1. Introduction

WIMSD-5B is a lattice code with a depletion capability for the analysis of reactor physics problems related to a design and safety. It is released from the OECD/NEA Data Bank in 1998 and is now being used widely for thermal research and power reactor calculations. The purpose of this study is to assess and improve the resonance treatment method in WIMSD-5B, through the introduction of a new method with a high accuracy in treating the resonance, as one of the development works for WIMS/CANDU, which is being developed for replacing WIMS-AECL, for the physics analysis of CANDU reactors. In this article, we specifically describe the recent improvements in the resonance integral method using the Carlvik’s approximation. As a result, a comparison for the resonance calculation on the CANDU-6 fuel lattice was performed between the WIMSD-5B code and the WIMS/CANDU code with the 69-energy groups of the ENDF/B-VI nuclear data library and the WIMS-AECL code with the 89-energy group of the ENDF/B-VI nuclear data library.

2. Pin-cell treatment for resonance integral calculation

The lattice code, WIMSD-5B, which was developed from a version of the WIMS code of Winfrith, solves the general multi-group neutron transport equation and treats a variety of geometric configurations. It uses the neutron cross-section data library with a distribution of 69 and 172 neutron energy groups which were completed by the WLUP (WIMS-D Library Update) project.

The resonance treatment in WIMSD-5B uses the intermediate resonance approximation to calculate the group-averaged resonance integrals as a function of the background scattering cross-section. Equivalence relations are used to relate the heterogeneous geometry to an equivalent homogeneous one in reactor lattices. The effective resonance cross-sections are calculated by the interpolation from the resonance integrals in the data library. To arrive at an equivalence relation to the fuel-to-fuel collision probability \( p_{FF} \) in the lattice, \( p_{FF} \) is expressed as a sum of rationals

\[
p_{FF} = \sum \beta_n \cdot \frac{x}{x+\alpha_n} \quad \text{with} \quad \sum \beta_n = 1,
\]

where \( x = 4V/\Sigma_p/\Sigma_f \) and in which case the equivalence relation for the resonance cross integral of the heterogeneous system becomes a sum of the homogeneous integrals

\[
RI = \sum \beta_n RI(\sigma_p + \alpha_n \sigma_e),
\]

where \( \sigma_p \) is the potential scattering cross-section of the fuel, per absorber atom, and \( \sigma_e = S_e/4V \) the escape cross-section, \( N \) being the absorber number density.

Tracking the neutrons born in the fuel, one obtains

\[
p_{FF} = p_{FF} + \frac{x(1-p_{FF})^2}{x(1-p_{FF})+A}, \tag{3}
\]

in terms of the single rod self-collision probability \( p_{FF} \) and a constant \( A \) of probability of a fuel and a cell boundary.

As an approximation method of \( p_{FF} \), WIMSD-5B uses the Wigner’s method to calculate the collision probability in an annular model of fuel, cladding, coolant and moderator. It approximates the single rod self-collision probability, \( p_{FF} \), by a single rational term

\[
p_{FF} = x/(x+a), \quad \text{where} \ a \ \text{is a Bell factor.}
\]

But it is well known that an approximation of \( p_{FF} \) by a single rational is never accurate, as can be seen in Figure 1. In fact, a Bell factor is interpreted as a correction parameter in the equivalence relation, which yields the proper value for the resonance integral. Consequently, instead of an introduction of a Bell factor, one could use an improved approximation for \( p_{FF} = 2x/(x+2) - x/(x+3) \), expressed by a sum of two rationals and proposed by Carlvik(1962). Its accuracy is shown in Figure 1.

\[
\begin{align*}
\text{Error in } p_{FF} \text{ for different approximations} \\
\text{Single rational } a=1.0 \\
\text{Single rational } a=1.16
\end{align*}
\]

Now a substitution of \( p_{FF} = 2x/(x+2) - x/(x+3) \) into the equation (3) gives

\[
p_{FF} = x(\beta(x+\alpha_j)+(1-\beta)/(x+\alpha_j)), \tag{4}
\]

with

\[
\alpha_{2,1} = \{(5A+6)z\sqrt{A^2+36A+36}/2(A+1),
\]

\[
\beta = \{(4A+6)/(A+1)- \alpha_j/(\alpha_j-\alpha_j).
\]

Also, it leads to the equation of a resonance integral with the parameters \( \alpha_1, \alpha_2, \text{and} \beta \), that is,

\[
RI = R(\sigma_p+\alpha, \sigma_e)+(1-\beta)RI(\sigma_p+\alpha_2, \sigma_e) \tag{7}
\]

Instead of the Wigner’s approximation, one applies and implements an improved method proposed by Carlvik with WIMSD-5B, in order to calculate the...
collision probability in an annular model of fuel, cladding and coolant, and, if relevant, moderator.

3. Cluster resonance treatment in WIMS/CANDU

When the geometry of a cluster-type lattice is described, as in the CANDU reactors, this type of geometry should be treated with the resonance shielding. The cluster model used in WIMS/CANDU is as follows:
- Composed of $N_{fuel}$ identical cylindrical pins
- Divided into an outer and inner zone
- Infinite lattice
- Moderator region outside of the fuel cluster.

In WIMS/CANDU, the resonance integral calculation is carried out by dividing the fuel cluster into two fuel zones: inner zone and outer zone. The resonance integral in the inner zone is calculated as if there were an infinite lattice of pins and that in the outer zone is calculated to preserve the cluster-averaged resonance effects. Then the cluster resonance is treated by defining the fuel-to-fuel collision probability for the fuel in the inner part and the outer part of a cluster respectively, in order to account for the heterogeneity of a lattice in WIMS/CANDU. The comparison of the resonance integrals was performed between WIMSD-5B and WIMS/CANDU using the same methodology in the resonance treatment. The results on the typical CANDU-6 fuel lattice condition: burn-up = 0, coolant temperature = 560 K, and moderator temperature = 340 K, are given in the Table 1.

Table 1. Resonance integrals in WIMSD-5B, WIMS/CANDU and WIMS-AECL (energy range: 5.530×10^3 eV–9.118×10^3 eV)

<table>
<thead>
<tr>
<th>Isope</th>
<th>Type</th>
<th>Zone</th>
<th>WIMSD-5B</th>
<th>WIMS/ CANDU</th>
<th>WIMS-AECL</th>
</tr>
</thead>
<tbody>
<tr>
<td>U^{234} Abs.</td>
<td>Inner</td>
<td>1.11658</td>
<td>1.11671</td>
<td>1.33427</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Outer</td>
<td>1.11664</td>
<td>1.11663</td>
<td>1.34041</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cluster</td>
<td>1.11669</td>
<td>1.11667</td>
<td>1.33753</td>
<td></td>
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<tr>
<td>U^{235} Abs.</td>
<td>Inner</td>
<td>4.53606</td>
<td>4.52861</td>
<td>4.44180</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Outer</td>
<td>4.53601</td>
<td>4.53604</td>
<td>4.46192</td>
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<tr>
<td></td>
<td>Cluster</td>
<td>4.53595</td>
<td>4.44690</td>
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<tr>
<td>U^{238} Abs.</td>
<td>Inner</td>
<td>0.64972</td>
<td>0.63098</td>
<td>0.61027</td>
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<tr>
<td></td>
<td>Outer</td>
<td>0.66113</td>
<td>0.18069</td>
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<tr>
<td></td>
<td>Cluster</td>
<td>0.67317</td>
<td>0.41193</td>
<td>0.62351</td>
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</tr>
</tbody>
</table>

4. Conclusion

The resonance treatment in WIMS/CANDU is improved based on the treatment developed by an equivalence relation in this study. And the difference of the resonance calculations in between WIMS/CANDU and WIMSD-5B is the calculation of a collision probability in an annular model: WIMSD-5B resonance treatment uses the Wigner’s method, while WIMS/CANDU uses the Carvick’s method. The accuracy of WIMS/CANDU with the Carvick’s method is better than the others. In addition, the resonance integral calculation was performed and compared between the WIMS/CANDU code, implemented with the new method of high accuracy, and the WIMSD-AECL code. In the future, the resonance treatment in WIMSD-5B and WIMS/CANDU will be assessed against the WIMSD-AECL and experimental data.

Acknowledgement

The authors would like to express their appreciation to the Ministry of Science Technology (MOST) of the Republic of Korea for support of their work through the mid- and long-term nuclear R&D Project.

References