rationale for excluding these studies is that, although they state that they utilize qualitative research methods, the evidence for their use is very weak. An in-depth discussion of what this means would be the subject of another new paper. What is clear, however, is that these studies were either conducted with little

understanding of qualitative research (4 papers were categorized as semi-qualitative research) or were mixed studies that just added interviews to the quantitative survey (3 papers were categorized as mixed research). Furthermore, only one of the three mixed research papers directly quoted interviews.

In the case of the journal related elementary mathematics education, the number of qualitative studies has exceeded the number of quantitative studies since the mid 2010s (Kim & Pang, 2017)[3], and in light of the above facts, it is believed that the discussion of qualitative research methods in the field of mathematics education needs to be continuously expanded in the future.

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