Effects of Earth’s Atmosphere on Terrestrial Reference Frame: A Review

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Abstract: Displacement of the Earth’s surface due to atmospheric loading has been recognized since a century years ago, and its accurate estimation is required in present day geodesy and surveying, particularly in space geodesy. Atmospheric load deformation in continental region can readily be calculated with the given atmospheric pressure field and the load Green’s function, and, in near coastal area, approximate model is used for the calculation. The changes in the Earth’s atmospheric circulation and the seasonal variation of atmospheric pressure on two hemispheres of the Earth are the each main causes of variation of the Earth’s spin angular velocity and polar motion respectively. Wind and atmospheric pressure do the major role in other periodic and non-periodic perturbations of the positions in the Earth’s reference frame and variations in the Earth’s spin rotational state. In this reviewing study, the developments of related theories and models are summarized along with brief description of phenomena, and the geodetic perturbing effects of a hypothetical typhoon passing Korea are shown as an example. Finally related existing problems and further necessary studies are discussed in general.

Keywords: Terrestrial reference frame, atmosphere, crustal displacement, earth rotation variation

Introduction

The atmosphere is composed mostly of nitrogen and oxygen and can be regarded as outer skin of the Earth. Compared with the whole Earth’s mass, that of the Earth’s atmosphere is quite small and only about one part of million - which is smaller than oceanic mass by a factor of 260. Nevertheless the atmosphere not only affects human civilization through its dynamic role in the weather and climate over the globe but also mechanically excites several kinds of perturbations in the Earth’s shape and its rotational state. Space geodesy nowadays is directly involved to assess the atmospherically driven deformation and rotational changes of the Earth. European Centre for Medium-Range Weather Forecasts (ECMWF; http://www.ecmwf.int/) and NCEP (National Centers for Environmental Prediction; http://www.ncep.noaa.gov/) are the two foremost organizations, which support various datasets of the...
Earth’s atmosphere. Salstein addressed how the atmospheric data are acquired and processed (Salstein, 2004). A sample plot of atmospheric pressure distribution from ECMWF and another plots based on $S_1$ and $S_2$ atmospheric tide models (Ray and Ponte, 2003) are drawn in Fig. 1.

G. H. Darwin, a pioneer of geophysics, is known to have speculated about the deformation of the Earth’s crust due to variations of atmospheric pressure on its surface. Investigators attempted to solve the problem of loading on the Earth, and made theoretical developments (for example, Love (1911), Longman (1962)). Two key concepts - i) Love number and ii) Green’s function are involved here to represent the response of Earth for given loading - either i) spherical harmonic distribution load or ii) point load. Farrell (1972) acquired very accurate load Love numbers and load Green’s function and also gave a good summary of theory developments. Although the atmosphere driven crustal deformation is only a few millimeters to centimeters, the recent fast advances in the space geodetic techniques necessitate accurate modeling of these phenomena. Two well-known periodicities of atmospheric pressure loading are the seasonal and daily cycles. van Dam and others have continually reported about the atmospheric

![atmospheric pressure (ECMWF) 2015. 1. 1. 0h UT](image1)

**Fig. 1.** Sample plots of the Earth’s atmosphere: (top) atmospheric pressure raw data from ECMWF on a particular date [unit: hPa], (left) Five diurnal $S_1$ atmospheric tidal pressure distributions of equal time interval of 0.2 day [unit: Pa], (right) Three semi-diurnal $S_2$ atmospheric tidal pressure distributions of equal time interval of 8 hours [unit: Pa].
loading effects on space geodetic measurements (for example, van Dam et al., 1985, 1994a-b, 2005, 2010, Petrov and Boy, 2004). Kim and Park (2009) detected annual signal of atmospheric loading in GPS height time series of Korea (a few millimeters) and attained enhancement using their model. To the best knowledge of the authors, theirs is the only previous study of this sort reported in Korea.

It is also remarkable that human beings noticed the seasonal variation of length of day (l.o.d.) as early as 1930s (Munk and MacDonald, 1960). The l.o.d. in July was found slightly shorter than that in January by two milliseconds. Later in 1950s, this difference inferred by using pendulum clocks has been corrected to be smaller after the advent of more accurate quartz clock. And also the semi-annual variation of l.o.d. has been detected as well. Thenceforth perturbation in the Earth’s spin rotation became quite interesting scientific object, and Earth’s spin rotation is no longer regarded as constant time keeper. Soon they suspected Earth’s atmosphere being the main cause of these periodic variations of l.o.d. and polar motion. However reliable confirmation of its effect on the seasonal variation of the Earth’s spin rotation came out only after several decades, when calculation of the angular momentum associated with the Earth’s atmospheric circulation become feasible by using the accumulated data. In the meantime, space geodetic observations, particularly by VLBI, enhanced measurement of Earth’s spin rotational state by more than two orders of magnitude. In fact, the observation of Earth’s rotation by combined space geodetic techniques; VLBI, GNSS, SLR, and DORIS resulted in thorough knowledge about precession, nutation, l.o.d. variation, and polar motion (Gross, 2009, Dehant and Mathews, 2009, and Petit and Luzum, 2010 and references therein). Like early conjectures, the atmosphere is found responsible for most of the perturbations of Earth’s spin rotation on the wide frequency range from sub-diurnal fluctuation to El Nino Southern Oscillation (ENSO).

The objective of this short reviewing study is to help understanding of the theory and existing models of the two kinds of effects of the Earth’s atmosphere; i) atmospheric loading on the Earth and ii) atmospheric excitation on the Earth’s spin rotation. As an example, a hypothetical typhoon passing the Korean peninsula is considered.

### Atmospheric Loading on the Earth’s Surface

Load deformation can be evaluated either by using load Love numbers or load Green’s function, as described below in this section. Green’s function is one of key concepts in applied mathematics, and it is frequently used in solving partial differential equations of science and technology. It may be regarded as ‘impulse response’ of a given system. Longman’s development of set of six first-order differential equations, which were converted from the equation of motion was an important step in attaining the load Green’s function, and he calculated Love numbers up to degree of a few dozen (Longman, 1962). It was Farrell, who first successfully evaluated the load Green’s function with desired accuracy using Gutenberg-Bullen Earth model (Farrell, 1972). As Farrell summarized in his article, the static deformation of flat Earth under surface loading has been well known pursued called ‘Boussinesq problem.’ Guo et al. (2004) computed the load Green’s function by using the PREM (Preliminary Earth Model) with some extended mathematical expressions, and Na & Baek (2011) did it by using the IASPEI Earth model and compared with former results. Two other quite elaborate works related are; i) inelastic load Green’s function by Pagiatakis (1990), and ii) more realistic boundary condition in atmospheric loading treated by Guo et al. (2004). However, due to the dominance of Earth’s elastic response and small thickness of the Earth’s atmospheric layer, those two advanced modifications are not adopted often. Following is a brief theory of loading on the spherically symmetric and elastic layered Earth.

When a load of spherical harmonic distribution \( W(\theta, \phi) = \sum_{n=0}^{n_{max}} (\cos \theta) e^{in\phi} \) is applied on the Earth, the resultant deformation \( \ddot{u} = \ddot{u}_r + \ddot{u}_r \) and the gravity potential \( V(\theta, \phi) \) associated with the deformation can be expressed as follows.

\[
\begin{align*}
\dot{u}_r &= \frac{h'}{g} W, \\
\dot{u}_\theta &= \frac{l'}{g} \frac{\partial W}{\partial \theta}, \\
\dot{u}_\phi &= \frac{l'}{g} \sin \theta \frac{\partial W}{\partial \phi}, \\
V &= k' W 
\end{align*}
\]

(1)

where \( h', k', l' \) are the three load Love numbers, and \( g \) is the Earth’s gravity. One can use Eq. (1) to evaluate the load deformation only when the loading is of spherical harmonic distribution, and so its application is quite limited. More often Earth’s deformation due to surface load is evaluated by convolving the load distribution \( \sigma(\theta', \phi') \) with the load Green’s function as follows.

\[
\begin{align*}
u_r(\theta, \phi) &= \int \sigma(\theta', \phi') G_r(\psi) a^2 \sin \theta' d\theta' d\phi' \\
u_\theta(\theta, \phi) &= \int \sigma(\theta', \phi') G_\theta(\psi) a^2 \sin \alpha \sin \theta' d\theta' d\phi' \\
u_\phi(\theta, \phi) &= \int \sigma(\theta', \phi') G_\phi(\psi) a^2 \cos \alpha \sin \theta' d\theta' d\phi'
\end{align*}
\]

(2)

where \( G_r \) and \( G_\phi \) are vertical and horizontal load Green’s
functions, while the angles $\alpha$ and $\psi$ are i) the bearing of loading point $(\theta', \phi')$ with respect to the field point $(\theta, \phi)$ and ii) the geocentric angle between the two points. In Fig. 2, the two Green’s functions $G_u$ and $G_v$, acquired for the IASPEI Earth model are illustrated. In case of atmospheric loading, the deviational pressure $\Delta P$ (in hPa) can be changed into surface load mass density $\sigma = 10.2 \Delta P$ (kg/m$^2$).

After the pioneering work of Farrell (1972), calculations of the atmospheric/ocean loading have been extensively carried. And they soon realized that several millimeters up to a few centimeter occur as displacement due to atmospheric pressure; 10 mm displacement at the center of circular area of 2000 km diameter loaded by 20 hPa and similarly 6 mm displacement for center of circular area of 1000 km diameter loaded by 20 hPa (van Dam and Wahr, 1987). If all the loading points are on in-land area, Eq. (2) suffices to represent the load deformation anywhere on the Earth. However, at near coastal area, different scheme is necessary, because the ocean water would be redistributed when surface atmospheric pressure changes. In fact, there is no complete solution at hand for this phenomenon. Wunsch and Stammer (1997) addressed this problem with certain description of possible resonant behavior of oceans. Usually the ‘inverted barometer assumption’ is taken as approximate solution, although its validity is uncertain when atmospheric disturbance varies with period less than a few days. According to the ‘inverted barometer assumption’, oceanic water subjected to an atmospheric pressure deviation $\Delta P$ would undergo such redistribution that the whole ocean bottom would exert the same average pressure $\Delta P_0$.

$$\Delta P_0 = \frac{\iiint_{\text{ocean}} \Delta P(\theta, \phi) \sin \theta d\theta d\phi}{\iiint_{\text{ocean}} \sin \theta d\theta d\phi}$$

As repeatedly pointed out (for example, Ponte and Ray, 2002; Petrov and Boy, 2004; Bock et al., 2005; Tregonning and van Dam, 2005), there still has not been found any practical method to accurately evaluate the oceanic response to atmospheric loading rather than ‘inverted barometer ocean.’ van Dam and Wahr (1987) early noted the importance of data coverage - at least 1000 km radius is necessary for reliable atmospheric loading estimation. They also noted an approximate relation between local atmospheric pressure and gravity as 0.42 $\mu$Gal/hPa. Petrov and Boy (2004) confirmed validity of the atmospheric loading by using the ‘admittance factor’, which is a ratio between observed and calculated atmospheric loading deformation. Tregonning and van Dam (2005) found that the ‘inverted barometer assumption’ cannot be applied for variation within two day time span. And they also found that the assumption is poor in areas either of i) high latitude, ii) shallow sea, or iii) the tropic zone.

Due to Earth rotation, solar irradiance upon its surface results in 24 hour-period variation on the Earth’s atmospheric pressure. IERS adopted the model of Ray and Ponte (2003) for diurnal and semidiurnal atmospheric pressure induced displacement on the globe (IERS, 2010). Both components are large (up to 1.5 mm) along tropical regions. Cold and heavy atmosphere of winter time exerts annual periodic loading on

\[Fig. 2. \text{Loading and the Load Green’s function: (left) Largely exaggerated schematic illustration for the deformed Earth under point loading. (right) Two Green’s functions } G_u \text{ and } G_v \text{ - which are the vertical and horizontal components of the Earth’s deformation due to a point load of 1 kg. The Green’s functions become quite smaller in magnitude with angular distance increase (Na and Baek, 2011).}\]
the continents, and the displacement is usually larger at higher latitude region and often exceeds 1 cm.

We calculated the deformation of the Korean peninsula induced by a hypothetical typhoon passing through it. The assumed path is drawn with five particular locations on its way (Fig. 3a). The central pressure and radius of the typhoon were taken constant as 950 hPa and 150 km respectively at all the five locations. The simulated deformation due to the low atmospheric pressure of the hypothetical typhoon placed at the each location was acquired repeatedly and illustrated (Fig. 3b-

![Fig. 3. Demonstration of the atmospheric loading induced deformation due to a hypothetical typhoon: (a) Path of the hypothetical typhoon passing Korea, (b-f) Calculated vertical and horizontal displacement induced by the hypothetical typhoon placed at 5 each locations marked in (a). The contour lines correspond to equal uplift (contour interval: 0.5 mm in (b) and 1 mm in (c-f)), and the pink arrows represent horizontal displacements at grid points.]
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f). Maximum uplift in the peninsula slightly exceeds 6 mm, when the main parts of the typhoon lie on the peninsula. We made a table of maximum uplift due to various size typhoon in continental area (Table 1). As can be seen from the Fig. 3b-f, the horizontal displacements are directed to the outer regions from the center, and their magnitudes are far less than vertical displacements. As the typhoon goes offshore region, the deformation is much reduced. The grid spacing used in our calculation is 0.25 degree in both the latitude and longitude.

### Table 1. Maximum uplift in continental area due to hypothetical typhoon of different size [unit: mm]

<table>
<thead>
<tr>
<th>$\Delta P_0$</th>
<th>$r_0$ = 100 km</th>
<th>$r_0$ = 150 km</th>
<th>$r_0$ = 200 km</th>
<th>$r_0$ = 250 km</th>
<th>$r_0$ = 300 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>−70 hPa</td>
<td>5.74</td>
<td>7.81</td>
<td>9.50</td>
<td>10.89</td>
<td>12.07</td>
</tr>
<tr>
<td>−60 hPa</td>
<td>4.92</td>
<td>6.86</td>
<td>8.14</td>
<td>9.34</td>
<td>10.35</td>
</tr>
<tr>
<td>−50 hPa</td>
<td>4.10</td>
<td>5.58</td>
<td>6.78</td>
<td>7.78</td>
<td>8.62</td>
</tr>
<tr>
<td>−40 hPa</td>
<td>3.28</td>
<td>4.46</td>
<td>5.43</td>
<td>6.23</td>
<td>6.90</td>
</tr>
</tbody>
</table>

§ The typhoon pressure anomaly distribution was assumed as $\Delta P = \Delta P_0 \exp\left(-\frac{r^2}{r_0^2}\right)$.

excitation has been first documented by Munk and MacDonald (1960) and again by Lambeck (1980). However, significant developments of both theory and observation have followed since then, and some of them are cited here. Two excellent summaries of such development in observation up to early 90s were reported by Rosen (1993) and also by Hide & Dickey (1991). Gross and Lindqwister (1992) found atmospheric excitation of Earth's polar motion. Gross et al. (1996) found that El Nino (ENSO) is related with l.o.d. perturbation. Further investigations about atmospheric excitation on polar motion were carried (Nastula and Salstein, 1999; Brzezinski et al., 2002; Nastula and Kolaczek, 2002). Chao (2003) reported latter developments attained by modern space geodetic analysis. Gross (2009) gave a wide and excellent review on both the theory and observations of the Earth rotational perturbation including atmospheric excitation. Dehant (2009) briefly addressed the atmospheric effect on the Earth’s nutation and others. Recently another excellent articles were reported (Schindelegger et al., 2013 and 2014). Schindelegger employed his hybrid torque estimation approach (this direct calculation of the torque exerted on the Earth’s surface by the winds was first done by de Viron et al. (1999)) rather than considering atmospheric angular momentum only, and found better match in his model for short period variation. Schindelegger gave detailed descriptions of his own formulation and related ones

**Atmospheric Excitation of the Earth’s Rotation**

The formulation on rotational response of the Earth to given
then, the total angular momentum is written as follows.

\[ L = m_1 \Delta I_{11} + m_2 \Delta I_{22} + m_3 \Delta I_{33} \]

\[ \frac{dL}{dt} + \frac{b}{h} \times L = \tau \]  \hspace{1cm} (4a)

Denote the Earth’s inertia tensor and its variable angular velocity as,

\[
\begin{bmatrix}
    I_{11} & I_{12} & I_{13} \\
    I_{21} & I_{22} & I_{23} \\
    I_{31} & I_{32} & I_{33}
\end{bmatrix}
\begin{bmatrix}
    \omega_1 \\
    \omega_2 \\
    \omega_3
\end{bmatrix}
= \begin{bmatrix}
    0 & 0 & 0 \\
    0 & 0 & 0 \\
    0 & 0 & 0
\end{bmatrix}
+ \begin{bmatrix}
    \Delta I_{11} & \Delta I_{12} & \Delta I_{13} \\
    \Delta I_{21} & \Delta I_{22} & \Delta I_{23} \\
    \Delta I_{31} & \Delta I_{32} & \Delta I_{33}
\end{bmatrix}
\begin{bmatrix}
    \omega_1 \\
    \omega_2 \\
    \omega_3
\end{bmatrix}
\]

\[
\begin{bmatrix}
    \omega_1 \\
    \omega_2 \\
    \omega_3
\end{bmatrix}
= \omega_0
\begin{bmatrix}
    1 \\
    1 \\
    1 + m_1 + m_2 + m_3
\end{bmatrix}
\]

then, the total angular momentum is written as follows.

\[
\begin{bmatrix}
    L_1 \\
    L_2 \\
    L_3
\end{bmatrix}
= \begin{bmatrix}
    A + \Delta I_{11} & \Delta I_{12} & \Delta I_{13} \\
    \Delta I_{21} & A + \Delta I_{22} & \Delta I_{23} \\
    \Delta I_{31} & \Delta I_{32} & C + \Delta I_{33}
\end{bmatrix}
\begin{bmatrix}
    \omega_0 \\
    \omega_0 \\
    \omega_0
\end{bmatrix}
+ \begin{bmatrix}
    h_1 \\
    h_2 \\
    h_3
\end{bmatrix}
\]

With rigid Earth assumption, we eventually find next three relations by inserting \( \dot{L} \) of Eq. (4b) into Eq. (4a) as the followings.

\[
\frac{1}{\Omega} \frac{dm_2}{dt} - m_3 = -\phi - \frac{1}{h_0} \frac{d\phi}{dt}, \quad \frac{1}{\Omega} \frac{dm_1}{dt} + m_2 = \phi - \frac{1}{h_0} \frac{d\phi}{dt}
\]

\[
m_3 = -\frac{h_0}{C} \frac{\Delta I_{33}}{C}
\]

where the excitation function and the Chandler wobble frequency are defined as

\[
\phi = \frac{h_0}{(C - A) h_0} + \frac{0.684 \Delta I_{33}}{C - A} (i = 1, 2),
\]

\[
m_3 = \frac{0.997 h_0}{C} \frac{0.748 \Delta I_{33}}{C}
\]

And the Chandler wobble frequency is also modified as

\[
\Omega = \frac{\omega_0}{433} \left( 1 + \frac{i}{2 \Omega} \right), \quad \text{while the two differential equations between } m_1 \text{ and } \phi (i = 1, 2) \text{ remain unchanged (first two of Eq. (4c)).}
\]

The value of Chandler wobble quality factor \( Q \) has been controversial and its former estimates are in the range of 50 ~ 200. The evident relation of \( m_1 \) with l.o.d. or UT1 is stated as follows.

\[
\frac{d}{dt}(\text{UT1} - \text{UTC}) = -\frac{\Delta \text{LOD}}{\text{LOD}} = m_3
\]  \hspace{1cm} (4f)

A part of \( \Delta \text{l.o.d.} \) time series and its Fourier amplitude spectrum are given in Fig. 5. Seasonal variation is prominent and easily recognized both in time and frequency domains, and other peaks of monthly and fortnightly are of solid/ocean tidal origin. Also a polar motion time series is illustrated with its power spectrum in Fig. 6. Major components in the spectra are labeled. While Chandler wobble is rather randomly excited by ocean bottom pressure of wide frequency range, seasonal
atmospheric pressure on certain regions are responsible for the annual wobble (Gross, 2009 and references therein).

We calculated the effect of typhoon on the Earth’s polar motion and l.o.d. by using the above formulation (Eqs. (4c), (e-f)). Assuming the magnitude of angular momentum associated with the hypothetical typhoon as in the former section (center pressure: 950 hPa, radius: 150 km) to be $h = 1.6 \times 10^{21}$ kgm$^2$s$^{-1}$, we found the pole shift would be $(\Delta x_p, \Delta y_p) \cong (-13.8, -17.5) \mu$as at the time of typhoon formation. This northwest pole displacement of 0.64 mm on the Earth’s surface should be almost completely restored into former position when the typhoon disappears, unless the typhoon maintains its strength over long distance travel. Corresponding change in l.o.d. due to this typhoon would be $\Delta l.o.d. = +0.014 \mu$s, which is definitely too small to be isolated and also should be almost restored with typhoon weakening.

**Discussion and Conclusion**

Due to the changes in the atmospheric pressure exerting on its surface, the Earth undergoes ceaseless deformation of a few millimeters to centimeter in most continental regions. Daily and yearly periodic variations were confirmed and modeled.

Effect of atmospheric non-tidal variation can be evaluated using convolution between the given pressure distribution and load Green’s function. A concise summary of the theory of loading was given in earlier section. ‘Inverted barometer assumption’ may be used without further consideration for certain atmospheric pressure variations of which time scale is longer than a few days, however, correct estimation for it needs full ocean dynamics with bathymetry information. It was found that a typhoon would result in several millimeters uplift in the Korean peninsula and so in other regions as well. We used the load Green’s function calculated for the IASPEI Earth model (Na and Baek, 2011), because the physical properties of the Earth’s crust and upper mantle around Korea
is closer to that one. Unlike the Yellow Sea and the South China Sea (called ‘South Sea’ in Korea), the Korea-Japan Sea (called ‘East Sea’ in Korea) has narrow channels to outer open oceans, therefore, the ‘inverted barometer’ concept need to be used differently with treating the Korea-Japan Sea as a large lake. According to van Dam et al. (2010), a few millimeter errors were found during the atmospheric loading calculation in area of highly variable topography. Further investigations are needed on such points.

Brief yet updated formulation on the Earth’s spin rotational perturbation is re-stated in the former section. We found that a typhoon of typical size cannot affect more than a small temporary displacement (less than one millimeter) of the pole position and tiny temporary change of l.o.d. (in order of 0.01 µs). Except the three mechanisms; i) secular deceleration by tidal friction, ii) decadal fluctuation associated with the Earth's core, iii) slow and constant change induced by post glacial rebound, the Earth’s atmosphere has been identified as main cause of all other perturbations in the Earth’s spin rotation (Gross, 2009 and Lambeck, 1980). Atmospheric angular momentum estimated through international collaboration is found successful to explain most of those perturbations in the Earth rotation except variations of period less than a few days. Torque approach has been employed to find better match between the model and dataset, and the reconciliation is somewhat plausible but not complete within reasonable accuracy (Shindelegger, 2013 and 2014). As the datasets of the atmosphere become of higher accuracy, such incompleteness will eventually decrease with further clarification. However, at present, there are limits to data acquisition/storage and time resolution on the atmospheric state information.

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