J. Appl. Math. & Informatics Vol. **42**(2024), No. 1, pp. 199 - 206
https://doi.org/10.14317/jami.2024.199

ENDOMORPHISMS, ANTI-ENDOMORPHISMS AND BI-SEMIDERIVATIONS ON RINGS

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Abstract. The goal of this study is to bring out the following conclusion: Let $\mathcal{R}$ be a non-commutative prime ring of characteristic not two and $\mathcal{D}$ be a bi-semiderivation on $\mathcal{R}$ with a function $\mathfrak{f}$ (surjective). If $\mathcal{D}$ acts as an endomorphism or as an anti-endomorphism, then $\mathcal{D} = 0$ on $\mathcal{R}$.

AMS Mathematics Subject Classification : 16W25, 16N60, 16N80, 46L40.
Key words and phrases : (Semi)-prime ring, bi-semiderivation, endomorphism.

1. Introduction

Over the past several years, a number of scholars have examined the relationship between particular distinctive types of mappings on a ring $\mathcal{R}$ and $\mathcal{R}$'s commutativity. The first achievement in this area was made possible by Divinsky, who proved that if an automorphism of an Artinian ring $\mathcal{R}$ is nontrivial and commutative, then $\mathcal{R}$ must also be commutative. Luh expanded Divinsky's argument to prime rings. Mayne proved that if a prime ring has a non-identity centralizing automorphism, then $\mathcal{R}$ must be a commutative ring. These results have now been applied to additional algebraic structures. Posner confirmed that once a derivation takes place on a prime ring that is centralizing and nonzero, the commutative structure of the prime ring must exist. Over the last few decades, a number of scholars, including Bresar, Luh, Mayne, Kharchenko, Vukman, etc. have modified and improved these findings in various ways (see, for instance,[1], [2], [4], [5], [6], [7], [8], [9] and [11] for further references).

$\mathcal{R}$ should be represented as associative ring, together with the center $\mathcal{Z}(\mathcal{R})$ throughout the paper. Make sure that n is always a positive integer. $\mathcal{R}$ is referred to be a "n-torsion-free ring" if $nb = 0$ indicates that $b = 0$ for each b in

Received August 3, 2023. Revised September 24, 2023. Accepted November 28, 2023.
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the ring. The expression $[b, d]$ defined as $[b, d] = bd - db$ and served as a representation of the commutator of $b, d \in \mathcal{R}$. Remember that if $bRb = 0$ implies that $b = 0$, $\mathcal{R}$ is semiprime, and if $cRb = 0$ shows that either $c = 0$ or $b = 0$, $\mathcal{R}$ is said to be prime. A mapping η from $\mathcal{R}$ to $\mathcal{R}$ is recognized as a derivation on $\mathcal{R}$, if it satisfies $\eta(ce) = \eta(c)e + c\eta(e)$, for every $c, e \in \mathcal{R}$.

Let us say a ring possesses automorphism β. If $\mathrm{h}(bd) = \mathrm{h}(b)\beta(d) + b\mathrm{h}(d)$ fulfills for all b, d in $\mathcal{R}$ and exhibiting additivity, then the map h on $\mathcal{R}$ will be known as β-derivation. Denote identity mapping by $\mathcal{I}$ on $\mathcal{R}$, then $\mathrm{h} = \beta - I$ functioned as β-derivation. According to Maksa [7], a function $\mathcal{D} : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ is thought to have symmetry if $\mathcal{D}(p, q) = \mathcal{D}(q, p)$ for each p, q in $\mathcal{R}$. A mapping from $\mathcal{R} \times \mathcal{R}$ into $\mathcal{R}$ is said to be bi-additive if $\mathcal{D}$ is additive in both slots. The following is an introduction to the bi-derivations theory: The mapping $\mathcal{D}$, additive in each tuple and possessing symmetric property is known as symmetric bi-derivation when the mappings $q \mapsto \mathcal{D}(p, q)$ and the map $p \mapsto \mathcal{D}(p, q)$ are both derivations of $\mathcal{R}$. [7, 12] might be cited for thought-provoking reading on the subject. A function $\mathfrak{h}$ on $\mathcal{R}$ is referred to as the trace of $\mathcal{D}$ for a symmetric mapping $\mathcal{D}$ when it is written as $\mathfrak{h}(p) = \mathcal{D}(p, p)$, p in $\mathcal{R}$. We can create these mappings, as seen in the example below:

Example 1.1. Consider a ring $\mathcal{R} = \left\{ \begin{pmatrix} l & 0 & 0 \\ t & l & 0 \\ j & p & l \end{pmatrix} \mid l, t, j, p \in \mathbb{R} \right\}$. Then $\mathcal{R}$ is a noncommutative associative ring under the usual operations on matrix like addition and multiplication. Next designed a map $\varrho : \mathcal{R} \to \mathcal{R}$ by $\varrho(r) = \begin{pmatrix} l & 0 & 0 \\ 0 & 0 & 0 \\ j & 0 & 0 \end{pmatrix}$ for all $r \in \mathcal{R}$. ϱ must be additive function, that much is certain. Now, introduce a map $\varpi : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ by $\varpi(r, e) = [r, \varrho(e)] + [e, \varrho(r)]$ for each $r, e \in \mathcal{R}$, The symmetry and bi-additiveness of ϖ can be verified with ease.

J. Bergen introduces the idea of semiderivations of a ring $\mathcal{R}$ in [3]. A mapping f that is additive from $\mathcal{R}$ to $\mathcal{R}$ is known as a semiderivation if there exists a function g on $\mathcal{R}$ such that $f(ab) = f(a)g(b) + af(b) = f(a)b + g(a)f(b)$ and $f(g(a)) = g(f(a))$ for each a, b in $\mathcal{R}$. All semiderivations associated with g are just normal derivations if g is an identity map of $\mathcal{R}$. However, if g is a homomorphism of R such that $g \neq I_{identity}$, then $f = g - I$ is a semiderivation rather than a derivation. Some remarkable results related to semiderivations found in [5].

A bi-additive and symmetric mapping $\mathcal{D}$ from $\mathcal{R} \times \mathcal{R}$ to $\mathcal{R}$ is recognized as a symmetric bi-semiderivation associated with a mapping $\mathfrak{f} : \mathcal{R} \longrightarrow \mathcal{R}$, if

$$\mathcal{D}(pq, r) = \mathcal{D}(p, r)\mathfrak{f}(q) + p\mathcal{D}(q, r) = \mathcal{D}(p, r)q + \mathfrak{f}(p)\mathcal{D}(q, r)$$

and $\mathfrak{h}(\mathfrak{f}) = \mathfrak{f}(\mathfrak{h})$ for each p, q, r in $\mathcal{R}$. One can look in [11] for conceptual facts. We will take $\mathfrak{f}$, a surjective function throughout in our theorems.

2. Main results

The present results are the generalization of the research done by Bell and Kappe in [2]. We start with the following lemmas:

Lemma 2.1. [6] *Let α be a nontrivial automorphism on a prime ring $\mathcal{R}$. If $[\alpha(t), t] = 0$, for every $t \in \mathcal{R}$, then $\mathcal{R}$ is a commutative ring.*

Lemma 2.2. *Let $\mathcal{R}$ be a semiprime ring and $\mathcal{D}$ be a bi-semiderivation on $\mathcal{R}$ with a function $\mathfrak{f}$. If $\mathcal{D}$ acts as an endomorphism, then $(\mathcal{D}(c,a) - c)e\mathcal{D}(d,a) = 0$, for each $a, c, d, e \in \mathcal{R}$.*

Proof. Since $\mathcal{D}$ acts as an endomorphism, we have for each a, c, b, d in R

$$\begin{aligned} \mathcal{D}(cd, a) = \mathcal{D}(c, a)\mathcal{D}(d, a) \\ \mathcal{D}(c, ab) = \mathcal{D}(c, a)\mathcal{D}(c, b). \end{aligned} \tag{1}$$

Take the first identity in (1)

$$\mathcal{D}(c, a)\mathfrak{f}(d) + c\mathcal{D}(d, a) = \mathcal{D}(c, a)\mathcal{D}(d, a) \text{ for every } a, c, d \text{ in } \mathcal{R}. \tag{2}$$

Simplify above expression to get

$$\mathcal{D}(c, a)\mathfrak{f}(d) + c\mathcal{D}(d, a) - \mathcal{D}(c, a)\mathcal{D}(d, a) = 0 \text{ for every } a, c, d \text{ in } \mathcal{R}. \tag{3}$$

Put ce in place of c in (3), we obtain

$$\begin{aligned} &\mathcal{D}(c, a)\mathcal{D}(e, a)\mathfrak{f}(d) + ce\mathcal{D}(d, a) \\ &-\mathcal{D}(c, a)\mathcal{D}(e, a)\mathcal{D}(d, a) = 0 \text{ for every } a, c, d, e \text{ in } \mathcal{R}. \end{aligned} \tag{4}$$

Rearranging the above expression and use (2) to find

$$\mathcal{D}(c, a)\{-e\mathcal{D}(d, a)\} + ce\mathcal{D}(d, a) = 0 \text{ for every } a, c, d, e \text{ in } \mathcal{R}. \tag{5}$$

This yields that

$$(\mathcal{D}(c, a) - c)e\mathcal{D}(d, a) = 0 \text{ for every } a, c, d, e \text{ in } \mathcal{R}. \tag{6}$$

Lemma 2.3. *Let $\mathcal{R}$ be a semiprime ring and $\mathcal{D}$ be a bi-semiderivation on $\mathcal{R}$ with a function $\mathfrak{f}$. If $\mathcal{D}$ acts as an anti-endomorphism, then $\mathcal{D}(b, e)r[\mathcal{D}(b, e), t] = 0$, for each $b, e, r, t \in \mathcal{R}$.*

Proof. Since $\mathcal{D}$ acts as an anti-endomorphism, we have for each a, b, e in R

$$\begin{aligned} \mathcal{D}(ba, e) = \mathcal{D}(a, e)\mathcal{D}(b, e) \\ \mathcal{D}(e, ba) = \mathcal{D}(e, a)\mathcal{D}(e, b). \end{aligned} \tag{7}$$

Assume the first identity in (7) to find

$$\mathcal{D}(b, e)\mathfrak{f}(a) + b\mathcal{D}(a, e) = \mathcal{D}(a, e)\mathcal{D}(b, e). \tag{8}$$

Replace b by b^2 in (8) to get for every a, b, e in $\mathcal{R}$

$$\mathcal{D}(b,e)\mathcal{D}(b,e)\mathfrak{f}(a) + b^2\mathcal{D}(a,e) = \{\mathcal{D}(a,e)\mathcal{D}(b,e)\}\mathcal{D}(b,e). \tag{9}$$

Rewrite the left hand side of (9) by use of (8), we observe

$$\mathcal{D}(b,e)\mathcal{D}(b,e)\mathfrak{f}(a) + b^2\mathcal{D}(a,e) = \{\mathcal{D}(b,e)\mathfrak{f}(a) + b\mathcal{D}(a,e)\}\mathcal{D}(b,e). \tag{10}$$

On simplification of above expression, we find

$$\mathcal{D}(b,e)[\mathcal{D}(b,e),\mathfrak{f}(a)] + b\{b\mathcal{D}(a,e) - \mathcal{D}(a,e)\mathcal{D}(b,e)\} = 0 \text{ for every } a, b, e \text{ in } \mathcal{R}. \tag{11}$$

Making use of (8), (11) takes the form below

$$\mathcal{D}(b,e)[\mathcal{D}(b,e),\mathfrak{f}(a)] + b\{-\mathcal{D}(b,e)\mathfrak{f}(a)\} = 0 \text{ for every } a, b, e \text{ in } \mathcal{R}. \tag{12}$$

Reword the last expression by changing $r = \mathfrak{f}(a)$, as $\mathfrak{f}(a)$ is a surjective map on $\mathcal{R}$

$$\mathcal{D}(b,e)[\mathcal{D}(b,e),r] - b\mathcal{D}(b,e)r = 0 \text{ for every } b, e, r \text{ in } \mathcal{R}. \tag{13}$$

Put rt for r in above equation and use it again to obtain

$$\mathcal{D}(b,e)r[\mathcal{D}(b,e),t] = 0 \text{ for every } b, e, r, t \text{ in } \mathcal{R}. \tag{14}$$

This is, what we required.

Theorem 2.4. *Let $\mathcal{R}$ be a non-commutative prime ring of characteristic not two and $\mathcal{D}$ be a bi-semiderivation on $\mathcal{R}$ with an automorphism function $\mathfrak{f}$. If $\mathcal{D}$ acts as an endomorphism or as an anti-endomorphism, then $\mathcal{D} = 0$ on $\mathcal{R}$.*

Proof. We assume first $\mathcal{D}$ is an endomorphism and use Lemma 2.2 to get

$$(\mathcal{D}(c,a) - c)e\mathcal{D}(d,a) = 0 \text{ for every } a, c, d, e \text{ in } \mathcal{R}. \tag{15}$$

Reword above expression by replacing tc for c, we have

$$\mathcal{D}(t,a)\mathfrak{f}(c)e\mathcal{D}(d,a) + t\mathcal{D}(c,a)e\mathcal{D}(d,a) - tce\mathcal{D}(d,a) = 0 \text{ for each } a, c, d, e, t \text{ in } \mathcal{R}. \tag{16}$$

Comparing the last two equations to obtain

$$\mathcal{D}(t,a)\mathfrak{f}(c)e\mathcal{D}(d,a) = 0 \text{ for each } a, c, d, e, t \text{ in } \mathcal{R}. \tag{17}$$

This implies that by using the surjectivity of $\mathfrak{f}$, $\mathcal{D}(t,a)re\mathcal{D}(d,a) = 0$ for every $a, d, e, r, t \in \mathcal{R}$. Hence we have $\mathcal{D}(d,a)re\mathcal{D}(d,a)r = 0$ for each $a, d, e, r \in \mathcal{R}$. Primeness of $\mathcal{R}$ intended us $\mathcal{D}(d,a)r = 0$ for each $a, d, r \in \mathcal{R}$. Also we can get by last expression $\mathcal{D}(d,a) = 0$ for each $a, d \in \mathcal{R}$. Next using the additivity in first slot as $\mathcal{D}(d,a) = \mathcal{D}(d+0,a) = \mathcal{D}(d,a) + \mathcal{D}(0,a)$ for every $a, d \in \mathcal{R}$. Use an analogous trick for second slot to find $\mathcal{D} = 0$ on $\mathcal{R}$.

Next suppose that $\mathcal{D}$ as an anti-endomorphism, use Lemma 2.3 to have

$$\mathcal{D}(b,e)r[\mathcal{D}(b,e),t] = 0 \text{ for every } b, e, r, t \text{ in } \mathcal{R}. \tag{18}$$

A simple manipulation by taking suitable multiplier from left and right in (18) and subtracting the two obtained equation, which gives us

$$[\mathcal{D}(b,e),t]r[\mathcal{D}(b,e),t] = 0 \text{ for every } b, e, r, t \text{ in } \mathcal{R}. \tag{19}$$

Primeness intended us $[\mathcal{D}(b,e),t]=0$ for every $b,e,t\in\mathcal{R}$. Put bc in place of b in the last expression and simply to find

$$\mathcal{D}(b,e)[\mathfrak{f}(c),t]+[b,t]\mathcal{D}(c,e)=0 \text{ for every } b,e,c,t \text{ in } \mathcal{R}. \tag{20}$$

This implies that

$$\mathcal{D}(t,e)[\mathfrak{f}(c),t]=0 \text{ for every } e,c,t \text{ in } \mathcal{R}. \tag{21}$$

Primeness of $\mathcal{R}$ yields that either $\mathcal{D}(t,e)=0$ or $[\mathfrak{f}(c),t]=0$ for all $c,e,t\in\mathcal{R}$. Take the second part if $[\mathfrak{f}(c),t]=0$ for all $c,t\in\mathcal{R}$, then application of Lemma 2.1 gives the commutativity of $\mathcal{R}$, that is a contradiction. Hence by first part, we say $\mathcal{D}(t,e)=0$ for all t,e in $\mathcal{R}$, and hence $\mathcal{D}=0$ on $\mathcal{R}$ as desired.

Corollary 2.5. *Let $\mathcal{R}$ be a non-commutative prime ring having char $\neq 2$, $\mathcal{L}$ be a nonzero ideal of $\mathcal{R}$ and $\mathcal{D}$ be a bi-derivation on $\mathcal{R}$. If $\mathcal{D}$ acts as an endomorphism or as an anti-endomorphism on $\mathcal{R}$, then $\mathcal{D}=0$ on $\mathcal{R}$.*

Proof. The proof is straight forward by above theorem by taking $\mathcal{L}=\mathcal{R}$.

Corollary 2.6. *Let $\mathcal{R}$ be a non-commutative prime ring having char $\neq 2$, $\mathcal{L}$ be a nonzero ideal of $\mathcal{R}$ and $\mathcal{D}$ be a bi-derivation on $\mathcal{R}$. If $\mathcal{D}$ acts as a homomorphism or as an anti-homomorphism on $\mathcal{R}$, then $\mathcal{D}=0$ on $\mathcal{R}$.*

Corollary 2.7. *Let $\mathcal{R}$ be a non-commutative prime ring, $\mathcal{L}$ be a nonzero ideal of $\mathcal{R}$ and $\mathcal{D}$ be a derivation on $\mathcal{R}$. If $\mathcal{D}$ acts as a homomorphism or as an anti-homomorphism on $\mathcal{R}$, then $\mathcal{D}=0$ on $\mathcal{R}$.*

Proof. For the detailed proof of this result, one can look in [2].

Theorem 2.8. *Let $\mathcal{R}$ be a semiprime ring with 2-torsion freeness, $\mathcal{L}$ be a nonzero ideal of $\mathcal{R}$ and $\mathcal{D}$ be a bi-semiderivation on $\mathcal{R}$ with a function $\mathfrak{f}$. If $\mathcal{D}$ acts as a homomorphism or as an anti-homomorphism, then one of the following conditions hold:*

(1) *$\mathcal{D}=0$ on $\mathcal{R}$.*
(2) *$\mathcal{R}$ contains a nonzero central ideal.*

Proof. First consider $\mathcal{D}$ acting as homomorphism, then we have from equation (17) $\mathcal{D}(t,a)\mathfrak{f}(c)e\mathcal{D}(d,a)=0$ for every $a,c,e,d,t\in\mathcal{R}$. Some replacement and surjectiveness of $\mathfrak{f}$ yielding that

$$[s,t]\mathcal{D}(b,e)=0 \text{ for every } b,e,t,s \text{ in } \mathcal{L}. \tag{22}$$

Semiprimeness of $\mathcal{R}$ ensure us the presence of a family of prime ideals say $\mathfrak{P}=\{\mathcal{P}_i \mid i\in\ltimes\}$ such that $\bigcap\mathcal{P}_i=\{0\}$. Let us suppose that $\mathcal{P}_1$ and $\mathcal{P}_2$ are some member of $\mathfrak{P}$. By (22), we have $[b,d]\in\mathcal{P}_1$ and $\mathcal{D}(c,c)\in\mathcal{P}_2$ for all $c,d,b\in\mathcal{L}$. Now designed the two subsets as $\mathrm{A}=\{d\in\mathcal{L}\mid[b,d]\subseteq\mathcal{P}_1\}$ and $\mathrm{C}=\{c\in\mathcal{L}\mid\mathcal{D}(c,c)\subseteq\mathcal{P}_2\}$. We observe that both A and C are additive subgroup of $\mathcal{R}$ such that $\mathcal{R}=\mathrm{A}\bigcup\mathrm{C}$. Being the property that a group cannot consist of joint of its appropriate subgroups. As a result, we determine either $\mathcal{R}=\mathrm{A}$ or $\mathcal{R}=\mathrm{C}$. First, take the situation $\mathcal{R}\neq\mathrm{A}$, this yields that $\mathcal{R}=\mathrm{C}$. That is, $\mathcal{D}(c,c) \in \mathcal{P}_2$ for all $c \in \mathcal{L}$. A simple manipulation gives that $\mathcal{D}(c,c)t \in \mathcal{P}_2$ for all $c \in \mathcal{L}$ and $t \in \mathcal{R}$. Using primeness of $\mathcal{P}_2$, we find either $\mathcal{D}(c,c) \in \mathcal{P}_2$ or $t \in \mathcal{P}_2$ for each $c \in \mathcal{L}$ and $t \in \mathcal{R}$. If $t \in \mathcal{P}_2$, then $[t, \mathcal{R}] \subseteq \mathcal{P}_2$, a contradiction occur to our expectation $\mathcal{R} \neq \mathrm{A}$. Therefore, we have $\mathcal{D}(c,c) \in \mathcal{P}_2$ for all $c \in \mathcal{R}$. Hence we get $\mathcal{D}(c,c) \subseteq \cap \mathcal{P}_2 = \{0\}$ for every $c \in \mathcal{R}$. This implies that $\mathcal{D}(c,c) = 0$ for every $c \in \mathcal{R}$. Similarly, we discard the case when $\mathcal{R} \neq \mathrm{C}$ and we obtain $\mathcal{R} = \mathrm{A}$. This implicit that $[c,d] = 0$ for each $c, d \in \mathcal{L}$. Hence $\mathcal{R}$ owns a central and nonzero ideal contained in itself.

Next think about the case, $\mathcal{D}$ acting as anti-homomorphism, then from equation (21) we observe $\mathcal{D}(t,e)[\mathfrak{f}(c),t] = 0$ for every $e, c, t \in \mathcal{R}$. Since $\mathfrak{f}$ is surjective, we have $\mathcal{D}(t,e)[s,t] = 0$ for every $e \in \mathcal{L}$ and s, t in $\mathcal{R}$. Repeat the same arguments and proceed in the same way as above, we find the desired conclusion.

Corollary 2.9. *Let $\mathcal{R}$ be a semiprime ring with 2-torsion freeness, $\mathcal{L}$ be a nonzero ideal of $\mathcal{R}$ and $\mathcal{D}$ be a bi-derivation on $\mathcal{R}$ with a function $\mathfrak{f}$. If $\mathcal{D}$ acts as a homomorphism or as an anti-homomorphism, then one of the following conditions hold:*

(1) $\mathcal{D} = 0$ *on* $\mathcal{R}$.
(2) $\mathcal{R}$ *contains a nonzero central ideal.*

Corollary 2.10. *Let $\mathcal{R}$ be a prime ring $\mathcal{L}$ be a nonzero ideal of $\mathcal{R}$ and $\mathcal{D}$ be a bi-derivation on $\mathcal{R}$. If $\mathcal{D}$ acts as a homomorphism or as an anti-homomorphism, then one of the following conditions hold:*

(1) $\mathcal{D} = 0$ *on* $\mathcal{R}$.
(2) $\mathcal{R}$ *is commutative.*

Proof. By applying Theorem 2.8, we get the required result.

Example 2.11. Let $\mathcal{R} = \left\{ \begin{pmatrix} m & 0 \\ 0 & n \end{pmatrix} \mid m, n \in 2\mathbb{Z}_8 \right\}$ is a ring under matrix addition and matrix multiplication. Define map h from $\mathcal{R}$ to itself by

$$h\left[\begin{pmatrix} m_1 & 0 \\ 0 & n_1 \end{pmatrix}\right] = \begin{pmatrix} 0 & 0 \\ 0 & n_1 \end{pmatrix}$$

and $\mathcal{K} : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ by $\mathcal{K}\left[\begin{pmatrix} m_1 & 0 \\ 0 & n_1 \end{pmatrix}, \begin{pmatrix} m_2 & 0 \\ 0 & n_2 \end{pmatrix}\right] = \begin{pmatrix} m_1 m_2 & 0 \\ 0 & 0 \end{pmatrix}$ for every $m_1, m_2, n_1, n_2 \in 2\mathbb{Z}_8$. Then $\mathcal{K}$ is a nonzero bi-semiderivation on $\mathcal{R}$ with associated function h. It is easy to verify that $\mathcal{K}$ acting as endomorphism and anti-endomorphism on $\mathcal{R}$ but $\mathcal{K} \neq 0$. Hence the condition of primeness in the hypothesis of above theorems can not be omitted.

3. Conclusion

We conclude by our computation and study that it would be interesting to analyze the problem using the linearity tools of operator theory instead of purely ring theoretic context as only algebraic concepts are used in formulation of the results given in the present paper.

Conflicts of interest : Authors declared that there is no conflict of interest.

Data availability : Not applicable

Acknowledgments : The authors of the paper extend their sincere gratitude to Islamic University of Madinah and Taibah University, Madinah, Saudi Arabia.

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