Disorientation angle influence on the thermal residual stresses in carbon multilayered composite plates of different thicknesses

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One of the most acute problems in studying composite stress-strain states is determining the initial residual stresses during the design of layered construction elements. The full stress–strain state of composite constructions consists of the initial (generated during the manufacturing process) and operating (generated during the working period) stages. A thorough investigation of the initial stage of the stress-strain state is very important to finding the total stresses and deformations, and determining the correct destruction time of composite structures, and preventing their destruction. Otherwise, if the initial stresses and deformations are not taken into account, the structures may be destroyed long before their expected service life.

Structural factors and conditions, such as the thickness, layup and technological errors, need to be analyzed to solve the initial stress-strain state relevant problem. Among them, one of the technological errors is the disorientation angle, which is expressed as a deviation of the composite layers from the initially given laying directory. Of particular interest is the influence of design factors on the face composite layers and the layers near them since those layers are strongly influenced by different loads and may be destroyed prematurely.

The disorientation angle of the face layers was investigated from this perspective. This paper takes into account the disorientation angle, structure types, and thickness, and using this information, we aim to provide more information about predicting composite destruction, by finding the residual thermal stress values for the whole composite construction.


By reviewing these previous studies, it becomes clear that nearly all of them focus primarily on relationships between the thermal stresses and the constituent/structure of the composites, the elastoplasticity and so on; however, it also becomes evident that the influence of structural factors on the stress-strain state of composites has not been not well studied. Given the lack of research in that area, one of our aims is to address this gap in knowledge. This paper focuses on the dependencies between the thermal residual stress and the disori-
Disorientation angle influence on the thermal residual stresses in composites

A calculation model was selected for analysis: a composite panel (500 mm × 500 mm) [7, 8] was manufactured using a curing temperature of 170°C from carbon composite KMU-4L, which is based on the LU-P carbon fiber and ENFB epoxy matrix. After curing, the panel was cooled at room-temperature to 23°C. The properties of the composite materials and the characteristics of the manufacturing process were used as they are given in the reference [7]. The longitudinal and transverse panel edges of the plate were hinge-supported. Hinge-support is a common type of bearing employed between two or more details, in locations where rotation is necessary (e.g., overlap fixing, various components of construction in space, radio telescopes, aircraft, helicopters, etc.).

In deriving the mathematical model, Kirchhoff’s hypothesis applies to the entire body of an anisotropic medium for the interconnected hogging and bending of the plane plate. Therefore, the proposed model [7] was divided into two parts as follows. The first part addressed the main characteristics of each layer of the composite (i.e., the stiffness, thermal expansion, and deformation coefficients). The second part of the model reduced the individual results from each layer to one panel using a geometric transformation. For more details on the analytical model, refer to [7,9]. In this mathematical model, the components of the displacement vector eq 1 are given by [7,9]:

\[ U = U_0(x,y) \frac{\partial W}{\partial x} z_i^* V = V_0(x,y) \frac{\partial W}{\partial y} z_i^* W = W(x,y) \]  

Components of the stress state of the \( K \)th layer in the whole panel like eq 2 were obtained by applying Cauchy equations, geometric transformation formulas, Hooke’s law with thermal effect, fibers tension, and transformation equations of stresses with rotated axes:

\[
\begin{align*}
\sigma\left(\begin{array}{l}
\sigma_x^{(K)} \\
\sigma_y^{(K)} \\
\tau_{xy}^{(K)} \\
\end{array}\right) &= \left(\begin{array}{ccc}
\tilde{Q}_{11} & \tilde{Q}_{12} & \tilde{Q}_{16} \\
\tilde{Q}_{21} & \tilde{Q}_{22} & \tilde{Q}_{26} \\
\tilde{Q}_{36} & \tilde{Q}_{66} & \tilde{Q}_{66} \\
\end{array}\right) \left(\begin{array}{l}
\sigma_x^{(K)} \\
\sigma_y^{(K)} \\
\tau_{xy}^{(K)} \\
\end{array}\right) - \frac{1}{2} \left(\begin{array}{l}
\epsilon_x^{(K)} + K_{xx}^{(K)} \epsilon_x^{(K)} - \alpha_x^{(K)} \Delta T - \epsilon_{H1}^{(K)} \\
\epsilon_y^{(K)} + K_{yy}^{(K)} \epsilon_y^{(K)} - \alpha_y^{(K)} \Delta T - \epsilon_{H2}^{(K)} \\
\gamma_{xy}^{(K)} + K_{xy}^{(K)} \gamma_{xy}^{(K)} - \alpha_{xy}^{(K)} \Delta T - \epsilon_{H3}^{(K)} \\
\end{array}\right)
\end{align*}
\]

Similar to eq 2, the transformations for the coefficients of thermal expansion are given by:

\[
\left(\begin{array}{c}
\alpha_{11}^{(K)} \\
\alpha_{22}^{(K)} \\
\alpha_{66}^{(K)} \\
\end{array}\right) = \left(\begin{array}{ccc}
m^2 & n^2 & m^2 \\
n^2 & m^2 & m^2 \\
2mn & -2mn & mn \\
\end{array}\right) \left(\begin{array}{c}
\alpha_{11}^{(K)} \\
\alpha_{22}^{(K)} \\
\alpha_{66}^{(K)} \\
\end{array}\right)
\]

The transformations for the deformation of layers tension are given by:

\[
\begin{align*}
\epsilon_x^{(K)} &= \frac{\sigma_x^{(K)}}{E_{xx}^{(K)}} \\
\epsilon_y^{(K)} &= \frac{\sigma_y^{(K)}}{E_{yy}^{(K)}} \\
\gamma_{xy}^{(K)} &= \frac{\tau_{xy}^{(K)}}{G_{xy}^{(K)}}
\end{align*}
\]

Eqs. 1 and 2 are given by [7,9]:

\[ \left(\begin{array}{c}
\epsilon_x^{(K)} \\
\epsilon_y^{(K)} \\
\gamma_{xy}^{(K)} \\
\epsilon_{H1}^{(K)} \\
\epsilon_{H2}^{(K)} \\
\epsilon_{H3}^{(K)} \\
\end{array}\right) = \left(\begin{array}{c}
m^2 \\
n^2 \\
2mn \\
\alpha_x^{(K)} \\
\alpha_y^{(K)} \\
\alpha_{xy}^{(K)} \\
\end{array}\right)
\]

The sequence of the calculation, which takes thermal stresses into account, was given in the paper [7]. The developed analytical method was verified with an experimental study [7], which included the manufacturing of composite plates under the curing process, the subsequent cooling, and then measuring the plate deformations that were obtained in the manufacturing process due to the distinction between the matrix and fiber properties at different manufacturing temperatures (curing state → cooled state). The parameters for the curing, manufacturing, and measuring processes are shown and fully elaborated on in the reference [7]. A verification of the theoretical and experimental results of the deformation (plate hogging) shows that the results match well with one another in the limits from −7.4% to 15.7% with 0% difference in several cases [7]. Thus, this confirms that the developed analytical method can be used for analyzing the stress-strain state of composite structures.

The most basic fabrication method for thermoset composites is the hand layup, which typically consists of laying dry fabric layers, or prepreg plies, by hand onto a tool to form a laminate stack [10]. Because this method can be easily impacted by human error, several faults can occur. One of the more common mistakes incurred by this method is the disorientation angle (\( Q \)). Although this is one of the more common mishaps associated with the hand layup method, the relationship between the disorientation angle and residual thermal stresses, which can cause the premature destruction of composite elements and constructions, has not been sufficiently studied.

Consider this issue, the most popular composite structures in the manufacturing process were used in this study: we analyzed the basic composite structures \( [0/90/0/90/0/90/0] \) and \( [0/45/-45/90/-45/0/0/0] \) with plate thicknesses \( (H) \) of 0.91 mm, 1.82 mm, and 2.73 mm. The disorientation angles were varied from 1 to 7 degrees for the layup \( [0/90/0/90/0/90/0] \) in the two layers closest to the face layers. Correspondences between the angle of disorientation and the basic composite layup designs \( [0/90/0/90/0/90/0] \) and \( [0/45/-45/90/-45/0/0/0] \) are shown in Table 1. All of the studied cases are shown in Table 2.

Table 1. All of the studied cases are shown in Table 2.
The research results for macro-stresses (shear stresses $\tau_{xy}$ and normal stresses $\sigma_x$, $\sigma_y$) depending on the disorientation angle and composite thickness are presented in Figs. 1-3. In the figures, points 1–12 correspond to the studied cases from Table 2.

In Figs. 1-3, it can be seen that the composite thickness influences thermal residual stresses more than the disorientation angle does. It can be presumed that this occurs due to the increased heating and cooling time that is necessary for a thicker plate in the same curing process. This leads to less of a temperature difference and correspondingly to a decrease in thermal stress. Furthermore, it was revealed that the disorientation angle becomes less influential with increasing composite structure thickness. However, the maximum influence of the disorientation angle on the layup $[0/45/-45/90/-45/45/0]$ caused the normal stresses to increase up to 106% (Fig. 2, cases 1–4), and the disorientation angle’s maximum influence on the layup $[0/90/0/90/0/90/0]$ caused the shear stresses to increase up to 577% (Fig. 3, points 1–4). The minimal plate thickness is 0.91 mm in both cases. In the other cases, the stresses do not experience much change with disorientation angle growth, so much so that the effects are nearly identical. Due to this, a detailed examination of these relationships is not included in the present study.

The analysis reveals that the maximal values of $\sigma_x$ (Fig. 1) correspond to the 0.91 mm-thick composite plates. In contrast, the minimal normal stress was obtained for a maximal composite thickness of 2.73 mm for each studied layup. In addition, it can be seen that for each layup, the stress levels for the thickest plates remained nearly unchanged at different disorientation angles; however, as the plates became thinner, the change to $\sigma_x$ became more pronounced (Figs. 1-3, cases 9–12 and 1–4). These results demonstrate that increasing the thickness of a

### Table 1. Disorientation angles and composite layups correspondence

<table>
<thead>
<tr>
<th>Disorientation angle, $\theta$ (degree)</th>
<th>Layers with disorientation angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[0/90/0/90/0/-\theta/0/\theta]$</td>
<td>$[0/45/-45/90/-45/45/0]$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
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<tr>
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<tr>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
</tr>
</tbody>
</table>

### Table 2. Studied cases

<table>
<thead>
<tr>
<th>Basic composite structures</th>
<th>$H$ (mm)</th>
<th>$Q$ (degree)</th>
<th>Case no.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$</td>
<td>0.91</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$45^\circ$</td>
<td>0.91</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$-45^\circ$</td>
<td>0.91</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>$90^\circ$</td>
<td>0.91</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>$-90^\circ$</td>
<td>0.91</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>$45^\circ$</td>
<td>1.82</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>$-45^\circ$</td>
<td>1.82</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>$90^\circ$</td>
<td>1.82</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>$-90^\circ$</td>
<td>1.82</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>$0^\circ$</td>
<td>2.73</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>$90^\circ$</td>
<td>2.73</td>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>$0^\circ$</td>
<td>2.73</td>
<td>7</td>
<td>12</td>
</tr>
</tbody>
</table>

The grey color indicates the layers with disorientation angle. $H$, plate thicknesses; $Q$, disorientation angle.
plate causes the normal stresses to decrease, and they also reveal that the disorientation angle becomes less influential with increasing composite thickness. The maximum growth of \( \sigma_z \) was about 600\% for [0/90/0/90/0/90/0] and 800\% for [0/45/-45/90/-45/45/0] between thicknesses of 0.91 mm to 2.73 mm (Fig. 1; curves A and B, cases 9–12 and 1–4). Furthermore, by comparing the basic structures of [0/90/0/90/0/90/0] and [0/45/-45/90/-45/45/0], it can be concluded that thickness is more important for decreasing stress for oblique composites (0/45) than for longitudinal-transverse composites (0/90).

In Fig. 2, the maximum growth of \( \sigma_z \) was about 860\% for [0/90/0/90/0/90/0] (curve B, cases 1–4 and 9–12) and 800\% for [0/45/-45/90/-45/45/0] (curve A, cases 1–4 and 9–12) between thicknesses of 0.91 mm to 2.73 mm. This is not a great difference, and the thickness has nearly the same significant influence for oblique composites (0/45) as for longitudinal-transverse composites (0/90).

Fig. 3 shows that the maximum growth of \( \tau_{xy} \) was about 650\% for [0/90/0/90/0/90/0] (curve B, cases 1–4 and 9–12) and 1000\% for [0/45/-45/90/-45/45/0] (curve A, cases 1–4 and 9–12) between thicknesses of 0.91 mm and 2.73 mm. For this case, thickness is highly important for decreasing stress for longitudinal-transverse composites (0/90).

In Figs. 1-3, it can be seen that the thermal stress values for \( \sigma_z \) and \( \tau_{xy} \) are much larger for the layup [0/45/-45/90/-45/45/0] than for the layup [0/90/0/90/0/90/0], but for \( \sigma_x \), the opposite is true.

An analysis was conducted to identify the effect of the disorientation angle on the thermal residual stresses in carbon squared composite plates of various thicknesses. Our results are summarized below:

The thickness is more influential on the thermal stresses than the disorientation angle.

Increasing thickness for each composite structure with disorientation angle causes decreasing residual thermal stresses.

The disorientation angle becomes less influential with increasing composite thickness for each composite structure.

The thickness with disorientation angle is more influential on \( \sigma_z \) for basic oblique composite structures, whereas it is more influential on \( \tau_{xy} \) for basic longitudinal-transverse composites, and its influence on \( \sigma_x \) is nearly the same for both composite structures.

In general, the obtained results are important for materials science because they further the understanding of composite stress-strain behavior and its dependence on the structure thickness and disorientation angle degree. From a practical viewpoint, these results are applicable to the engineering process of aerospace applications (e.g., airplane wing plating, nose cones of planes and helicopters, plating of radar stations, and space radio telescope reflectors) particularly to prevent early composite structure failure, improve working capacity, and extend working life.

**Conflict of Interest**

No potential conflict of interest relevant to this article was reported.

**Acknowledgements**

This study was supported by the National Research Foundation of Korea (award 2016R1A6A1A03013567), and the Korean Institute of Energy Technology Evaluation and Planning (award 2014030200590) and the Korean Ministry of Trade, Industry and Energy.

**References**


Nomenclature

\( A_{11}, B_{12}, D_{12} \) - stiffness matrices

\( K_{10} \) - nondimensional coefficient of the fiber tension level

\( K'_{1} \) - the curvature of the panel

\( M_{L}^{\theta} \) - thermal moments

\( M_{T}^{\theta} \) - tension moments

\( \theta^{(K)} \) - trigonometric function of the rotation angle of the coordinate axes of the K-th layer relative to the Cartesian coordinate system (x, y) if there is a contiguous side

\( w^{(K)} \) - trigonometric function of the rotation angle of the coordinate axes of the K-th layer relative to the Cartesian coordinate system (x, y) if there is an opposed side

\( N_{T}^{\theta} \) - thermal forces

\( N_{T}^{\phi} \) - tension forces

\( Q_{U}^{(}\theta) \) - layer stiffness for unidirectional fiber-reinforced plastic in elastic symmetry axes

\( Q_{U}^{(}\phi) \) - layer stiffness reduced to arbitrary x and y axes of the Cartesian coordinate system

\( U_{0}(x, y, z) \) - displacement of the reference plane, if \( z = 0 \)

\( V_{0}(x, y, z) \) - displacement of the reference plane, if \( z = 0 \)

\( W \) - hogging of the composite panel

\( a^{(K)}_{L} \) - linear expansion coefficient in the reinforcement direction

\( a^{(K)}_{T} \) - linear expansion coefficient perpendicular to the reinforcement direction

\( a^{(K)}_{\theta} \) - tangent expansion coefficient

\( \xi^{(K)}_{T} \) - deformation in the reference plane (tangent)

\( \xi^{(K)}_{\theta} \) - admissible deformation of the K-th composite layer

\( \xi^{(K)}_{\phi} \) - deformation in the reference plane

\( \sigma^{(K)}_{C} \) - admissible compression stresses

\( \sigma^{(K)}_{T} \) - admissible tension stresses

\( \sigma^{(K)}_{\phi} \) - admissible stresses (tangent)