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POWER CORDIAL GRAPHS

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Abstract. A power cordial labeling of a graph $G = (V(G), E(G))$ is a bijection $f : V(G) \rightarrow \{1, 2, ..., |V(G)|\}$ such that an edge $e = uv$ is assigned the label 1 if $f(u) = (f(v))^n$ or $f(v) = (f(u))^n$, for some $n \in \mathbb{N} \cup \{0\}$ and the label 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. In this paper, we study power cordial labeling and investigate power cordial labeling for some standard graph families.

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1. Introduction

In this paper, we consider simple, finite, connected and undirected graph $G = (V(G), E(G))$. For all standard terminologies and notations we follow Clark and Holton [1]. We will give brief summary of definitions which are useful for the present investigations.

Definition 1.1. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges) then the labeling is called a vertex labeling (an edge labeling).

For an extensive survey on graph labeling and bibliographic references we refer to Gallian [2].

In 1987, Cahit [3] introduced cordial labeling as a weaker version of graceful labeling and harmonious labeling. Many variants of cordial labeling like prime cordial labeling [4], divisor cordial labeling [5], product cordial labeling [6], edge product cordial labeling [7] etc. were also introduced.

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Motivated from this we define power cordial labeling of graph as follows:

Definition 1.2. For a graph $G = (V(G), E(G))$, the vertex labeling function is defined as a bijection $f : V(G) \to \{1, 2, \dots, |V(G)|\}$ and induced edge labeling function $f^* : E(G) \to \{0, 1\}$ is given by

$$f^*(e = uv) = \begin{cases} 1, & \text{if } f(u) = (f(v))^n \text{ or } f(v) = (f(u))^n, \text{ for some } n \in \mathbb{N} \cup \{0\}; \\ 0, & \text{otherwise.} \end{cases}$$

The number of edges labeled with 0 and 1 is denoted by $e_f(0)$ and $e_f(1)$ respectively. f is called *power cordial labeling* of graph G if $|e_f(0) - e_f(1)| \leq 1$. The graph that admits a power cordial labeling is called a *power cordial graph*.

In this paper, we discuss power cordial labeling for some standard graph families.

2. Main results

Theorem 2.1. *The path P_n is a power cordial graph for $n \leq 12$ and not a power cordial graph for $n > 12$.*

Proof. Let P_n be the path with vertices $v_1, v_2, v_3, ..., v_n$ and edges $e_1, e_2, e_3, ..., e_{n-1}$. Then $|V(P_n)| = n$ and $|E(P_n)| = n - 1$.

We define the power cordial labeling $f : V(P_n) \to \{1, 2, \dots, n\}$ by following six cases.

Case 1: For $n \leq 4$.

$$f(v_i) = i; \quad \text{for } 1 \leq i \leq n.$$

In view of above defined labeling pattern, we have $e_f(0) = 0$ and $e_f(1) = 1$ for $n = 2$, $e_f(0) = 1$ and $e_f(1) = 1$ for $n = 3$ and $e_f(0) = 2$ and $e_f(1) = 1$ for $n = 4$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Case 2: For $n = 5, 6$.

$$\begin{aligned} f(v_1) &= 2, \\ f(v_2) &= 1, \\ f(v_i) &= i; \quad \text{for } 3 \leq i \leq n. \end{aligned}$$

In view of above defined labeling pattern, we have $e_f(0) = 2$ and $e_f(1) = 2$ for $n = 5$ and $e_f(0) = 3$ and $e_f(1) = 2$ for $n = 6$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Case 3: For $n = 7, 8$.

$$\begin{aligned} f(v_1) &= 3, \\ f(v_2) &= 1, \\ f(v_3) &= 2, \\ f(v_i) &= i; \quad \text{for } 4 \leq i \leq n. \end{aligned}$$

In view of above defined labeling pattern, we have $e_f(0) = 3$ and $e_f(1) = 3$ for $n = 7$ and $e_f(0) = 4$ and $e_f(1) = 3$ for $n = 8$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Case 4: For $n = 9, 10$.

$$\begin{array}{ll} f(v_1) = 3, & \\ f(v_2) = 1, & \\ f(v_3) = 8, & \\ f(v_{3+i}) = 2^i; & \text{for } 1 \leq i \leq 2, \\ f(v_{5+i}) = 4 + i; & \text{for } 1 \leq i \leq 3, \\ f(v_i) = i; & \text{for } 9 \leq i \leq n. \end{array}$$

In view of above defined labeling pattern, we have $e_f(0) = 4$ and $e_f(1) = 4$ for $n = 9$ and $e_f(0) = 5$ and $e_f(1) = 4$ for $n = 10$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Case 5: For $n = 11, 12$.

$$\begin{array}{ll} f(v_i) = 3^i; & \text{for } 1 \leq i \leq 2, \\ f(v_3) = 1, & \\ f(v_4) = 8, & \\ f(v_{4+i}) = 2^i; & \text{for } 1 \leq i \leq 2, \\ f(v_{6+i}) = 4 + i; & \text{for } 1 \leq i \leq 3, \\ f(v_i) = i; & \text{for } 10 \leq i \leq n. \end{array}$$

In view of above defined labeling pattern, we have $e_f(0) = 5$ and $e_f(1) = 5$ for $n = 11$ and $e_f(0) = 6$ and $e_f(1) = 5$ for $n = 12$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Case 6: For $n \geq 13$.

In order to satisfy the edge condition for power cordial labeling of graph it is essential to label at least $\left\lfloor \frac{n-1}{2} \right\rfloor$ edges with label 0 as well as with label 1 out of total $n - 1$ edges. All the possible assignment of vertex labels will give rise at least $\left\lfloor \frac{n-1}{2} \right\rfloor + 1$ edges with label 0 and at most $\left\lfloor \frac{n-1}{2} \right\rfloor - 1$ edges with label 1. Therefore, $|e_f(0) - e_f(1)|$ is at least two. Thus, the path P_n is not a power cordial graph for $n \geq 13$.

Hence, the path P_n is a power cordial graph for $n \leq 12$ and not a power cordial graph for $n > 12$. □

Theorem 2.2. *The cycle C_n is a power cordial graph for $n \leq 11$ and not a power cordial graph for $n > 11$.*

Proof. Let C_n be the cycle with vertices $v_1, v_2, v_3, ..., v_n$ and edges $e_1, e_2, e_3, ..., e_n$. Then $|V(C_n)| = n$ and $|E(C_n)| = n$.

We define the power cordial labeling $f : V(C_n) \to \{1, 2, \dots, n\}$ by following five cases.

Case 1: For $n \leq 5$.

$$f(v_i) = i; \quad \text{for } 1 \leq i \leq n.$$

In view of above defined labeling pattern, we have $e_f(0) = 1$ and $e_f(1) = 2$ for $n = 3$, $e_f(0) = 2$ and $e_f(1) = 2$ for $n = 4$ and $e_f(0) = 3$ and $e_f(1) = 2$ for $n = 5$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Case 2: For $n = 6, 7$.

$$\begin{aligned} &f(v_i) = 2^{i-1}; && \text{for } 1 \leq i \leq 3,\\ &f(v_4) = 3, &&\\ &f(v_i) = i; && \text{for } 5 \leq i \leq n. \end{aligned}$$

In view of above defined labeling pattern, we have $e_f(0) = 3$ and $e_f(1) = 3$ for $n = 6$ and $e_f(0) = 4$ and $e_f(1) = 3$ for $n = 7$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Case 3: For $n = 8, 9$.

$$\begin{aligned} &f(v_1) = 1, &&\\ &f(v_2) = 8, &&\\ &f(v_{2+i}) = 2^i; && \text{for } 1 \leq i \leq 2,\\ &f(v_5) = 3, &&\\ &f(v_i) = i - 1; && \text{for } 6 \leq i \leq 8,\\ &f(v_i) = i; && \text{for } i = 9. \end{aligned}$$

In view of above defined labeling pattern, we have $e_f(0) = 4$ and $e_f(1) = 4$ for $n = 8$ and $e_f(0) = 5$ and $e_f(1) = 4$ for $n = 9$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Case 4: For $n = 10, 11$.

$$\begin{aligned} &f(v_1) = 1, &&\\ &f(v_2) = 8, &&\\ &f(v_{2+i}) = 2^i; && \text{for } 1 \leq i \leq 2,\\ &f(v_{4+i}) = 3^i; && \text{for } 1 \leq i \leq 2,\\ &f(v_i) = i - 2; && \text{for } 7 \leq i \leq 9,\\ &f(v_i) = i; && \text{for } 10 \leq i \leq n. \end{aligned}$$

In view of above defined labeling pattern, we have $e_f(0) = 5$ and $e_f(1) = 5$ for $n = 10$ and $e_f(0) = 6$ and $e_f(1) = 5$ for $n = 11$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Case 5: For $n \geq 12$.

In order to satisfy the edge condition for power cordial labeling of graph it is essential to label at least $\left\lfloor \frac{n}{2} \right\rfloor$ edges with label 0 as well as with label 1 out of total n edges. All the possible assignment of vertex labels will give rise at least

$\left\lfloor \frac{n}{2} \right\rfloor + 1$ edges with label 0 and at most $\left\lfloor \frac{n}{2} \right\rfloor - 1$ edges with label 1. Therefore, $|e_f(0) - e_f(1)|$ is at least two. Thus, the cycle C_n is not a power cordial graph for $n \geq 12$.

Hence, the cycle C_n is a power cordial graph for $n \leq 11$ and not a power cordial graph for $n > 11$. □

Theorem 2.3. *The complete graph K_n is a power cordial graph for $n = 1, 2, 3$ and 5 and not a power cordial graph for $n = 4$ and $n \geq 6$.*

Proof. Let K_n be the complete graph with vertices $v_1, v_2, ..., v_n$. Then $|V(K_n)| = n$ and $|E(K_n)| = \frac{n(n-1)}{2}$.

We define the power cordial labeling $f : V(K_n) \to \{1, 2, \ldots, n\}$ by following two cases.

Case 1: For $n = 1, 2, 3$ and 5.

$$f(v_i) = i; \quad \text{for } 1 \leq i \leq n.$$

In view of above defined labeling pattern, we have $e_f(0) = 0$ and $e_f(1) = 0$ for $n = 1$, $e_f(0) = 0$ and $e_f(1) = 1$ for $n = 2$, $e_f(0) = 1$ and $e_f(1) = 1$ for $n = 3$ and $e_f(0) = 5$ and $e_f(1) = 5$ for $n = 5$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Case 2: For $n = 4$ and $n \geq 6$.

In order to satisfy the edge condition for power cordial labeling of graph it is essential to label at least $\left\lfloor \frac{n(n-1)}{4} \right\rfloor$ edges with label 0 as well as with label 1 out of total $\frac{n(n-1)}{2}$ edges. All the possible assignment of vertex labels will give rise 2 edges with label 0 and 4 edges with label 1 out of total 6 edges for $n = 4$ and All the possible assignment of vertex labels will give rise at least $\left\lfloor \frac{n(n-1)}{4} \right\rfloor + 1$ edges with label 0 and at most $\left\lfloor \frac{n(n-1)}{4} \right\rfloor - 1$ edges with label 1 for $n \geq 6$. Therefore, $|e_f(0) - e_f(1)|$ is at least two. Thus, the complete graph K_n is not a power cordial graph for $n = 4$ and $n \geq 6$.

Hence, the complete graph K_n is a power cordial graph for $n = 1, 2, 3$ and 5 and not a power cordial graph for $n = 4$ and $n \geq 6$. □

Definition 2.4. The *wheel* W_n is defined to be the join $K_1 + C_n$, the vertex corresponding to K_1 is known as an apex vertex and vertices corresponding to C_n are known as rim vertices.

Theorem 2.5. *The wheel W_n is a power cordial graph for $n \geq 4$ and not a power cordial graph for $n = 3$.*

Proof. Let W_n be the wheel with rim vertices $v_1, v_2, ..., v_n$ and an apex vertex v. Then $|V(W_n)| = n + 1$ and $|E(W_n)| = 2n$.

We define the power cordial labeling $f : V(W_n) \to \{1, 2, \ldots, n+1\}$ by following two cases.

Case 1: For $n \geq 4$.

$$\begin{aligned} &f(v) = 1, \\ &f(v_1) = p; && \text{where } p \text{ is the largest prime number less than } n + 2, \\ &f(v_i) = i; && \text{for } 2 \leq i \leq p - 1, \\ &f(v_i) = i + 1; && \text{for } p \leq i \leq n. \end{aligned}$$

In view of above defined labeling pattern, we have $e_f(0) = n$ and $e_f(1) = n$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| = 0$.

Case 2: For $n = 3$.

The wheel W_3 is same as K_4 and by Theorem 2.3. it is not a power cordial graph.

Hence, the wheel W_n is a power cordial graph for $n \geq 4$ and not a power cordial graph for $n = 3$. □

Example 2.6. The wheel W_8 and its power cordial labeling is shown in Fig 1.

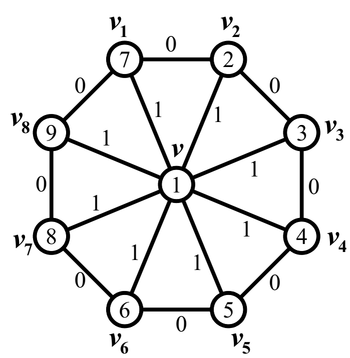

Fig 1: The wheel W_8 and its power cordial labeling.

Definition 2.7. The *tadpole* $T(n, m)$ is a graph in which path P_m is join by an edge with cycle C_n.

Theorem 2.8. *The tadpole $T(n, m)$ is a power cordial graph for $n + m \leq 13$ and not a power cordial graph for $n + m > 13$.*

Proof. Let $e_1, e_2, ..., e_n$ be edges, $v_1, v_2, ..., v_n$ be vertices of cycle C_n and e_{n+2}, $e_{n+3}, ..., e_{n+m}$ be edges, $v_{n+1}, v_{n+2}, ..., v_{n+m}$ be vertices of path P_m. Now to construct tadpole $T(n, m)$ join path P_m to the vertex v_n of cycle C_n by an edge e_{n+1}. Then $|V(T(n, m))| = n + m$ and $|E(T(n, m))| = n + m$.

We define the power cordial labeling $f : V(T(n,m)) \rightarrow \{1, 2, \dots, n+m\}$ by following six cases.

Case 1: For $n + m \leq 5$.

$$f(v_i) = i; \quad \text{for } 1 \leq i \leq n+m.$$

In view of above defined labeling pattern, we have $e_f(0) = 2$ and $e_f(1) = 2$ for $n + m = 4$ and $e_f(0) = 3$ and $e_f(1) = 2$ for $n + m = 5$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Case 2: For $n + m = 6, 7$.

$$\begin{aligned} &f(v_i) = i+1; && \text{for } 1 \leq i \leq n-1, \\ &f(v_n) = 1, && \\ &f(v_i) = i; && \text{for } n+1 \leq i \leq n+m. \end{aligned}$$

In view of above defined labeling pattern, we have $e_f(0) = 3$ and $e_f(1) = 3$ for $n + m = 6$ and $e_f(0) = 4$ and $e_f(1) = 3$ for $n + m = 7$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Case 3: For $n + m = 8, 9$.

$$\begin{aligned} &f(v_n) = 1, && \\ &f(v_i) = 2^i; && \text{for } 1 \leq i \leq 2, \\ &f(v_4) = 3, && \\ &f(v_i) = i; && \text{for } 5 \leq i \leq n+m. \end{aligned}$$

In view of above defined labeling pattern, we have $e_f(0) = 4$ and $e_f(1) = 4$ for $n + m = 8$ and $e_f(0) = 5$ and $e_f(1) = 4$ for $n + m = 9$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Case 4: For $n + m = 10, 11$.

$$\begin{aligned} &f(v_1) = 1, && \\ &f(v_2) = 8, && \\ &f(v_{2+i}) = 2^i; && \text{for } 1 \leq i \leq 2. \\ &f(v_{4+i}) = 3^i; && \text{for } 1 \leq i \leq 2, \\ &f(v_i) = i-2; && \text{for } 7 \leq i \leq 9, \\ &f(v_i) = i; && \text{for } 10 \leq i \leq n+m. \end{aligned}$$

In view of above defined labeling pattern, we have $e_f(0) = 5$ and $e_f(1) = 5$ for $n + m = 10$ and $e_f(0) = 6$ and $e_f(1) = 5$ for $n + m = 11$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Case 5: For $n + m = 12, 13$.
Subcase 1: For $n \leq 5$.

$$\begin{aligned} &f(v_n) = 1, && \\ &f(v_{n+1}) = 8, && \\ &f(v_{n+1+i}) = 2^i; && \text{for } 1 \leq i \leq 2, \\ &f(v_{n+3+i}) = 3^i; && \text{for } 1 \leq i \leq 2. \end{aligned}$$

Now label the remaining vertices in such a way that they neither be powers nor roots of the label of adjacent vertices (except when one of the vertex label is 1).

Subcase 2: For $n > 5$.

$$
\begin{aligned}
&f(v_n) = 1,\\
&f(v_1) = 8,\\
&f(v_{1+i}) = 2^i; \quad \text{for } 1 \le i \le 2,\\
&f(v_{3+i}) = 3^i; \quad \text{for } 1 \le i \le 2.
\end{aligned}
$$

Now label the remaining vertices in such a way that they neither be powers nor roots of the label of adjacent vertices (except when one of the vertex label is 1).

In view of above defined labeling pattern, we have $e_f(0) = 6$ and $e_f(1) = 6$ for $n + m = 12$ and $e_f(0) = 7$ and $e_f(1) = 6$ for $n + m = 13$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Case 6: For $n + m \ge 14$.

In order to satisfy the edge condition for power cordial labeling of graph it is essential to label at least $\left\lfloor \frac{n+m}{2} \right\rfloor$ edges with label 0 as well as with label 1 out of total $n + m$ edges. All the possible assignment of vertex labels will give rise at least $\left\lfloor \frac{n+m}{2} \right\rfloor + 1$ edges with label 0 and at most $\left\lfloor \frac{n+m}{2} \right\rfloor - 1$ edges with label 1. Therefore, $|e_f(0) - e_f(1)|$ is at least two. Thus, the tadpole $T(n,m)$ is not a power cordial graph for $n + m \ge 14$.

Hence, the tadpole $T(n,m)$ is a power cordial graph for $n + m \le 13$ and not a power cordial graph for $n + m > 13$. □

Example 2.9. The tadpole $T(6,4)$ and its power cordial labeling is shown in Fig 2.

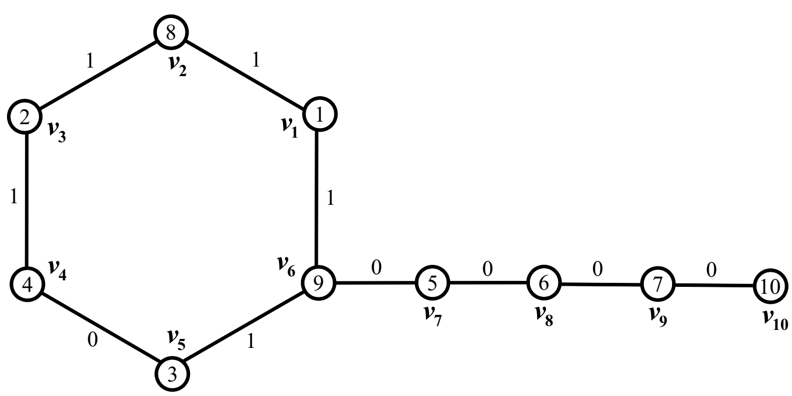

Fig 2: The tadpole $T(6,4)$ and its power cordial labeling.

Definition 2.10. A complete bipartite graph $K_{1,n}$ is called *star*. A vertex corresponding to partition having one vertex is called apex vertex.

Theorem 2.11. *The star $K_{1,n}$ is a power cordial graph for $n \leq 5$ & $n = 7$ and not a power cordial graph for $n = 6$ & $n \geq 8$.*

Proof. Let $K_{1,n}$ be the star with an apex vertex v, pendant vertices $v_1, v_2, v_3, ..., v_n$ and edges $e_1, e_2, e_3, ..., e_n$. Then $|V(K_{1,n})| = n + 1$ and $|E(K_{1,n})| = n$.

We define the power cordial labeling $f : V(K_{1,n}) \rightarrow \{1, 2, \ldots, n + 1\}$ by following two cases.

Case 1: For $n \leq 5$ & $n = 7$.

$$
\begin{aligned}
&f(v) = 2,\\
&f(v_1) = 1,\\
&f(v_i) = i + 1; \quad \text{for } 2 \leq i \leq n.
\end{aligned}
$$

In view of above defined labeling pattern, we have $e_f(0) = 0$ and $e_f(1) = 1$ for $n = 1$, $e_f(0) = 1$ and $e_f(1) = 1$ for $n = 2$, $e_f(0) = 1$ and $e_f(1) = 2$ for $n = 3$, $e_f(0) = 2$ and $e_f(1) = 2$ for $n = 4$, $e_f(0) = 3$ and $e_f(1) = 2$ for $n = 5$ and $e_f(0) = 4$ and $e_f(1) = 3$ for $n = 7$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Case 2: For $n = 6$ & $n \geq 8$.

In order to satisfy the edge condition for power cordial labeling of graph it is essential to label at least $\left\lfloor \frac{n}{2} \right\rfloor$ edges with label 0 as well as with label 1 out of total n edges. All the possible assignment of vertex labels will give rise at least $\left\lfloor \frac{n}{2} \right\rfloor + 1$ edges with label 0 and at most $\left\lfloor \frac{n}{2} \right\rfloor - 1$ edges with label 1. Therefore, $|e_f(0) - e_f(1)|$ is at least two. Thus, the star $K_{1,n}$ is not a power cordial graph for $n = 6$ & $n \geq 8$.

Hence, the star $K_{1,n}$ is a power cordial graph for $n \leq 5$ & $n = 7$ and not a power cordial graph for $n = 6$ & $n \geq 8$. □

Theorem 2.12. *The complete bipartite graph $K_{2,n}$ is a power cordial graph.*

Proof. Let $V = X \cup Y$ be the partition of the vertex set of graph $K_{2,n}$ such that $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2, y_3, ..., y_n\}$. Then $|V(K_{2,n})| = n + 2$ and $|E(K_{2,n})| = 2n$.

We define the power cordial labeling $f : V(K_{2,n}) \rightarrow \{1, 2, \ldots, n+2\}$ as follows:

$$
\begin{aligned}
&f(x_1) = 1, &&\\
&f(x_2) = p; &&\text{where } p \text{ is the largest prime number less than } n + 3,\\
&f(y_i) = i + 1; &&\text{for } 1 \leq i \leq p - 2,\\
&f(y_i) = i + 2; &&\text{for } p - 1 \leq i \leq n.
\end{aligned}
$$

In view of above defined labeling pattern, we have $e_f(0) = n$ and $e_f(1) = n$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| = 0$.

Hence, the complete bipartite graph $K_{2,n}$ is a power cordial graph. □

Example 2.13. The complete bipartite graph $K_{2,7}$ and its power cordial labeling is shown in Fig 3.

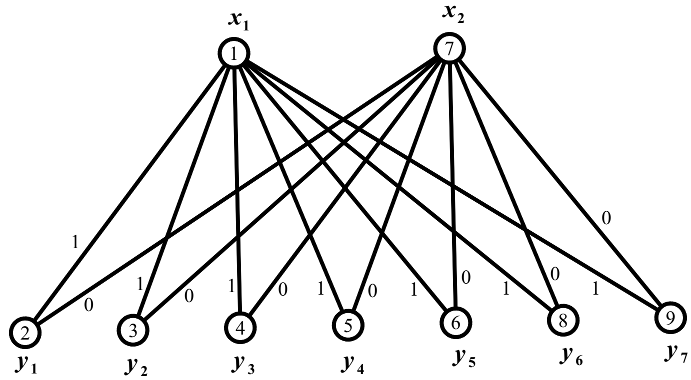

Fig 3: The complete bipartite graph $K_{2,7}$ and its power cordial labeling.

Theorem 2.14. *The complete bipartite graph $K_{3,n}$ is a power cordial graph for $n \leq 7$ & $n \neq 4$ and not a power cordial graph for $n = 4$ & $n \geq 8$.*

Proof. Let $V = X \cup Y$ be the partition of the vertex set of graph $K_{3,n}$ such that $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2, y_3, ..., y_n\}$. Then $|V(K_{3,n})| = n + 3$ and $|E(K_{3,n})| = 3n$.

We define the power cordial labeling $f : V(K_{3,n}) \rightarrow \{1, 2, \ldots, n + 3\}$ by following two cases.

Case 1: For $n \leq 7$ & $n \neq 4$.

$$\begin{aligned} f(x_i) &= i; && \text{for } 1 \leq i \leq 3, \\ f(y_i) &= i + 3; && \text{for } 1 \leq i \leq n. \end{aligned}$$

In view of above defined labeling pattern, we have $e_f(0) = 1$ and $e_f(1) = 2$ for $n = 1$, $e_f(0) = 3$ and $e_f(1) = 3$ for $n = 2$, $e_f(0) = 5$ and $e_f(1) = 4$ for $n = 3$, $e_f(0) = 8$ and $e_f(1) = 7$ for $n = 5$, $e_f(0) = 9$ and $e_f(1) = 9$ for $n = 6$ and $e_f(0) = 11$ and $e_f(1) = 10$ for $n = 7$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Case 2: For $n = 4$ & $n \geq 8$.

In order to satisfy the edge condition for power cordial labeling of graph it is essential to label at least $\left\lfloor \dfrac{3n}{2} \right\rfloor$ edges with label 0 as well as with label 1 out of total $3n$ edges. All the possible assignment of vertex labels will give rise at least $\left\lfloor \dfrac{3n}{2} \right\rfloor + 1$ edges with label 0 and at most $\left\lfloor \dfrac{3n}{2} \right\rfloor - 1$ edges with label 1. Therefore,

$|e_f(0) - e_f(1)|$ is at least two. Thus, the complete bipartite graph $K_{3,n}$ is not a power cordial graph for $n = 4$ & $n \geq 8$.

Hence, the complete bipartite graph $K_{3,n}$ is a power cordial graph for $n \leq 7$ & $n \neq 4$ and not a power cordial graph for $n = 4$ & $n \geq 8$. □

Definition 2.15. The *bistar* $B_{n,n}$ is graph obtained by joining the apex vertices of two copies of stars $K_{1,n}$ by an edge.

Theorem 2.16. *The bistar $B_{n,n}$ is a power cordial graph.*

Proof. Let u be the apex vertex and $u_1, u_2, ..., u_n$ be the other vertices of first $K_{1,n}$ and v be the apex vertex and $v_1, v_2, ..., v_n$ be the other vertices of second $K_{1,n}$ of the Bistar $B_{n,n}$. Then $|V(B_{n,n})| = 2n + 2$ and $|E(B_{n,n})| = 2n + 1$.

We define the power cordial labeling $f : V(B_{n,n}) \rightarrow \{1, 2, \dots, 2n + 2\}$ as follows:

$$\begin{aligned}
&f(u) = 1, \\
&f(u_i) = 2i + 2; \quad \text{for } 1 \leq i \leq n, \\
&f(v) = 2, \\
&f(v_i) = 2i + 1; \quad \text{for } 1 \leq i \leq n.
\end{aligned}$$

In view of above defined labeling pattern, we have $e_f(0) = n$ and $e_f(1) = n+1$. Thus, f satisfies the condition $|e_f(0) - e_f(1)| = 1$.

Hence, the bistar $B_{n,n}$ is a power cordial graph. □

Example 2.17. The bistar $B_{8,8}$ and its power cordial labeling is shown in Fig 4.

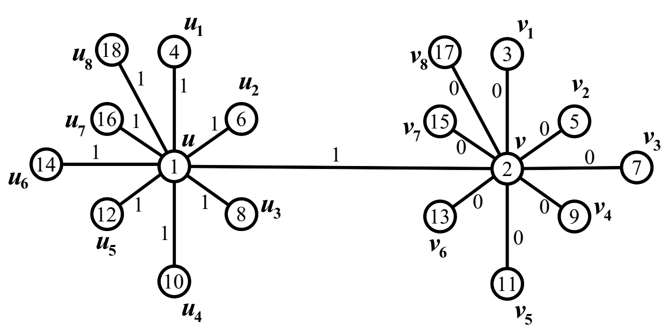

Fig 4: The bistar $B_{8,8}$ and its power cordial labeling.

3. Concluding remarks and Future scope

By the definitions of power cordial graph and divisor cordial graph, we can conclude that every power cordial graph is a divisor cordial graph. But the converse is not true as P_{15} is divisor cordial graph [5] but not a power cordial graph.

To investigate power cordial labeling for new graph families or relate graph theoretical properties with power cordial labeling are open areas of research.

Conflicts of interest : The authors declare no conflict of interest.

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