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# The Macroeconomic Production Model in Business Environment - Analying with a Static and Dynamic Equations

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## Abstract

**Purpose:** The purpose of this research is to explore the macroeconomic model through both static and dynamic equations. The primary objective of this study is to investigate the variations in the elasticity of substitution across changing economic variables within the framework of the Allen-Uzawa production functions. **Research, design, data and methodology:** The data were drawn from the World Bank's annual central statistical office database from 2010 to 2021 in the United States of America. The level of expenditures and of the public finance sector, macroeconomic data like output, inflation rates, and labor are examined. **Results:** This study demonstrates the interaction of two equations, clarifying that the macroeconomic model is practical to determining the stability of both static and dynamic equation systems analytically. The Allen-Uzawa equations allow for the verification of macroeconomic model properties, and study results demonstrate an increase in the range of capital uses as a form of mechanization. A constant elasticity of substitution function is derived from the macroeconomic variables. **Conclusion:** The macroeconomic model, though the analysis of the static and dynamic Allen - Uzawa model, not only facilitates the examination of long-term trends in crucial endogenous variables but also overcomes challenges commonly associated with other mathematical methods. Overall, the analysis promotes economic growth, investment, and employment. The levels of expenditures and the public finance sector, along with macroeconomic data such as output, inflation rates, and labor, are examined.

**Keywords:** Constant Elasticity of Substitution, Static Equation, Dynamic Equation

**JEL Classification Code:** C61, C3, C5

## 1. Introduction

There are many types of macroeconomic models, and one of them considers the relationship between static equation and dynamic equation models. When the correlation between these two types are examined it becomes readily apparent that various postulates are equally plausible. Thus, this brings into question the need for the contemplation that perhaps criteria other than economic efficiency may be far more important in the determination of the macroeconomic model (Barnichon & Mester, 2020).

Economists have generally attempted to maintain a division between static and dynamic equation of the macroeconomic model. The favored economic methodology requires that the economist limit discussion to ethically neutral statements in which a dispassionate observer would

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be able to make and conclude. One main aspect to the formation of this paper was in the basis that the normative dimension is a requisite part of any meaningful discussion of this issue and will always find expression, implicit1y or explicit1y, in the workings of the model. In light of this reality, it would appear that many intellectual insights can be gained by a thorough airing of both the nonnative and positive issues (De Jong & Kumar, 1972).

The current study has utilized the extension of the Allen elasticity of substitution variables to arbitrary factors (Allen, 1938). The macroeconomic model is enlarged to encompass the linear homogeneous production function in terms of the cost equations. Taking this into consideration, the research takes adoption of the cost functions by Allen for linear homogenous equation functions (Li, 2000). This study obviates the need to rely on problematical assumptions and devices which are typically used to derive positive factors on the equation is stable (Maestas et al., 2023). In the macroeconomic model, the Allen-Uzawa elasticities of substitution will show statics and dynamic equations in empirical studies of production function.

Accordingly, this is why the elasticity of substitution differs by changing economic variables in time series analysis of the Allen-Uzawa production functions. The production function exhibited here is the Allen-Uzawa (Allen, 1938; Uzawa, 1962). The distinctive feature of the current study, in contrast to previous ones on the regulation of minimum real wages, lies in the utilization of this specific production function. The distinguishing features of this type of production function can be stated as follows: (1) It is a type of CES(constant elasticity of substitution) production function, which is subject to constant returns to scale and to diminishing marginal rates of substitution. (2) It can deal with the multi-category, multi-input production case. (3) While the partial elasticities of substitution between any two factors of production remain constant regardless of factor prices, variations may exist among these elasticities for different pairs of input factors. (4) More specifically, Uzawa (1962) used an assumption that the partial elasticities of substitution between any two input factors do not belong to the same category; but the partial elasticities of substitution can be any non-negative real number if the two input factors belong to the same category (Birdsall, 1989).

## 2. Literature Review

The macroeconomic model is significantly altered when both factors of labor and capital sectors are incorporated, as opposed to considering only one of them (Beaudry et al., 2018). The Allen-Uzawa production function has been assumed that the government deficit-income ratio is held constant. Also, the production equation is confined to a closed economy with no foreign factors considered (Uzawa, 1964). As the Allen-Uzawa elasticity of substitution has been extensively employed in empirical studies on production functions, economic theory implies certain consequences in relation to prices, labor, capital, interest rates, and inflation (Uzawa, 1964).

The macroeconomic model produces a distinct outcome for the Allen-Uzawa partials, especially in instances of homogeneous functional forms, and this generalization applies to functional forms with varying elasticities of substitution. Importantly, insights into the macroeconomic function's curvature are gained through the Allen-Uzawa partial elasticity of substitution. As per the Allen-Uzawa economic theory, cost functions exhibiting proper behavior must display concavity in relation to prices (Klump & Grandville, 2000).

According to the macroeconomic model, for given values of minimum real wage and monetary growth rate, a higher government deficit-income ratio can lead the economy to unstable growth patterns. This can be translated into explosive growths for unemployment rates for both labor sectors and might even asymptotically converge to 100% (Arrow et al., 1961). Additionally, the static production equation will be situated beneath the natural growth path, corresponding to the full employment growth trajectory of the skilled sector (Grossman & Helpman, 1991). In order to consider a more general case, both equation models assume two labor sectors are not the perfect substitutes. Particular attention will need to be focused on a case in which a variation might divulge whether the minimum real wage remains effective in just one labor sector or both labor sectors (Keller, 1975).

Both the static and dynamic equation are kept constant for various simulations; therefore, the differences among the unemployment paths originate from different values of economic growth rate (Rutherford, 1998). If the minimum real wage becomes ineffective in the skilled sector, due to a sufficient amount of money growth, the unemployment paths will be stabilized (Matsumoto & Szidarovsky, 2011). The constancy of both the minimum real wage and the macroeconomic growth rate contributes to the variations in unemployment paths, stemming from distinct deficit-income ratios. Importantly, the effectiveness of the minimum real wage persists in both labor sectors (Judd, 1985).

According to static equation, for a given monetary growth rate, a larger deficit-income ratio necessitates a greater increase in the stock of government bonds. As more bonds are sold to finance the current government deficit, private investment will become more crowded (Coeurdacier & Rey, 2013). The first is the wealth effect in the consumption function, through which commodity prices will rise. This will make the actual money supply smaller, driving up the interest rates, and crowding out investments. The other channel is the wealth effect in the money demand function, through which the interest rate will be raised (Iwaisako & Futagami, 2013). When the deficit-income ratio reaches a point where a substantial issuance of bonds occurs, leading to a notable crowding out of private investment, the marginal products of both labor types will decrease significantly. If these reductions are substantial, the minimum real wage can become influential in both sectors (Candore & Levine, 2012).

Connected to these unstable unemployment trends, it can be observed that the paths of income growth will consistently be positioned below the anticipated natural growth trajectory if the skilled labor sector were to reach full employment in the long run. Conversely, in scenarios where the deficit-income ratio is low, a relatively ample capital stock will be produced (Park, 2018). This ensures that the marginal productivity of skilled labor will remain sufficiently high, preventing the minimum real wage from exerting its influence in the skilled sector and thus avoiding instability (Sensier & Dijk, 2004).

## 3. Research Design, Data and Methodology

### 3.1. Data Sample

The data utilized in this study were from the World Bank Statistical Database. The panel data period was from 2010 to 2021 in the United States of America. The parameter values used for the macroeconomic Allen and Uzawa production model was conducted referencing the following: Blackkorby and Russell (1981), Edding and Marchenko (2012), Rutherford (1998) as well as Klump et al. (2012). This study used the secondary parameter values to investigate the growth of the macroeconomic Allen and Uzawa production model.

In addition, an extensive website search was conducted by using keywords such as average values, nominal money stock, interest elasticity of demand for money, and parameter of investment, all within a time frame range of the years 2010 to 2021. The macroeconomic Allen and Uzawa model investigated the interaction of the two equations. Accordingly, there were four input factors in the macroeconomic production function which were: capital, labor, interests and inflation.

The practicality of the macroeconomic model is evident in its ability to analytically ascertain the stability of both static and dynamic equation systems (Allen, 1932; Uzawa, 1962).

### 3.2. Methodology

In the macroeconomic model, the introduction of increased capital utilization is regarded as a form of mechanization in both static and dynamic equation systems. The production equation encompasses three input factors, categorized into two distinct groups: capital and labor. The capital category contains only one input: capital and labor. The capital category contains two input: skilled labor (L1) and unskilled labor (L2). The partial elasticities of substitution between any two inputs are all constant and are equal to unity if the two inputs are from two different categories (e.g., K and L1, or K and L2). However, they can be any non-negative real number if the two inputs belong to the same category (e.g., L1 and L2). The following equation expresses the production function:

$$Y = AK\ \alpha\text{-}\{[\gamma L1\text{-}\beta+(1\text{-}\gamma)L2\text{-}\beta]\text{-}1/\beta\ \}p$$
$$A>0;\ \alpha>0;\ \beta\geq-1;\ \beta\neq 0;\ 1>\gamma>0;\ \alpha+\beta=1$$

Y represents real income and the parameter A is an indicator of the state of technology. The parameters α, β, p indicates relative factor shares. The parameter β is what determines the value of the (constant) elasticity of substitution between the labor sectors. The following equation defines the partial elasticity of substitution between two types of labor:

$$\theta = 1 / (1+\beta)\neq 1$$

If $\beta= 0$, $\theta= 1$: The CES function can be simplifed to the (three-input) Allen-Uzawa production function

If $\beta = -1$, $\theta = \theta =\infty$: The skilled and unskilled labor sector are perfect substitutes.

If $\beta =\infty$, $\theta = \theta =0$: The skilled and unskilled labor sector are a perfect complement.

If $-1< \beta <\infty$ ($\beta \neq 0$): In this intermediate case, the two labor sectors can, to a certain extent, be substituted for each other. This case is the primary concern of this present study.

The Allen-Uzawa type of CES production function is adopted for the present study for two reasons. First, this research deals with a three-input and two-category production function, as indicated in the preceding paragraph. With these presumptions, the conventional two-input (capital and a unique type of labor) CES production function cannot serve this purpose. However, the Allen-Uzawa type CES production function contains a wide range of values for the elasticity or substitution between the two labor sectors (from 0 to positive infinity). This kind of generality is very crucial for the analysis of the effects of a minimum real wage in a two-labor-sector model. It is this two-labor-sector assumption that differentiates this study from previous studies conducted in the past. Two other properties associated with the Allen-Uzawa type CES production function make the model compatible with the present study:

(1) It is subject to diminishing marginal returns with additions to each input.

(2) It can be shown mathematically that the marginal product of any input approaches zero when the amount employed of that particular input approaches positive infinity; on the other extreme, if the amount employed of one particular input is zero, the marginal product of that input approaches positive infinity.

The macroeconomic model—which includes both the production function and the other equation, along with a list of notation—is presented below in Figure 1. Next, the parameter values for the present study will be given.

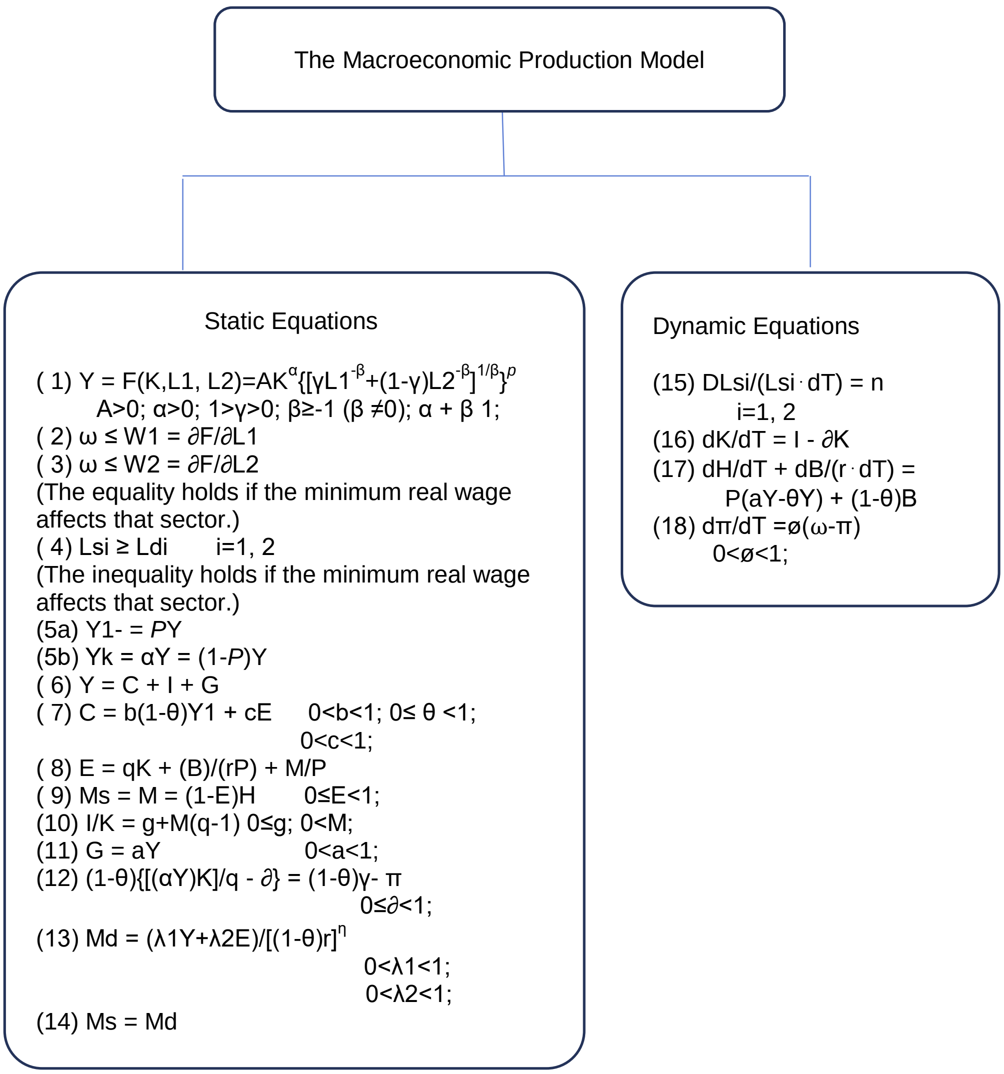


**Figure 1:** Tree diagram of the Allen and Uzawa model

## 3.3. Explanation of Variables

Below are the symbols of variables that were used for this study in the macroeconomic production model. Most was referenced from mathematical symbols in economics.

**Table 1:** Explanation of Mathematical Symbols

| Symbol of Variables | Definition | Symbol of Variables | Definition |
|---|---|---|---|
| Y | real income | Y1 | real labor income |
| Yk | real capital income | Lsi | effective labor supply of the ith labor sector, where I=1, 2 |
| Ld | effective total labor demand | K | capital stock |
| A | indicator of the state of technology | α | capital's share of income |
| P | labor's share of income | γ | unskilled labor's share of labor income |
| β | substitution parameter, which determines the elasticity of substitution between the two labor sector (∂), where ∂=1/(1+β) | ∂F/∂Li | partial derivative of the production function F(K, L1, L2) |
| Wi | real wage in the ith labor sector where I=1,2 | ω | minimum real wage imposed by government |
| C | real consumption | I | real gross investment |
| G | real government spending | b | marginal propensity to consume out of labor income |
| θ | proportional tax rate—applied to labor income, property income, and interest income | c | marginal propensity to consume out of wealth |
| E | real assets (wealth) | q | value of capital |
| r | nominal interest rate | P | price level |
| B | number of interest-bearing government bonds; each bond is a perpetuity paying $1 per year | μ | adjustment parameter of investment, with respect to the "q" ratio |
| a | ratio of government purchases of goods and services to income | σ | depreciation rate of capital |
| π | expected rate of inflation | Md | total money demand |
| $\lambda^1$ | income coefficient in the money demand function | $\lambda^2$ | wealth coefficient in the money demand function |
| η | the absolute value of the interest elasticity of demand for money (where the interest is net of taxes) | T | time (in terms of years) |
| n | exogenously determined population growth rate | ø | adjustment parameter of inflationary expectation |
| ω | actual rate of inflation | | |

## 3.4. The Parameter Values Used for Macroeconomic Production Model

Below are the parameter values that were used for this study in the macroeconomic production model. Most was referenced from the World Bank Database and the parameter values were adopted from the previous studies of Klump et al. (2012), Edding and Marchenko (2012), Blackkorby and Russell (1981), and Rutherford (1998).

**Table 2:** Parameter Values for Macroeconomic Production Model

| parameter values | parameter measure methods |
|---|---|
| ρ= .61 | This is the average value in the U.S. A. from 2010 to 2021.<br>Hence, capital's share of income is =.39. |
| γ= .25 | |
| b = .7 | This is an empirical value estimated by Klump, Mcadam & Willman (2012). |
| θ= .207 | This is the average value in the U.S. from 2010 to 2021. |
| c= .06 | This is an empirical estimation made by Klump, Mcadam & Willman (2012). |
| E= .23 | This is the average of the ratios of H to M occurring in the U.S. from 2010 to 2021, where M is measured as (M1+M2)/2. |
| g= .09 | Parameter in investment function by Edding & Marchenko (2012) |
| μ= .17 | This is an empirical value estimated by Blackkorby & Russell (1981). |
| a= .215 | This is the average value in the U.S. from 2010 to 2021. |
| ∂= .06 | |
| η= .4 | This is an empirical estimation given by Rutherford (1998). |
| n= .03 | |
| ø= .1 | |

## 4. Results

There are eighteen equations in this macroeconomic model as shown in Figure 1; the first fourteen are static equations and the last four are dynamic equations. Equation (1), the production function, was discussed in the preceding section. The next three equations, (2), (3), and (4), illustrate the demand for unskilled labor, the demand for skilled labor, and the short-run equilibrium situation in each labor is exogenously determined. Each labor sector is assumed to be growing at a constant rate over time, as shown in equation (15).

The demand for labor equations, specified in equations (2) and (3), include three possible cases;

(i) If the inequality holds for both (2) and (3), the minimum real wage, W, is not effective in either labor sector. Given a certain capital stock, the real wage in each sector, W1 and W2, depends on the marginal product of each type of labor at the full-employment level. Presumably, the equilibrium real wage in the skilled sector (W2) is greater than that of the unskilled sector (W1). In this case both labor sectors remain fully employed.

(ii) If the equality condition holds for both (2) and (3), the minimum real wage becomes effective for both labor sectors. The immediate impact of an effective minimum real wage with a given capital stock is that the quantity of labor demanded is indicated by a movement along the labor-demand curve toward the left side (Tsuzuki, 2016). Therefore, in each labor sector the labor supply exceeds the effective labor demand. This implies that the inequality in equation (4) holds for both sectors.

(iii) If the equality condition in (2) coexists with the inequality condition in (3), the minimum real wage remains effective only in the unskilled sector. Full employment is maintained for the skilled sector, but the unskilled sector will experience unemployment. Hence, in equation (4), the inequality condition holds for the unskilled sector but the equality condition holds for the skilled sector.

Equation (6) states the equilibrium condition in the commodity market. This is the conventional Keynesian formulation for a closed economy: the demand side of the commodity market consists of three components—consumption, investment, and government expenditure (Tsuzuki, 2016). Equation (7) specifies the consumption function. Real consumption depends on both real labor income, which is defined in equation (5a), and real private wealth. The latter consists of three assets: the capital stock, interest-bearing government bonds, and real money balances, as shown in equation (8). Since private wealth is one of the arguments in the consumption function, a change in private wealth (e.g., caused by a change in bonds or money) can cause the private sector's consumption to change accordingly via the wealth effect (Abbring, 2013).

In equation (12), q is the ratio of the market value if existing capital relative to the replacement cost of newly produced capital. Capital and bonds are assumed to be perfect substitutes in this study. The market-clearing condition for both bonds and capital is such that the real return (net of tax) on capital equals the real return (also net of tax) on government bonds (Klump & Preissler, 2000). This last market-clearing condition is shown in equation (12).

All government bonds are assumed to be perpetuities paying $1 every year. In addition, it is assumed that the public does not discount its future tax liability associated with the government bonds. In other words, government bonds in this study are considered by the public as net wealth. The money supply is assumed to be determined exogenously by the monetary authority as specified in equation (9). Demand for money is defined by equation (13). As in the case of the demand for consumption goods, the demand for real balances also depends on both real income and real wealth. Furthermore, in this model, the interest elasticity for money demand is assumed to be constant as shown in the denominator of equation (13).

Finally, equation (14) specifies the market-clearing condition for the money market.

Economists have debated whether wealth should be incorporated as an element in the money-demand function. A recent empirical study provides evidence that there is a significant wealth effect in the money-demand function (Tsuzuki, 2016). Furthermore, it should be pointed out that even if there were no wealth effect in the money demand function, the wealth effect in the consumption function alone would be sufficient to induce some of the crowding out effect associated with government bonds. This is true since government bonds, via the consumption function, have a tendency to elevate the price level, consequently reducing real money holdings. This, in turn, raises interest rates, thereby restraining investment. The set of dynamic equation, (15), (16), (17), and (18), illustrates how some of the important endogenous variables change over time. Equation (15) shows the rate of growth of the total effective labor force. Presumably, the two labor sectors grow at the same rate. Equation (16) shows the change over time in capital stock. Note that it is surmised that capital accumulation comes solely from private investment, and not directly from government spending. Equation (17) specifies the government budget constraint. The right-hand side of this equation indicates the government deficit occurring during a certain period of time. This equation states that the government deficit has to be financed by either printing more high-powered money or by issuing more government bonds. The last equation of this macroeconomic model, (18), describes the change of inflationary expectation, which is assumed to be adjusted adaptively.

## 5. Conclusions

### 5.1. Summary

This research obtained a CES production function by automating the process through the utilization of the Allen-Uzawa equations. In the absence of guaranteed stability in the long-run equilibrium, this study relied on the simulation of World Bank data as an analytical method to explore the long-run equilibrium solution. By changing one policy variable in each equation, the long-run movement of the important endogenous variables could be examined. Although many different equations have to be run in order for the macroeconomic model to be well understood, the difficulties frequently encountered in using other mathematical methods are overcome by utilizing this model. By analyzing a complicated, yet dynamic macroeconomic model, this current study was able to explore the interaction between several government macroeconomic policies. Those include a money financed deficit policy, a bond financed deficit policy, a tax financed policy and minimum real wage regulation in a set economy.

Although there is a lack of empirical evidence regarding the value of the Allen Uzawa partial elasticity of substitution between the two labor sectors, it is probably safe to argue that skilled and the unskilled workers are very unlikely to be perfect substitutes. Thus, as was one of the primary goals of this research, a more realistic model should consider the less extreme cases in which skilled labor and unskilled labor are not perfect substitutes.

When the two labor sectors are not perfect substitutes, there is a dramatic change of results. The unskilled labor, in the long run, does not necessarily converge to one of the two extreme cases—either 100% unemployment or 0% unemployment. Rather, there is a wide range of values of minimum real wage for the unskilled sector. On the other hand, if the minimum real wage affects both labor sectors, both sectors will converge to 100% unemployment just as in the one-labor-sector model. Conversely, if the minimum real wage is set too low to affect either labor sector, both sectors will maintain in full employment and the economy will behave as in the conventional neo-classical growth model.

This research employs mechanization to derive a CES production function from the Allen-Uzawa functions, and the challenges of mechanization are reflected in the elasticity of substitution between capital and labor. Consequently, the study explores the reasons behind variations in the elasticity of substitution among economies in the time series analysis. By investigating the ratio of the growth rate of output per labor unit, the economic growth rate becomes the range of labor use. As capital and labor act as substitutes in the long run, the Allen-Uzawa production mechanization model raises the coefficient to elucidate the economic growth rate.

### 5.2. Limitations and Future Studies

There are a large number of endogenous variables involved in this present model. Analyzing the current model using algebraic or differential equation methods is not feasible. Given the complexity of the model, which includes four dynamic equations, it is impractical to assess the stability of the system analytically. For example, if one uses differential equation analysis, with four dynamic equations, it requires four Allen-Uzawa necessary and sufficient conditions to determine whether the system is stable. This is almost an impossible task because most of the stability conditions contain various types of long-run equilibrium multipliers whose signs are indeterminate. Furthermore, the Allen-Uzawa production model does not incorporate a CES production function without technological changes. Consequently, this study is limited in its ability to examine empirical studies as technological changes play a crucial role in representing the overall production function.

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