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RESEARCH ARTICLE

Exploring Opportunities for Mathematical Modeling in Korean High School Textbooks: An Analysis of Exponential and Logarithmic Function Tasks

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Abstract

This study aims to investigate the extent to which Korean high school textbooks incorporate opportunities for students to engage in the mathematical modeling process through tasks related to exponential and logarithmic functions. The tasks in three textbooks were analyzed based on the actions required for each stage in the mathematical modeling process, which includes identifying essential variables, formulating models, performing operations, interpreting results, and validating the outcomes. The study identified 324 units across the three textbooks, and the reliability coefficient was 0.869, indicating a high level of agreement in the coding process. The analysis revealed that the distribution of tasks requiring engagement in each of the five stages was similar in all three textbooks, reflecting the 2015 revised curriculum and national curriculum system. Among the 324 analyzed tasks, the highest proportion of the units required performing operations found in the mathematical modeling process. The findings suggest a need to include high-quality tasks that allow students to experience the entire process of mathematical modeling and to acknowledge the limitations of textbooks in providing appropriate opportunities for mathematical modeling with a heavy emphasis on performing operations. These results provide implications for the development of mathematical modeling activities and the reconstruction of textbook tasks in school mathematics, emphasizing the need to enhance opportunities for students to engage in mathematical modeling tasks and for teachers to provide support for students in the tasks.

Keywords: textbook analysis, mathematical modeling, exponential and logarithmic functions

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I. INTRODUCTION

The Common Core State Standards Mathematics (CCSSM; National Governors Association Center for Best Practices [NGACBP] & Council of Chief State School Officers [CCSSO], 2010) in the United States suggested mathematical modeling as one of the eight standards for mathematics practices. While the 2015 revised mathematics curriculum in Korea includes mathematical modeling as a sub-element of problem-solving competency, the explicit mention of mathematical modeling in the mathematics curriculum documents highlights its significance in school mathematics. The 2022 revised mathematics curriculum also places a strong emphasis on mathematical modeling as a teaching and learning approach (Ministry of Education, 2022). This approach encourages students to use various mathematical expressions to create models that are relevant to their lives and then apply these models to real-life situations, including social and natural phenomena. Furthermore, mathematical modeling can be used to explore mathematical concepts, principles, and laws through connections with other subjects.

In emphasizing mathematical modeling as a teaching and learning approach, the Korean recent mathematics curriculum addresses the need for creative and interdisciplinary individuals who can apply their knowledge in new and unfamiliar situations. Mathematical modeling involves formalizing real-world problems using mathematical symbols and expressions to construct mathematical models and then interpreting these models using mathematical reasoning to solve real-world problems (Kaiser, 2017). This process of mathematical modeling allows students to learn reasoning and communication skills that can be applied in their daily lives and provides them with diverse mathematical experiences.

Despite the importance of mathematical modeling, many students wonder why they need to learn math and how they can use math in their everyday life (Park & Lee, 2008). Due to the heavy emphasis on problem-solving skills in the curriculum and teaching methods that prioritize understanding problems and finding solution strategies, mathematics education has so far focused on improving students' problem-solving capabilities (Kim, 2012). However, using mathematical modeling tasks actively in classrooms can help improve students' mathematical communication and reasoning skills, and develop their ability to apply and utilize math. It is widely recognized that students often do not have the opportunity to engage in mathematical modeling tasks during their mathematics classes. This could be due to a lack of understanding and experience in mathematical modeling (Shin & Kwon, 2001). by both teachers and students, which ultimately leads to a lack of educational value for mathematical modeling.

As the curriculum places increased emphasis on mathematical modeling, it is essential to examine students' opportunities to engage in this practice. In many schools, textbooks serve as the primary teaching material (Kim, 2013), and teachers design lessons around the tasks presented in the textbook, with students using the textbook to learn. Consequently, a thorough analysis of the textbook is necessary to gain insight into the extent of students' opportunities to learn mathematical modeling in school mathematics. Mathematical modeling tasks in textbooks should reflect non-mathematical aspects of reality, and the results obtained through the mathematical modeling process should be interpreted and verified in given real world contexts.

For this reason, this study aims to analyze tasks on exponential and logarithmic functions in three Korean high school textbooks to identify opportunities for students' learning experiences in the mathematical modeling processes. In the 2015 revised mathematics curriculum, exponential and logarithmic functions are organized as a core conceptual area in the unit Mathematics1. The curriculum content framework suggests that they should be used to represent and explain natural and social phenomena (Ministry of Education, 2015). Furthermore, mathematical modeling aims to provide students with opportunities to engage with and explore real-life situations, recognize the practicality of mathematics, and understand and predict real-life phenomena (Maaß, 2010). Therefore, tasks involving exponential and logarithmic functions are expected to offer more opportunities for mathematical modeling compared to other mathematical concepts. Thus, we expect that tasks on exponential and logarithmic functions are more likely to have opportunities for mathematical modeling than other mathematics concepts. Based on these findings, the study will propose implications for the development of mathematical modeling activities, including the development of textbook assignments that include mathematical modeling and the reconstruction of assignments in school mathematics. To achieve this, the following research questions have been formulated: (1) To what extent do Korean high school textbooks incorporate mathematical modeling tasks? And (2) What proportion of tasks included in Korean high school textbooks provided opportunities for each stage of the mathematical modeling process?

II. LITERATURE REVIEW

Perspectives on Mathematical Modeling

Mathematical modeling has various perspectives in defining it based on the way of explaining the relationship between mathematics and "the rest of the world" (Pollak, 1968; Kaiser, 2017). Cai et al. (2020) defined modeling as the technology or process of forming a model of an existing system as a part of reality. With this definition, a model refers to the representation of an object that is not an object in, but of itself. In particular, Pelesko defines a mathematical model to mean a purely mathematical model or representation (Cai et al., 2020). Thus, mathematical modeling is defined as the technology or process of forming mathematical models.

In the field of mathematics education, mathematical modeling is defined as a process that involves converting a real-world situation into a mathematical model, applying mathematical knowledge to the model, and then translating the results back into the real-world situation. Pollak (2011) explained the mathematical modeling process as follows: deciding what aspect is most important in a real situation and converting it into an ideal situation that can be converted into mathematical terms, forming a mathematical model. Then, mathematical knowledge is applied to the mathematical model, and the results obtained are translated back into the real situation. The results are judged as being practical, rational and acceptable, and if not, the process is repeated. In addition, the CCSSM (NGACBP & CCSSO, 2010) defines mathematical modeling as the process of selecting and using appropriate mathematics and statistics to analyze and better understand real-world experiences in order to improve decision making. Blum and Ferri (2009) define mathematical modeling as a process that transforms both the real world and mathematics into each other, with the real world including the natural, social, everyday life, and other scientific fields according to Pollak (1979).

Mathematical Modeling Process

As discussed above, mathematical modeling is defined differently by different researchers, and there are various opinions on the use of related terms. Hence, to provide an accurate definition of mathematical modeling, it is crucial to first analyze the underlying process (Jung et al., 2018). That there can be different emphasized stages depending on different perspectives on mathematical modeling (Kaiser, 2017; Kaiser & Sriraman, 2006). In practical or applied modeling perspectives, the focus is on understanding the real world and finding solutions to real-world problems. In contrast, cognitive or theoretical modeling perspectives focus on developing mathematical concepts or algorithms based on real-world context. In educational modeling perspectives, the emphasis is on structuring the learning process, improving students' modeling skills and promoting social learning. In modeling derivation perspectives, the focus is on using models derived from solving original problems and applying them to new problems, as well as the stimulation of mathematical activities.

The modeling cycle developed by Blum (1985) is based particularly on the work of Pollak (1968, 1969; Kaiser, 2017). This cycle includes features where real-life situations are simplified to create real models, and in this process, assumptions are necessary, and the required factors must be identified. The real model is transformed into a mathematical model through mathematical transformation, called the process of mathematization. Then, mathematical results are obtained through the mathematical model. The results should be applied to the real model, interpreted, and verified. Based on this verification, the entire or a part of the process can be repeated. Other mathematical modeling approaches prioritize cognitive analysis, which results in inclusion of the step of students' understanding of the situation in the modeling process. In these approaches, students develop a situation model, which is then transformed into a real model (Blum, 2011). The mathematical modeling cycle by Kaiser and Stender (2013) emphasizes the need to interpret and verify mathematical results to obtain real results in students' modeling activities from an educational modeling perspective. From a modeling derivation perspective, Lesh and Doerr (2003) proposed another perspective on modeling activities and a modeling cycle that separates the real world and the model. This mathematical model is developed through explanation in the real word, and it is possible to predict real-life phenomena that require validation through this model.

There are some projects strongly reference educational modeling perspectives from a modeling standpoint. For example, the Consortium for Mathematics and its Applications (COMAP, n.d.) is a curriculum project in the US that promotes interdisciplinary approaches to solving real-world problems through mathematical modeling and technology use. Two US projects – Connected Mathematics Project (CMP) and Core-Plus Mathematics Project (CPMP; Senk & Thompson, 2003) – emphasize the inclusion of real-world applications and modeling as a part of the project goals. More recently, the mathematical modeling cycle described in the CCSSM (NGACBP & CCSSO, 2010) includes (1) identifying variables in a situation and selecting variables that represent essential functions, (2) formalizing the model by creating and selecting geometric, graphic, table, algebraic, or statistical representations of the relationships between variables, (3) performing work on these relationships to draw conclusions, (4) interpreting the results, (5) verifying the conclusions by comparing them with the situation, and (6) improving the model when allowed. Selecting, assuming, and approximating exist throughout the entire cycle.

Mathematical Modeling Tasks

The difficulties students experience in solving problems are simply finding ways to move from a given state to a goal state although the problems and their solutions are clearly defined and the steps to solve them are well structured. In general, students apply previously learned problem-solving strategies to find the answer, which can be relevant to real-life situations. However, solving these problems rarely provides an opportunity for students to generalize and reapply their learning (English, 2006).

Modeling problems, on the other hand, provide rich opportunities for students to engage in various mathematical thinking, including the development of mathematical structures within the problem, and interpretation and reinterpretation of problem information, making decisions, legitimizing their own inferences, posing hypotheses and problems, presenting arguments and counterarguments, applying prior learning, and engaging in metacognitive behavior (English, 2006).

According to Blum and Ferri's (2009) study, through examining the modeling tasks used, it is found that an appropriate modeling task should have a problem situation that reflects a mathematical and real-life situation, and students should be able to understand the problem situation, construct a situation model, simplify and structure it, and turn it into a real model. Furthermore, the mathematical model should be transformed and mathematical results should be obtained through mathematical operations. The results are then interpreted and verified in the real world. This process can be repeated depending on the factors considered, and the starting point and focus at certain stages of the mathematical modeling process can be reconstructed for individuals. Different models can be created through the mathematical modeling process. To sum up, an appropriate modeling task allows students to experience the modeling process and enhance their modeling skills through a problem situation that reflects reality, and leads to diverse solutions through individual modeling processes.

Textbook Analysis with the Mathematical Modeling Perspective

Previous studies on the analysis of textbooks tasks for mathematical modeling have mainly compared Korean and other countries' textbooks, focusing on tasks presented in a specific area of mathematics (Jung et al., 2020; Kim, 2021; Park and Ko, 2022). These studies have selected tasks that include real-life context in the textbook where students can experience the mathematical modeling process. Park and Choi-koh (2022) classified tasks into mathematical modeling tasks or word problems. According to them, mathematical modeling tasks include ones with clear mathematical models that can enhance the stages of the mathematical modeling process, while word problems are those that do not require setting up a mathematical model. Kim (2021) also analyzed math tasks presented in the statistics section of middle school textbooks in Korea and Singapore from the perspective of mathematical modeling, considering the reflection of mathematical modeling process, type of data provided, form of representation, problem context, and mathematical activities. The results suggested the need for providing balanced experiences in the mathematical modeling process and presenting tasks in various forms of representation, as well as tasks with high contextual realism that supports students to engage in the mathematical modeling process.

Mathematics educators have highlighted limited opportunities for mathematical modeling provided in mathematics textbooks. Park and Ko (2022) analyzed the number and characteristics of modeling problems in the function section of three International Baccalaureate Diploma (IBDP) mathematics textbooks and nine Korean high school mathematics textbooks. The findings indicated that the proportion of modeling tasks differed among publishers in Korean textbooks. Furthermore, all 12 textbooks exhibited a significant bias towards mathematical modeling components that were restricted to mathematical domains, leading to limited learning opportunities for all the stages of the mathematical modeling process.

Moreover, Jung et al. (2020) compared the diversity of mathematical modeling opportunities given to students in South Korea and the United States by analyzing real-life context tasks presented in geometry textbooks. They analyzed the tasks presented in the textbooks of both countries based on three aspects: the mathematical modeling process, data, and types of representation. The mathematical modeling process was classified based on the seven stages proposed by Blum and Leiß (2007): understanding the situation, establishing a real-world model, mathematization, obtaining mathematical results, interpreting, validating, and reporting. The study found common and differentiated characteristics in the tasks presented in the textbooks of the two countries, allowing for an understanding of the steps and directions of mathematical modeling activities in geometry. Their analysis results showed that both countries' textbook tasks were concentrated on obtaining mathematical results, with a low proportion of tasks focusing on other stages. This indicates a need for expanding the exploration of real-life contexts and increasing opportunities for iterative validation, which are inherent features of mathematical modeling tasks. Based on these results, Jung et al. (2020) argue that textbook tasks should be modified to reflect these characteristics when utilized by teachers. Additionally, to enable the experience of realism and complexity in mathematical modeling, an appropriate increase in the proportion of non-routine tasks is required. The common claims made in the previous studies are that: first, students need more opportunities to experience the mathematical modeling processes, such as exploring real-life contexts and repeatedly evaluating the validity of students' models. Second, students should be presented with tasks that reflect the reality and complexity of mathematical modeling. Students also need to involve various forms of expression. In order to enhance students' mathematical modeling competency, teachers should modify textbook tasks to reflect the characteristics of mathematical modeling tasks. To revise the tasks, teachers should be provided with instructional materials such as teacher's guide or supplementary materials for mathematical modeling.

The objective of this research is to analyze the mathematical modeling tasks presented in high school math textbooks. Previous studies have indicated that the proportions of such tasks vary in Korean textbooks, with a strong focus on obtaining mathematical results using only text and illustrations. As a result, there is a need for appropriate mathematical modeling tasks that can help students enhance their cognitive ability and develop their mathematical modeling competency. Building on the conclusions and recommendations of earlier studies, we conducted an examination of the opportunities presented in all tasks related to exponential and logarithmic functions in high school mathematics textbooks for each stage of mathematical modeling.

III. METHODS

Data Sources

Three high school mathematics textbooks were selected for the analysis: the Textbook K (Kwon et al., 2019), the Textbook D (Park et al., 2019), and the Textbook B (Kim et al., 2019). The rationale for the textbook selection is that these textbooks were widely adopted in high schools and the research team have experience to use these textbooks. All three textbooks include the chapters for exponential and logarithmic functions. The specific information about the textbooks were shown in Table 1.

Table 1. Selected High School Mathematics Textbooks

Textbook	Publisher	Number of Pages	Number of Units	Analyzed Chapter
Textbook K	Kyohak	24	111	I-2
Textbook D	DongA	21	93	I-2
Textbook B	VISANG Education	26	120	I-3

Textbooks used in mathematics education do not typically contain tasks specifically designated as "mathematical modeling tasks." This means that students may not have a comprehensive experience of going through all stages of mathematical modeling. However, it can be assumed that the tasks included in the textbook provide opportunities for students to engage in certain stages of mathematical modeling. Previous studies (e.g., Park & Han, 2018) have analyzed the opportunities for mathematical modeling within textbook tasks that have real-world contexts. This study takes a different approach by examining each exercise problem in the textbook as a separate unit of analysis, regardless of its context. This is because each problem in the textbook has the potential to offer unique opportunities for students to engage in different aspects of the mathematical modeling process."

We identified 324 units across the three textbooks, and the number of units in each textbook is presented in Table 1. The analysis of these textbooks typically involves counting the frequency of a particular topic within the text. These counts are typically given equal weight, as stated by Ding (2016), Polikoff (2015), and Smith et al. (2016), as it can be challenging to determine which topics are more critical for students' learning. Figure 1 provides an illustration of the unit used in this study. The task presented had two sub-questions, and we treated each sub-question as a separate unit of analysis. This was done because sub-questions within one task can require different stages of the mathematical modeling process, despite sharing a common context.

The price of a tablet computer that starts at 1,250,000 won is said to decrease by 20% per year.
(1) How many years does it take for the price to reach 800,000 won?
(2) What is the minimum number of years it takes for the price to become less than 640,000 won?

문제 03

처음 가격이 125만 원인 어느 태블릿 컴퓨터의 가격이 매년 20%씩 하락한다고 한다.
(1) 가격이 80만 원이 되는 것은 몇 년 후인지 구하시오.
(2) 가격이 64만 원 이하가 되는 것은 최소 몇 년 후인지 구하시오.

Figure 1. Example of analysis units

Framework of Analysis

Tasks were coded with what actions in the mathematical modeling process are required to engage in. These five stages in Table 2 were based on the criteria of mathematical modeling suggested in the CCSSM (Meyer, 2015). CCSSM (NGACBP & CCSSO, 2010) originally suggested the six stages of mathematical modeling. In this study, we analyzed textbook tasks to investigate how students engage in mathematical modeling through their learning. To accomplish this, we applied an analytical framework that includes five activities for each stage of mathematical modeling, excluding the conclusion (report) stage according to Meyer (2015).

Table 2. Analysis Framework of the Mathematical Modeling Process

Actions	Descriptions
Identifying Essential Variables	This action involves finding the necessary variables in the problem situation, and distinguishing and selecting the important information that is necessary to solve the problem.
Formulating Models	This action involves describing the problem situation using the found variables and creating a mathematical model using equations, functions, tables, graphs, etc.
Performing Operations	This action involves solving the problem by performing mathematical operations on the created model, and obtaining the result.
Interpreting the results	This action involves checking whether the results obtained in the previous step make sense in the context of the problem situation.
Validating	This action involves comparing the results obtained in the previous step with data from other sources or by other methods to confirm their validity.

문제 2 추론

어떤 유물에 들어 있는 탄소 동위원소 ^{14}C는 5700년마다 그 양이 반으로 줄어든다고 한다. 어떤 유물의 ^{14}C가 처음 들어 있던 양의 $\frac{1}{8}$만 남아 있다면 이 유물은 몇 년 전의 것이라고 추정할 수 있는지 말해 보시오.

It is said that the amount of ^{14}C in a certain artifact decreases by half every 5700 years. If only 1/8 of the initial amount of ^{14}C remains in an artifact, how many years ago can it be estimated to have existed?

Figure 2. Example of tasks for identifying

The frameworks in Table 2 are used to analyze all questions in the mathematics textbooks on exponential functions and logarithmic functions. Tasks designated as "Identifying Essential Variables" entail identifying the necessary variables in the problem situation and selecting the crucial information needed to solve the problem. Tasks coded as "Formulating Models" involve representing the problem using the identified variables and constructing a mathematical model through the use of equations, functions, tables, and graphs. Tasks designated as "Performing Operations" involve carrying out mathematical operations, while "Interpreting the Results" requires evaluating the computation outcomes to see if they align with the problem context. Lastly, tasks labeled as "Validating" involve checking the accuracy of the computation results by comparing them to data from other sources or through alternative methods. Lastly, tasks labeled with “Others” requires actions other than the above five activities.

In the mathematical modeling process, the first stage of *identifying* is a crucial aspect. This is because the process involves understanding various factors that impact the raw actual phenomena and given situations, and going through the appropriate entire mathematical modeling process. As a result, by identifying variables, students can approach the mathematical modeling tasks in a variety of ways and find different answers. Figure 2 is an example that includes the stage of *identifying*. The textbook task in Figure 2 shows a situation where students must find the amount of carbon isotopes that were initially present in an artifact. In this task, students can identify the period of time during which the artifact existed as a variable and find an answer using an exponential equation with the variable.

Coding Procedure

All three researchers were involved in establishing codes, define and creating analysis units, and coding the textbooks to establish the validity of coding through researcher triangulation (Creswell, 2013). After careful discussion of the analytic framework in Table 2, the research team decided to code all items together considering the total number of the items. Each item was given a maximum of five codes if an item requires students to perform all stages of the mathematical modeling process. This indicates that it is impossible to apply a typical reliability analysis like Cronbach’s alpha. Thus, the reliability was determined using a generalizability theory D study (Alkhrausi, 2012). The reliable coefficient in the first round of coding was 0.869 (see Table 2), which is acceptable. After the first two authors independently coded the items and achieved a high inter-rater reliability, the authors jointly coded all items to create a final set of tables for analysis, resolving any coding discrepancies that arose.

Table 3. Interrater Reliability based on a generalizability theory D study

	Sum of Square	Degree of freedom	Mean Square	Estimated Variance	% of Total Variance	Reliability
Item	443.0	323	1.4	0.021	2	
Coder	7.6	1	7.6	0.338	32	0.869
Residual	347.9	499	0.7	0.697	66	

IV. RESULTS

Mathematical Modeling Tasks

In our analysis of the three textbooks of the 2015 revised curriculum, we discovered only one high-level task that allowed students to experience the entire process of mathematical modeling. In other words, the textbooks do not provide students with the opportunity to experience the whole process of mathematical modeling, but only at certain stages.

The stars visible in the sky are classified according to their brightness. The Greek astronomer Hipparchos (B.C. 190?-B.C. 125?) was the first to classify the stars visible to the naked eye into six brightness categories based on their brightness. He classified the 1800 stars visible in the sky with the brightest being first magnitude, the darkest which can barely be identified with the naked eye as sixth magnitude, and stars with intermediate brightness as second to fifth magnitude based on their brightness.

하늘에 보이는 별들은 겉으로 보이는 밝기에 따라서 그 밝기의 등급을 매긴다.

기원전 그리스의 히파르코스(Hipparchos, B.C. 190?~B.C. 125?)가 눈에 보이는 별들을 밝기에 따라서 6개의 등급으로 분류한 것이 시초이다. 그는 하늘에 보이는 1800여 개의 별들에 대하여 가장 밝은 별을 1등성, 육안으로 겨우 식별이 가능한 어두운 별을 6등성, 그 사이 밝기인 별들은 2~5 등성으로 밝기에 따라 등급을 분류하였다.

In the 19th century, it was found that the brightness of first magnitude stars is about 100 times brighter than that of sixth magnitude stars through the astronomer Herschel J. (1792-1871). If the brightness ratio between each category is constant, the brightness of the star increases by about 2.5 times as the magnitude decreases from first to second magnitude. According to Pogson's formula derived in 1856, the relationship between a star's magnitude and brightness is as follows.

그 후 19세기에 이르러 천문학자 허셜(Herschel, J., 1792~1871)에 의해서 1등성의 밝기가 6등성의 밝기보다 약 100배임을 알게 되었는데, 각 등급 간의 밝기의 비가 일정하다면 별의 등급이 1등급 작아질 때마다 별의 밝기는 약 2.5배 밝아지게 된다.

1856년에 유도된 포그슨의 공식에 의하면 별의 등급 m과 별의 밝기 I 사이의 관계는

$$m = -k\log I + C \quad (\text{단, } C, k\text{는 상수})$$

라고 한다.

Determine the value of the constant k based on the information that 1st magnitude star's brightness is 100 times brighter than 6th magnitude star's brightness

1. 1등성의 밝기가 6등성의 밝기의 100배임을 이용하여 상수 k의 값을 구해 보자.

2. 포그슨의 공식을 적용하면 태양은 −26.7등급, 보름달은 −12.5등급이라고 한다. 이들의 밝기가 1등급인 별의 밝기에 비해 얼마나 밝은지 토의해 보자.
(단, $\log 1.2 = 0.08$, $\log 2.51 = 0.4$로 계산한다.)

According to Pogson's formula, the sun is -26.7 magnitude and the full moon is -12.5 magnitude. Let's discuss how bright they are compared to a star of 1st magnitude.
($\log 1.2 = 0.08$, $\log 2.51 = 0.4$)

3. 육안으로 볼 수 있는 별의 밝기는 6등성까지이고 6등성보다 어두운 별을 보려면 망원경과 같은 장비가 필요하다. 어떤 천체 망원경을 통하여 본 별의 밝기가 육안으로 본 별의 밝기보다 200배 밝을 때, 이 망원경으로 관측 가능한 가장 어두운 별의 등급이 어떻게 될지 토의해 보자. (단, $\log 2 = 0.3010$으로 계산한다.)

It's possible to see stars up to 6th magnitude with the naked eye, and a telescope or other equipment is necessary to see stars that are darker than 6th magnitude. When viewing a star through a telescope, if the brightness of the star is 200 times brighter than what can be seen with the naked eye, what is the darkest star that can be observed through this telescope? Let's discuss. ($\log 2 = 0.3010$)

Figure 3. Textbook problem requiring all mathematical modeling stages (Textbook K; Kwon et al., 2019, p. 65)

Figure 3 shows a task from the textbooks in the exponential function and logarithmic function units, which shows all five stages of mathematical modeling presented in the analysis framework. By examining the possible process of mathematical modeling in the task, it starts from *identifying* to understand what is important and what is being sought by considering the context that the brightness of the first magnitude is 100 times less than the brightness of the sixth magnitude given in the task. Then, it goes through *formulating* where a new equation is composed by comparing the brightness of the sun and the brightness of the full moon by grade, using the already given Pogson formula. And, in the stage of *performing*, the solution needs to be computed. Additionally, the process goes through the next stage of *interpreting* where the darkest star rank that can be confirmed through the given telescope equipment is confirmed. Through the stage of *validating*, where the validity of their interpretation is reviewed by comparing the initial situation of the task and the obtained results, and the results obtained in *interpreting* are determined as the appropriate answer for the question in the task, experiencing all five stages of the mathematical modeling process. Tasks that perform all the steps, as shown in Figure 3, should be included in the textbook to give students the opportunity to experience all the steps of mathematical modelling.

Coding Results

The results of the analysis of three textbooks show that the majority of the tasks presented to students emphasize the performance of basic computations. Over 80% of the tasks require students to perform mathematical operations using algorithms in order to obtain the desired result. This suggests that the focus of these textbooks is on mastering the fundamental computational skills and applying algorithms to solve problems.

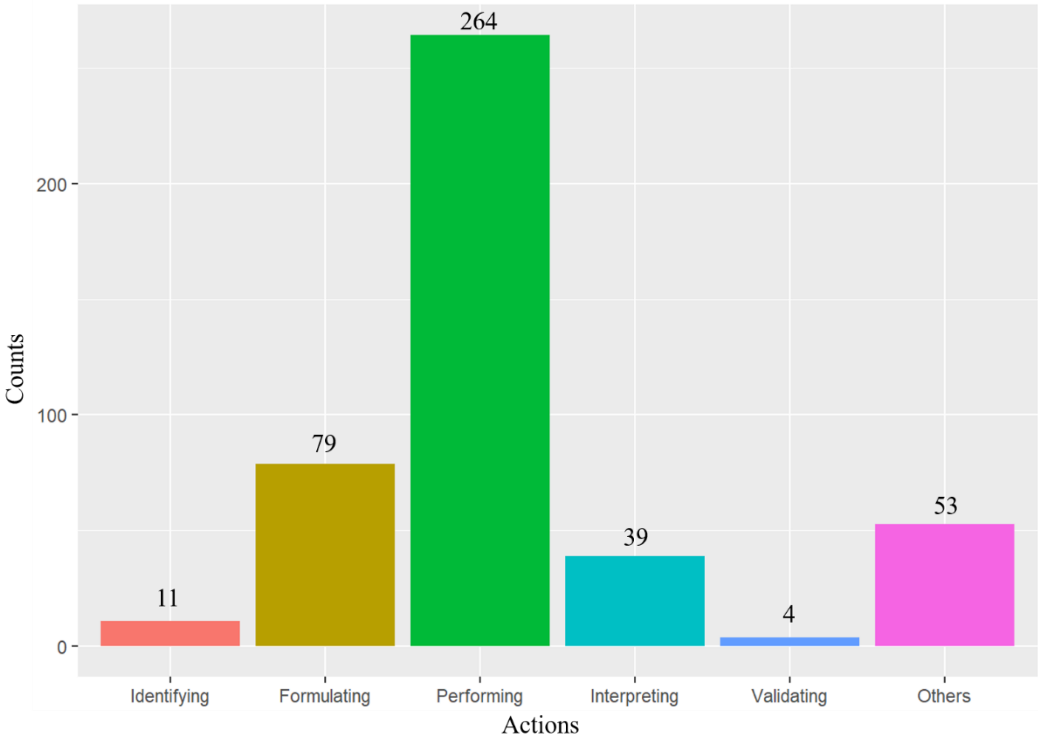

Figure 4. The Number of tasks requiring each stage in the mathematical modeling process

However, the results also indicate that only a quarter of the tasks require students to formulate models using equations, functions, tables, and graphs. In addition, the textbook tasks do not provide opportunities for students to validate their results using data from other sources or by other methods. This limitation may hinder the development of the entire mathematical modeling process. This unbalanced proportions were somewhat expected because there were no tasks explicitly labeled as “mathematical modeling tasks.” Because the Korean national curriculum highlighted mathematical modeling as what students should experience in mathematics classrooms, it is important for textbooks to provide a balance of both computational tasks and modeling tasks that encourage students to think beyond the algorithm and analyze real-world data.

Table 3. Proportion of Tasks Requiring each Stage by Textbook

Textbook	Textbook K	Textbook D	Textbook B	Total
Identifying	4.0%	3.2%	3.6%	3.4%
Formulating	28.3%	25.8%	24.3%	24.4%
Performing	97.0%	83.9%	81.1%	81.5%
Interpreting	15.2%	8.6%	14.4%	12.0%
Validating	2.0%	1.1%	0.9%	1.2%
Others	23.2%	11.8%	17.1%	16.4%

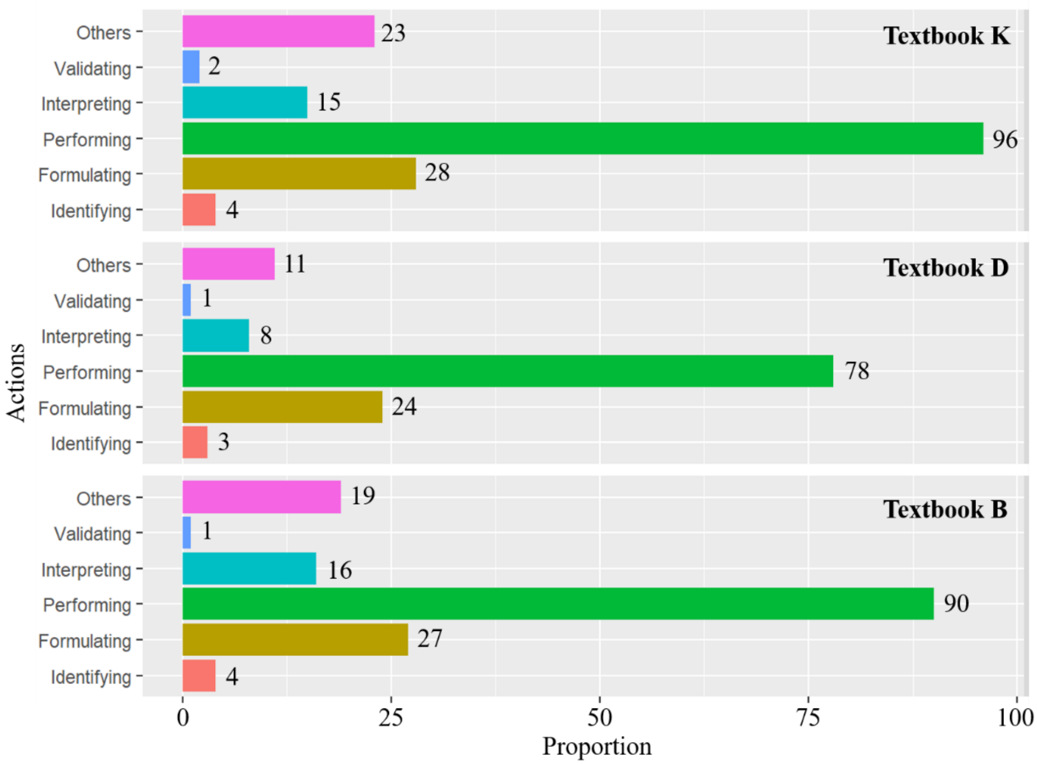

Figure 5. Proportion of tasks requiring each stage by textbook

The results of the analysis revealed that the differences among the three textbooks are minimal. However, it is not possible to conduct any statistical tests to evaluate these differences due to the fact that multiple codes were assigned to some units. Despite this limitation, the results provide insight into the overall trends about the opportunities for mathematical modeling across the textbooks.

One interesting finding in Table 3 and Figure 5 is that the majority of tasks in the textbook K require students to perform computations. This textbook has 97% of tasks that require students to perform computations compared to the other textbooks. At the same time, the proportion of tasks for each action in the textbook K is also higher than the others. This indicates that the tasks in the textbook K requires various combinations of the actions. Considering the finding that the differences among the textbooks are minimal, all textbooks do not have balance of tasks offered in each textbook to ensure that students have opportunities to develop the mathematical modeling process.

V. DISCUSSIONS AND CONCLUSION

The purpose of this research was to study all tasks for the exponential and logarithmic function in the three 2015 revised high school mathematics textbooks, resulting in a total of 324 tasks analyzed. Using the analytics framework based on the five stages of mathematical modeling proposed by CCSSM, the tasks were evaluated through five activities per stage. The analysis found that the distribution of tasks experiencing each of the five stages was similar in all three textbooks, reflecting the 2015 revised curriculum and national curriculum system. Of the 324 questions analyzed, the highest proportion, 81.48%, required *performing* in the mathematical modeling process. This result may be due to the traditional emphasis on entrance exam-oriented education and problem solving as a key mathematics subject competency in Korean high schools (Park & Han, 2018). In mathematics instruction aimed at developing problem-solving skills, emphasis is placed on understanding problems, exploring solution strategies, implementing solutions, verifying results, and reflecting on the process. The ability to solve mathematics problems using mathematical concepts and knowledge can be considered a problem-solving skill, which corresponds to the performing stage in the mathematical modeling process.

Similarly to the study of Meyer (2015), most of the tasks in Korean textbooks provide all the necessary information, presenting mathematical models such as equations, function formulas, tables, graphs, and other tools to help students find answers to the problems. However, there is a lack of tasks in the textbook to experience *identifying* and *validating* in the mathematical modeling process. This means that there is limited opportunity to experience all five stages of mathematical modeling. When students gain experience with all five stages of mathematical modeling in a balanced way, they can fully develop their competency in mathematical modeling. However, the textbooks do not provide students with the opportunity to find a mathematical model that fits the situation or provide the information needed to solve the task. Tasks labeled as "others" include tasks for "discussing problems," "observing changes," and creating problems similar to previous tasks, where students do not independently determine how to solve the problems.

The results of this study provide implications for the need to incorporate more diverse mathematical modeling tasks in high school textbooks to enhance students' problem-solving competency. First, textbooks play a vital role in the education of mathematical modeling. To provide students with a comprehensive development of the mathematical modeling competency, textbooks should offer a diverse range of tasks that encompass all stages of the mathematical modeling process. The findings show that most tasks provided by the textbooks enable students to experience at least one stage of the mathematical modeling process. The textbooks offer students the chance to partially or indirectly experience the mathematical modeling process, but There are only a limited number of tasks that provide the opportunity to experience the complete process.

As Meyer (2015) described, during the *identifying* stage, students can develop their skills in distinguishing between essential and non-essential information in a given context. By identifying the essential variables, students can recognize the necessity of the model and move on to the next stage of the process naturally. According to Park and Han (2018), mathematical modeling tasks need to include variable search activities in mathematics classrooms, so students can develop their own problem-solving skills. Park and Han suggested that textbooks should include questions that require students to think and make decisions independently, rather than providing tasks that already have all the values and conditions needed to solve them. Through the stage of *validating*, Students can verify the accuracy of the model and confirm its predicted outcomes. As Meyer (2015) notes, students may also recognize that mathematical models used to represent real-world situations are not always 100% accurate. During the *validating* stage, students should focus on the process of modifying the model, rather than finding the correct answer. To promote the mathematical modeling competency and a deeper understanding of mathematical modeling itself, textbooks should include tasks that allow students to experience both *identifying* and *validating.*

Second, the analysis revealed that over 80% of the tasks in the three types of textbooks were classified as performing tasks. This finding indicates that the textbooks do not provide students with the opportunity to engage in the complete process of mathematical modeling but rather focus on specific aspects. Consequently, it is crucial for teachers to recognize the importance of mathematical modeling and reorganize textbook tasks accordingly, allowing students to have a broader range of opportunities to experience all the steps involved in mathematical modeling. The findings showed that there was only one task (see Figure 3) that allowed for a complete mathematical modeling process although it was not labeled as a mathematical modeling task. As discussed above, the importance of the mathematical modeling competency is recognized in many mathematics curricula worldwide and numerous projects. Although there is extensive research demonstrating the high potential of mathematical modeling in the classroom, it has become clear that it is important for mathematical modeling to be addressed every day, whether or not modeling competencies are being developed (Kaiser, 2017). Mathematical modeling tasks offer ample opportunities for students to participate in various epistemic actions in mathematics (Hwang et al., 2023), including interpreting problem information, making informed decisions, justifying reasoning, formulating hypotheses and problems, presenting arguments, applying prior learning, and thinking metacognitively (English, 2006). The results indicate that while textbooks partially provide opportunities for students to perform the actions required in the process of mathematical modeling, they do not explicitly present them. Therefore, it is necessary to provide students with a comprehensive experience of mathematical modeling.

Incorporating mathematical modeling into a structured curriculum and textbook tasks is crucial for students' better opportunity for learning mathematics. The findings of this study can be used to inform the development of textbook tasks that facilitate task reconstruction and effectively integrate mathematical modeling into school mathematics.

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